Meet #1

Solutions – Oct 2010

Arithmetic with "*" Operations

1. $x * (-4) = 2 \rightarrow |x| - 4 = \pm 2 \rightarrow x = \pm 6$ or ± 2. **Ans. 4**

2. $2 * 4 = \frac{2^4 - 4^2}{2-4} = \frac{16-16}{-2} = 0$. $2 * 3 = \frac{2^3 - 3^2}{2-3} = \frac{8-9}{-1} = 1$. $0 * 1 = \frac{0^1 - 1^0}{0-1} = \frac{0-1}{0-1} = 1$. **Ans. 1**

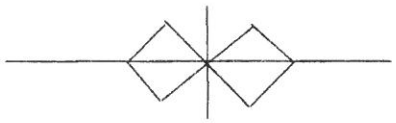
3. $994^2 + 989^2 = (1000 \cdot 988 + 6^2) + (1000 \cdot 966 + 11^2) = (1000 \cdot 1966 + (36 + 121) =$
$1000 \cdot 1966 + (36 + 121) = 1,966,157$. **Ans. 1, 966, 157**

Inequalities and Absolute Values

1. The only point inside the triangle formed by the boundaries is (-1, 0). **Ans. (-1, 0)**

2. The critical points are at (1) $4x - 7 = 5x - 6 \rightarrow x = -1$ and at (2) $4x - 7 = 6 - 5x \rightarrow$ $x = 13/9 = 1\frac{4}{9}$. Plugging in interval points into $|4x - 7| \le |5x - 6|$: -2 yields $1 \le 14 \rightarrow$ yes; 0 yields $7 \le 6 \rightarrow$ no; 2 yields $1 \le 4 \rightarrow$ yes. **Ans.** $x \le -1$ or $x \ge 13/9$

3. Rearrange to get $|y| = 1 - |1 - |x||$. The right side is non-negative only when $-2 \le x \le 2$.
The graph is a double-diamond with vertices at (-2, 0), (-1, ± 1), (0, 0), (1, ± 1) and (2, 0). **Ans. 7**



Matrices, Determinants and Systems of Equations

1. The product of the matrices is $\begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix}$. **Ans.** $\begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix}$

2. Let the width of each rectangle be a and length b. Then (1) $5a + 4b = 176$.
Also (2) $3a = 2b$. In (2): $b = 3/2\ a$. Subbing into (1): $5a + 4(3/2\ b) = 176 \rightarrow 11a = 176$.
Thus $a = 16$. In (2): $b = 3/2\ (16) = 24$. Perimeter is $2(16 + 24) = 80$. **Ans. 80**

3. $\begin{vmatrix} 2 & k & 1 \\ 3 & -3 & -k \\ -4 & 2 & k \end{vmatrix} = 9 \rightarrow -12 - 3k^2 + 4k - 6k + 4k^2 + 6 = -9 \rightarrow k^2 - 2k - 15 = 0 \rightarrow$
$(k - 5)(k + 3) = 0$ so $k = 5$ or -3. **Ans. 5 or -3**

Number Theory

1. Notice that when dividing by 33 the remainder is 9. $33 - 9 = 24$. **Ans. 24**

2. Only perfect squares have an odd number of factors and thus one "in the middle". The square root is the one in the middle. **Ans. 20**

3. z must have 5 as a factor so that 72z is divisible by 30. z must also have $3 \cdot 2^2$ as factors so that 30z is divisible by 72. So $z = 5 \cdot 2^2 \cdot 3 = 60$. **Ans. 60**

Geometric Similarities

1. The triangles are similar so $\frac{5}{7} = \frac{DE}{6} \rightarrow DE = \frac{30}{7} = 4\frac{2}{7}$ **Ans. 30/7 or $4\frac{2}{7}$**

2. Students should check out the triangles with three sides that can be reduced: 5, 8, 13, 21. 5 is a 13-14-19; 8 is a 20-21 29; 13 is a 20, 21, 30; 21 is a 20-21-29, which is the same as 8. $8 + 21 = 29$. **Ans. 29**

3. ΔADE is a right triangle and has a 8-15-17 ratio. Therefore $AE = 68$. $\Delta ACB \approx \Delta EFB$. Therefore $\frac{7}{68 + AB} = \frac{5}{AB} \rightarrow 7AB = 340 + 5AB \rightarrow 2AB = 340$, so $AB = 170$. **Ans. 170**

Team

1. Critical points are at 0 and 2. All numbers from 0 to 2 work. **Ans. $0 \leq x \leq 2$**

2. $2^4 - 1$ is not prime. $2^5 - 1$ is. $31(16) = 496$. **Ans. 496**

3. Factoring out the 1/9 means 9 is the determinant of A, then switching the resulting elements on the principle diagonal and then negating the elements on the other diagonal produces the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$. **Ans.** $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

4. Searching for critical points by taking the positive and negative values of $x + 3$ and $x - 3$, produces $x = 2\frac{1}{2}$ and $x = -2\frac{1}{2}$. Plugging numbers in each of the intervals into the inequality we find that $x < 2\frac{1}{2}$. **Ans. $x < 2\frac{1}{2}$**

5. Find the LCM of all 24 numbers:
LCM $\{2, 3, 2^2, 5, 2 \cdot 3, 7, 2^3, 3^2, 2 \cdot 5, 11, 2^2 \cdot 3, 13, 2 \cdot 7, 3 \cdot 5, 2^4, 17, 2 \cdot 3^2, 19, 2^2 \cdot 5, 3 \cdot 7, 2 \cdot 11, 23, 2^3 \cdot 3\} = \{2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 = 5354228880$. This is the only common multiple with 10 digits. The sum of the digits is 45. **Ans. 45**

6. The two digit's sum has to be even so this eliminates any two digit number starting with an even number. Those remaining are 11, 13, 17, 19, 31, 37, 53, 59, 71, 73, 79, and 97. The sum is 560. In groups of 4 there are many 10's. **Ans. 560**

7.

1st digit	middle digit	last digit			
Odd	odd	odd	$5 \cdot 4 \cdot 3 = 60$		
Odd	even	odd	$5 \cdot 5 \cdot 4 = 100$		
Even	odd	odd	$4 \cdot 5 \cdot 4 = 80$		
Even	even	odd	$4 \cdot 4 \cdot 5 = 80$	Total = 320	**Ans. 320**

8. $2^3 \cdot 3^3 \cdot 5^2 = 5400$. So the numbers for a, b, c have to be small with a and b since they are cubed should be the smaller primes. (1): If a =2, b =5, and c = 3, $2^3 \cdot 5^3 \cdot 3^2 = 9{,}000$. (2): If a and b are 2 and 3 and c = 7, $2^3 \cdot 3^3 \cdot 7^2 = 10{,}584$. If c = 11 ➔ 26,136. (3): If a = 3, b = 5 and c = 2, $3^3 \cdot 5^3 \cdot 2^2 = 13{,}500$. If a = 2, b = 7 and c = 3 ➔ 24,696. Any other factors would produce a larger product. Adding the three smallest:

Ans. 33,084

9. The ones that are not:
   - The unit 1 (1)
   - The primes (25)
   - The powers ≥ 2 of the primes ($2^2, 2^3, 2^4, 2^5, 2^6, 3^2, 3^3, 3^4, 5^2, 7^2$ (10)
   - The numbers that have 3 prime factors ($2 \cdot 3 \cdot 5, 2 \cdot 3 \cdot 7, 2 \cdot 3 \cdot 11, 2 \cdot 3 \cdot 13, 2 \cdot 5 \cdot 7, 2^2 \cdot 3 \cdot 5, 2 \cdot 3^2 \cdot 5, 2^2 \cdot 3 \cdot 7$) (8)

   $100 - 1 - 25 - 10 - 8 = 56$.

Ans. 56

Answer Sheet Oct 2010

Arithmetic with "*" Operations

1. 4
2. 1
3. 1,966,157

Inequalities and Absolute Values

1. (-1, 0)
   or
   $x = -1, y = 0$
2. $x \le -1$ or $x \ge 13/9$
   or
   $x \le -1$ or $x \ge 1\frac{4}{9}$
3. 7

Team

1. $0 \le x \le 2$
2. 496
3. $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$
4. $x < 2\frac{1}{2}$
5. 45
6. 560
7. 320
8. 33, 084
9. 56

Matrices, Determinants and Systems of Equations

1. $\begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix}$
2. 80
3. 5 or -3 or k = 5 or -3

Number Theory

1. 24
2. 20
3. 60

Geometric Similarities

1. 30/7 or $4\frac{2}{7}$ or $4.\overline{285714}$
2. 29
3. 170