6 Team (You may use Calculators) Oct 2011

3 pts 1. {2, 3, 4, 5, 6} is the replacement set for x.
Find the elements of the set for $x - 3 > 1$.

Ans. _____

3 pts 2. P and Q are prime numbers and $T > 6$. Find the smallest natural number T, such that $P + Q = T$ is impossible.

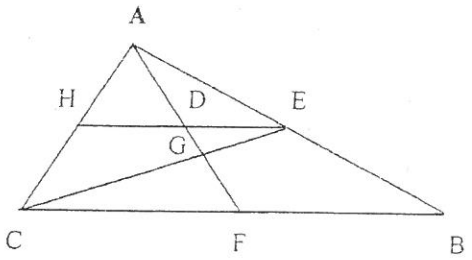
Ans. _____

3 pts 3. Lily has to count 80 quarters, 100 dimes, 80 nickels and 50 pennies. She starts with the quarters. How many quarters are still uncounted when Lily has counted half the value of the coins?

Ans. _____

4 pts 4. Given $\overline{AE} \cong \overline{EB}$,
GF = 8,
$\overline{CF} \cong \overline{FB}$
$\overline{DE} \parallel \overline{CB}$

Find DG.



Ans. _____

4 pts 5. Find (x, y), if $y = 2^{2x}$ and $y = 2^{x} + 12$.

Ans. _____

4 pts 6. Find the area of the triangle whose vertices are
(5, 7), (-4, 2) and (3, -6).

Ans. _____

5 pts 7. The volumes of two spheres are 4500π and 2304π. What is the ratio of the surface area of the smaller to that of the larger?

Ans. _____

5 pts 8. The sum of the digits of the integer that equals $(475)^2(291)^2(867)^2$ is between 65 and 80, inclusive. Find the sum.

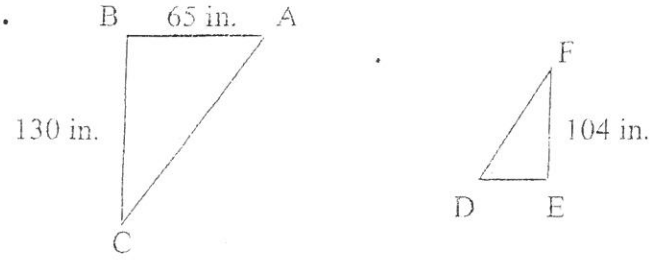
Ans. _____

5 pts 9. Consider the decimal (base 10) number $4^{64} - 1$. If this number is written as a base 4 number and the digits are all added together, what is their sum in base 4?

Ans. _____

5 Geometric Similarities (You may use Calculators) Oct 2011

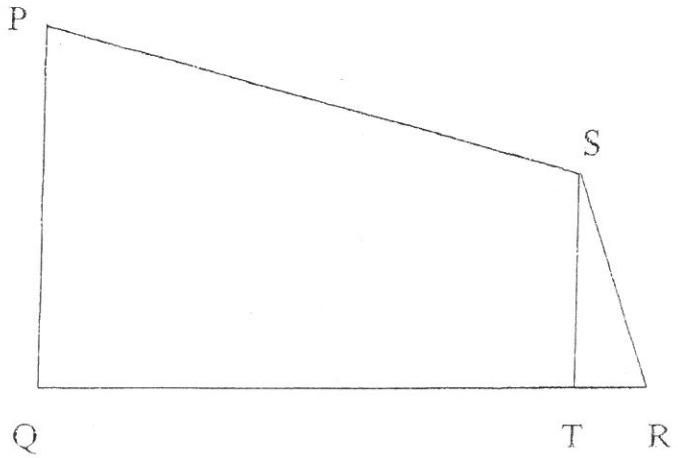
3 pts 1.



Given $\triangle ABC \approx \triangle DEF$ with side lengths as given, find the length of side DE. Express your answer in feet and inches.

Ans. _____

4 pts 2. In the figure $PQ \perp QR$, $ST \perp QR$, $\angle P \cong \angle R$, PQ = 15, QT = 24 and ST = 8. Find the perimeter of quadrilateral PQRS.



Ans. _____

5 pts 3. Triangle ABC is a right triangle with right angle at B. AB = 12 and BC = 9. A segment is drawn from B perpendicular to side AC at D and extends through D to a point E, such that $\overline{DE} \cong \overline{BD}$. A segment is then drawn from E perpendicular to $\overline{BC}$ (extended), meeting at point F. Find the length of $\overline{CF}$. Express answer in as a decimal.

Ans. _____

Meet #1

Solutions – Arithmetic with "*" Operations

1. $3 * 2 = 2(3) - 2(2) = 2$. $2 * 3 = 2(2) - 2(3) = -2$. $2 \nabla -2 = 3(2) + 3(-2) = 0$. **Ans. 0**

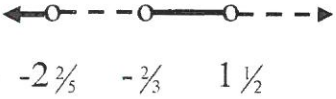
2. $336\left(\frac{7}{6}\right)\left(\frac{5}{4}\right)\left(\frac{3}{2}\right) = 7(105) = 735$. **Ans. 735**

3. $c * d = c^2 + d$. $d * c = d^2 + \mathbf{c}$. $c^2 + d = d^2 + \mathbf{c}$. Thus $c^2 - d^2 = c - d$ or $(c + d)(c - d) = c - d$. $c + d = 1$, so $d = 1 - c$. **Ans. 1 – c**

Inequalities and Absolute Values

1. $5(2x - 3) + 8 \geq 6(x - 5) + 7 \rightarrow 10x - 15 + 8 \geq 6x - 30 + 7 \rightarrow 10x - 7 \geq 6x - 23 \rightarrow$ $4x \geq -16$. $x \geq -4$. **Ans. $x \geq -4$**

2. $\frac{3}{2x-3} \leq \frac{2}{3x+2}$. Critical points are at $x = 1\frac{1}{2}$, $x = -2/3$ and



where $3(3x - 2) = 2(2x - 3) \rightarrow 9x + 6 = 4x - 6 \rightarrow 5x = -12$

$x = -2\frac{2}{5}$. Plugging in interval points: $-3 \rightarrow -\frac{3}{9} \leq -\frac{2}{7}$, yes. $-1 \rightarrow -\frac{3}{5} \leq -2$, no.

$0 \rightarrow -1 \leq 1$, yes. $2 \rightarrow 3 \leq \frac{2}{8}$, no. **Ans. $x \leq -2\frac{2}{5}$ or $-\frac{2}{3} < x < 1\frac{1}{2}$**

3. $\left|\frac{4}{x-2}\right| < x + 1$. Critical points are at $x = 2$, (1) $\frac{4}{x-2} = x + 1$ and (2) $\frac{4}{x-2} = -x - 1$.

In (1): $4 = x^2 - x - 2 \rightarrow 0 = x^2 - x - 6 \rightarrow 0 = (x - 3)(x + 2)$. So $x = -2$ and 3.

In (2): $4 = -x^2 + x + 2 \rightarrow x^2 - x + 2 = 0 \rightarrow (x - 2)(x + 1) = 0$. So $x = 2$ or -1.

Plugging in interval points for

Will only produce a usable solution when $x = 4 \rightarrow 2 < 3$. **Ans. $x > 3$**

Matrices, determinants, and Systems of Equations

1. $3x + 2y = 3 \rightarrow 9x + 6y = 9$ Thus $13x = 65$ and $x = 5$. $15 + 2y = 3$,
   $2x - 3y = 28 \rightarrow 4x - 6y = 56$ thus $y = -6$. **Ans. $x = 5, y = -6$**

2. $\begin{bmatrix} 3 & 5 & 2 \\ -1 & 6 & 7 \end{bmatrix}\begin{bmatrix} 9 & 4 & -8 \\ 2 & -5 & 6 \\ -3 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 27+10-6 & 12-25+14 & -24+30+8 \\ -9+12-21 & -4-30+49 & 8+36+28 \end{bmatrix}$ **Ans.** $\begin{bmatrix} 31 & 1 & 14 \\ -18 & 15 & 72 \end{bmatrix}$

3. $\begin{vmatrix} x & -x & 3 \\ 1 & x+1 & x \\ -x & 1 & 4 \end{vmatrix} = 57 \rightarrow 4x^2 + 4x + x^3 + 3 + 3x^2 + 3x - x^2 + 4x = 57.$

$x^3 + 6x^2 + 11x - 54 = 0$. By syn div:

2	1	6	11	-54
		2	16	54
	1	8	27	0

$x^2 + 8 + 27 = 0$ will not yield a real number. **Ans. 2**

Number Theory

1. Most students will get this by common sense. Others may factor the quadratic $x(x - 23) = 50$. The larger of the two is 25. **Ans. 25**

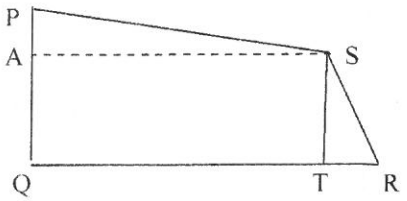
2. 180 = 1(180), 2(90), 3(60), 4(45), 5(36), 6(30), 9 (20), 10(18), 12(15). The numbers on the left get larger as the numbers on the right get smaller. Left sum = 52. Right sum = 494. Total = 546. **Ans. 546**

3. 23 has a remainder of 3 when divided by 5. 47 has a remainder of 2 when divided by 5. $23^{15} \cdot 47^{17} = 3^{15} \cdot 2^{17} = 6^{15} \cdot 2^2 = 1^{15} \cdot 4 = 4$. **Ans. 4**

Geometric Similarities

1. $\frac{65}{130} = \frac{DE}{104}$, DE = 52. 4 ft. 4 in. **Ans. 4 ft. 4 in.**

2. If you draw a segment through S parallel to QR intersecting PQ at A, it will form rectangle AQTS. AQ = 8, so AP = 7. Through the Pyth. Thm., PS = 25. Triangle APS is similar to triangle TRS.



Thus $\frac{SR}{ST} = \frac{PS}{AS}$ or $\frac{SR}{8} = \frac{25}{24}$ → SR = 25/3 = 8⅓. $\frac{TR}{ST} = \frac{AP}{AS}$ → $\frac{TR}{8} = \frac{7}{24}$, so TR = 7/3 or 2⅓.

The perimeter = 25 + 15 + 24 + 2⅓ + 8⅓ = 74⅔. **Ans. 74⅔**

3. $\frac{BD}{9} = \frac{12}{15}$, so BD = $\frac{36}{5}$. Thus BE = $\frac{72}{5}$. Since $\Delta ABC \approx \Delta BFE$

Then $\frac{BF}{BE} = \frac{AB}{AC}$ or $\frac{9 + CF}{\frac{72}{5}} = \frac{4}{5}$ → $9 + CF = \frac{72 \cdot 4}{25}$ → $CF = \frac{288 - 225}{25} = \frac{63}{25}$. **Ans. 2.52**

Team

1. Only the numbers 5 and 6 satisfy the inequality. **Ans. 5, 6**

2. T = 7: (5 + 2), T = 8: (5 + 3), T = 9: (7 + 2), T = 10: (3 + 7), T = 11: (not). **Ans. 11**

3. The sum of all the coins is 34.50. Half is 17.25 → 69 quarters. **Ans. 11**

4. HE is half as long as BC, because the segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long. $\Delta DGE \approx \Delta FGC$, the smaller being half the larger since DE = ½ CF. Thus DG = 4. **Ans. 4**

5. (1): $y = 2^{2x}$ and (2): $y = 2^x + 12$ In (1): $y = (2^x)^2$, then $\sqrt{y} = 2^x$.
In (2): $y = \sqrt{y} + 12 \rightarrow y - \sqrt{y} - 12 = 0 \rightarrow (\sqrt{y} + 3)(\sqrt{y} - 4) = 0$, so $\sqrt{y} = 4$. Thus y = 16.
In (1): $16 = 2^{2x} \rightarrow 16 = (2^x)^2 \rightarrow 4 = 2^x$. Thus x = 2. **Ans. (2, 16)**

6. Area through determinants is $\frac{1}{2}\begin{Vmatrix} 5 & 7 & 1 \\ -4 & 2 & 1 \\ 3 & -6 & 1 \end{Vmatrix} = ½ (10 + 24 + 21 - 6 + 30 + 28) = ½ (107)$

Ans. 53½

7. $\frac{2304\pi}{4500\pi} = \frac{576}{1125} = \frac{64}{125}$. Thus the radii are in a ratio of 4/5. So area is 16/25. **Ans. 16/25**

8. Since 3 is a factor of 291, 9 is a factor of 291^2, and therefore a factor of $(475)^2(291)^2(867)^2$. Any number that is a multiple of 9 has digits that sum to a multiple of 9. The only multiple of 9 between 65 and 80 is 72. Just for the record the product is 14,361,883,257,155,625. **Ans. 72**

9. 4^{64} in base 4 is a 1 followed by 64 zeroes. Therefore $4^{64} - 1$ in base four is 64 3's.
$64_{10} \cdot 3_{10} = 1000_4 \cdot 3_4 = 3000_4$. **Ans. 3000_4**

Answer Sheet – Oct 2011

Arithmetic with "*" Operations

1. 0
2. 735 or 735 bales
3. 1 – c

Inequalities and absolute values

1. $x \geq -4$
2. $x \leq -2\frac{2}{5}$ or $-\frac{2}{3} < x < 1\frac{1}{2}$

   or

   $x \leq -12/5$ or $-2/3 < x < 3/2$
3. $x > 3$

Matrices, Determinants and Absolute Values

1. (5, -6)
2. $\begin{bmatrix} 31 & 1 & 14 \\ -18 & 15 & 72 \end{bmatrix}$
3. 2 or $x = 2$

Number Theory

1. 25
2. 546
3. 4

Geometric Similarities

1. 4 ft. 4 in.
2. $74\frac{2}{3}$ or $74.\overline{6}$
3. 2.52

Team

1. 5, 6
2. 11
3. 11
4. 4 or 4 in.
5. (2, 16)
6. 53½ or 53.5
7. 16/25
8. 72
9. 3000_4