Meet #1

Solutions – Arithmetic with "*" Operations   Oct 07

1. $6 * 4 = 3(6) - 2^4 = 2$. $2 * 2 = 3(2) - 2^2 = 2$. **Ans. 2**

2. $3C^2 - 4D = 3D^2 - 4C \Rightarrow 3C^2 - 3D^2 = 4D - 4C \Rightarrow 3(C^2 - D^2) = -4(C - D) \Rightarrow$
$3(C - D)(C + D) = -4(C - D)$. Thus $C + D = -4/3$. **Ans. -4/3**

3. $\frac{14}{10} \cdot \frac{3}{4} \cdot \frac{4}{5} A = 37.80 \Rightarrow \frac{42}{50} A = 37.80 \Rightarrow \frac{84}{100} A = 37.80 \Rightarrow 84A = 3780$ **Ans. $45**

Inequalities and Absolute Values

1. $\frac{x+2}{3} - \frac{2}{5}(3x-1) < \frac{4(2x+1)}{15} \Rightarrow 5(x + 2) - 6(3x - 1) = 4(2x + 1) \Rightarrow$
$5x + 10 - 18x + 6 < 8x + 4 \Rightarrow -13x + 16 < 8x + 4 \Rightarrow -21x < -12 \Rightarrow x > 4/7$. **Ans. 1**

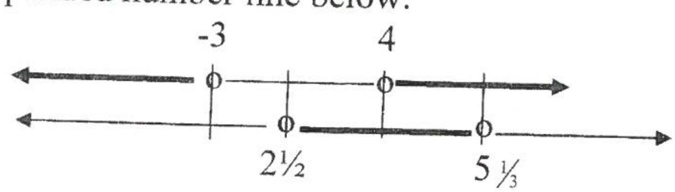
2. For critical points: (1) $3x - 5 = 5x + 8$ or (2) $3x - 5 = -5x - 8$. In (1) $-13 = 2x$ or $x = -6½$. In (2) $8x = -3$ or $x = -3/8$.

-------o---------o⟶
-6½ -3/8

Checking interval points into $|3x - 5| \le 5x + 8$:
$-7 \Rightarrow 26 \le -27$, not $-1 \Rightarrow 8 \le -3$, not $0 \Rightarrow 5 \le 8$, yes. **Ans. $x \ge -3/8$**

3. The solution for $x^2 - x - 12 > 0$: $(x - 4)(x + 3) > 0$. Critical points are at 4 and -3. Checking interval points: $-4 \Rightarrow (-)(-) > 0$, yes. $0 \Rightarrow (+)(-) > 0$, no. $5 \Rightarrow (+)(+) > 0$, yes.

The solution for $6x^2 - 47x + 80 < 0$: $(2x - 5)(3x - 16) < 0$. Critical points are at 2½ and $5\frac{1}{3}$. Checking interval points: $0 \Rightarrow (-)(-) < 0$, no. $3 \Rightarrow (+)(-) < 0$, yes. $6 \Rightarrow (+)(+) < 0$, no. Graphing on the expanded number line below:



Ans. $4 < x < 5\frac{1}{3}$

Matrices, Determinants, and systems of Equations

1. $2\begin{vmatrix} 7 & 9 \\ 5 & 7 \end{vmatrix} - \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 2 \\ -1 & 2 & -3 \end{vmatrix} = 2(49 - 45) - (-18 - 6 + 32 + 12 - 8 + 36) = 8 - 48 = -40$ **Ans. -40**

2. $\begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 3 \\ 5 & -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 2 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 6+10+9 & -4+15-6 \\ 15-2+18 & -10-3-12 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 25 & 5 \\ 31 & -25 \end{bmatrix} =$

Ans. $\begin{bmatrix} 28 & 10 \\ 38 & -23 \end{bmatrix}$

3. (1) $5x + 3y - 2z = -1$ $\quad 2(3) + (2) \Rightarrow 11x - 11z = 11$ or (4) $x - z = 1$
(2) $3x + 2y + 3z = 7$ $\quad 3(3) + (1) \Rightarrow$ (5) $17x - 23z = 5$
(3) $4x - y - 7z = 2$ $\quad -17(4) + (5) \Rightarrow -6z = -12$, thus $z = 2$. In (4): $x - 2 = 1$, $x = 3$.

In (1) $5(3) + 3y - 2(2) = -1 \Rightarrow 3y + 11 = -1 \Rightarrow 3y = -12$, $y = -4$. **Ans. (3, -4, 2)**

Number Theory

1. $41 + 43 + 47 + 53 + 59 + 61 = 304$ **Ans. 304**

2.
$$\begin{array}{r} 624_8 \\ \underline{56_8} \\ 4570 \\ \underline{3744} \\ 44,230 \end{array}$$

Ans. $44{,}230_8$

3. The first positive integer N is found: $N = 6a + 5$ and $N = 7b + 6$.
Thus $6a + 5 = 7b + 6 \Rightarrow 6a = 7b + 1$. Plugging in 5 for b will yield 6 for a. Thus $N = 41$. This is the first and each multiple of 42 added to 41 will give another. $500 \div 42 = 11^+$. $11(42) + 41 = 462 + 41 = 503$. This is the least integer. $1000 \div 42 = 23^+$. $42(23) - 1 = 966 - 1 = 965$. This is the greatest integer. **Ans. 503 and 965**

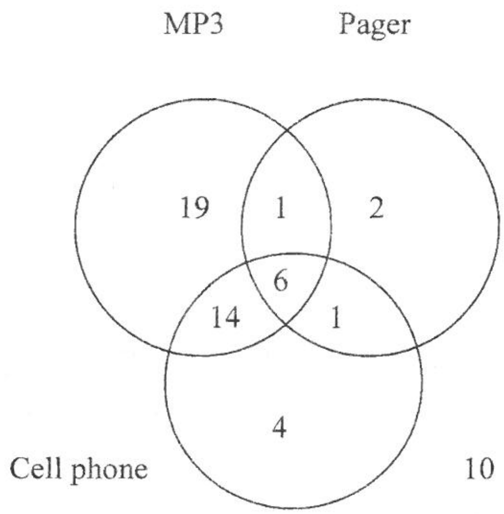
Geometric Similarities

1. The smaller is ¼ as large as the larger. The sum of the other sides of the larger triangle is 104. The sum of the three sides of the smaller is 26. **Ans. 26**

2. Side AD has to be split into a ratio of 36 to 39 or 12 to 13. AD = 13. **Ans. 13**

3. The volume ratio equals $\frac{1280}{14580} = \frac{128}{1458} = \frac{64}{729}$. Side ratio is $\frac{4}{9}$. Area ratio is $\frac{16}{81} = \frac{64}{x}$
Therefore $x = 4(81) = 324$. **Ans. 324**

9.



$19 + 1 + 2 + 6 + 14 + 1 + 4 + 10 = 57$

Ans. 57

Team

1. $(2+1)(3+1)(1+1) = 24$ **Ans. 24**

2. $\frac{5ft}{6ft8in} = \frac{60in}{80in} = \frac{3}{4}$. The weights in 3 dimension are in the ratio of $\frac{27}{64}$. $\frac{27}{64} = \frac{x}{192}$.
Thus x = 108. **Ans. 108 lbs.**

3. Students will find -3, 2, and -1 as critical points using calculators. So
$x^3 + 2x^2 - 5x - 6 \le 0$ will equal $(x+3)(x-2)(x+1) \le 0$. Plugging in interval points:

-4 ➔	–	–	–	≤ 0, yes
-2 ➔	+	–	–	≤ 0, no
0 ➔	+	–	+	≤ 0, yes
3 ➔	+	+	+	≤ 0, no

Ans. $x \le -3$ or $-1 \le x \le 2$

4. (1) $x^2 + y + z = 2$
(2) $x - y - z = 0$
(3) $3x - y = -2$

eq 1 + eq 2: $x^2 + x = 2$ ➔ $x^2 + x - 2 = 0$ ➔
$(x+2)(x-1) = 0$. $x = -2$ or $x = 1$.
Thus: $x = -2, y = -4, z = 2$ or $x = 1, y = 5, z = -4$

Ans. (-2, -4, 2) or (1, 5, -4)

5. Let $\frac{1}{2x+y} = a$, and $\frac{1}{x-2y} = b$. Then (1) $4a + 5b = -4\frac{1}{2}$ and (2) $8a - 3b = 4$.
-2(1) + (2): $-13b = 13$, thus $b = -1$. In (2): $8a - 3(-1) = 4$, thus $a = 1/8$. Therefore
(3) $2x + y = 8$ and (4) $x - 2y = -1$. 2(3) + (4): $5x = 15$ ➔ $x = 3$. In (3) $2(3) + y = 8$ ➔
$y = 2$. **Ans. (3,2)**

6. The first number beyond 13 that cannot be made is 15. From here it is trial and error incorporated with logic. The largest number that cannot be made is 30. **Ans. 30**

7. Eliminating every other leaves 50. Eliminating every 3rd leaves 34. Every 4th, 26. Every 5th, 21. Every 6th, 18. Every 7th, 16. Every 8th, 14. Every 9th, 13. Every 10th, 12. Every 11th, 11. **Ans. 11**

8. $2222_3 = 10000_3 - 1 = 80_{10}$. $80^2 = 6400 = 64(100) = (100_8)(144_8) =$ **Ans. $14{,}400_8$**