1 Arithmetic with "*" Operations Oct 2012 (No Calculators)

3 pts 1. If $a \& b = a^b - 3ba$, find $(2 \& 5) \& 3$.

Ans. ________________

4 pts 2. The digits 1, 2, 3, and 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are listed in order from smallest to largest, in what position is 3142?

Ans. ________________

5 pts 3. If $x * y = \dfrac{(x^2 - 2x + 1) - (y^2 - 4y + 4)}{x - y + 1}$, find $5 * (3 * 2)$.

Ans. ________________

2 Inequalities and Absolute Values Oct 2012 (No Calculators)

3 pts 1. Solve for x: $|x - 3| < 2$.

Ans. ________________________________

4 pts 2. Solve for *m*: $(m - 4)^3 > (1/8)^{-1}$

Ans. ________________________________

5 pts 3. Solve for x: $\dfrac{|2x - 5|}{x^2 - 4} > 0$.

Ans. ________________________________

3 Matrices, Determinants and Systems of Equations Oct 2012 (No Calculators)

3 pts 1. Given that $\begin{vmatrix} x & -7 \\ y & 9 \end{vmatrix} = -31$ and $\begin{vmatrix} 5 & x \\ -3 & y \end{vmatrix} = -29$, find (x,y).

Ans. ____________________.

4 pts 2. Given the equations $3x + y = 17$, $5y + z = 14$, and $3x + 5z = 41$, what is the value of the sum $x + y + z$?

Ans. ____________________.

5 pts 3. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ 0 & 4 \\ 3 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 22 \end{bmatrix}$, and $D = \begin{bmatrix} x \\ y \end{bmatrix}$,

Find D, if ABD = C.

Ans. ____________________

4 Number Theory Oct 2012 (No Calculators)

3 pts 1. Change the base ten number, 777, to a base seven number.

Ans. ____________________

4 pts 2. Find the number of digits in the number that results from the expansion of the following product $(5^{2012})(2^{2021})$.

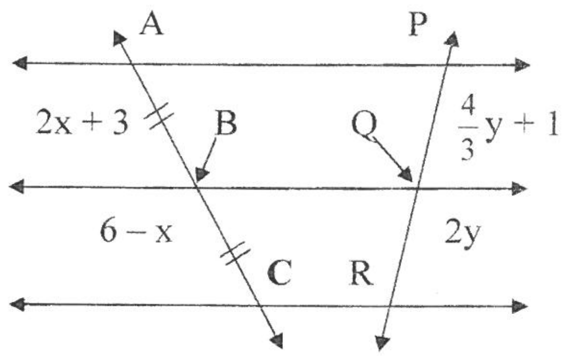
Ans. ____________________

5 pts 3. Four distinct, positive integers *a, b, c* and N exist, such that $N = 5a + 3b + 5c$. Also, $N = 4a + 5b + 4c$, and N is between 131 and 150. What is the numerical value of $a + b + c$?

Ans. ____________________

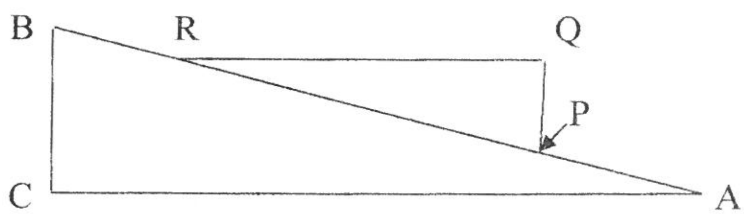
5 Geometric Similarities Oct 2012 (You may use calculators)

3 pts 1. Find x and y for the segments AB, BC, PQ, and QR in the following diagram. The horizontal lines are parallel.



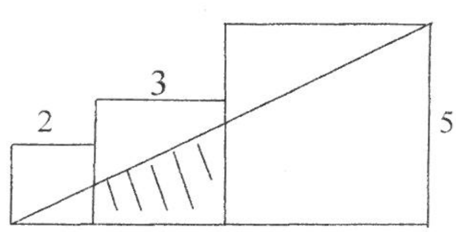
Ans. ____________________

4 pts 2. In the diagram, AB = 300, PQ = 20, and QR = 100. $\overline{QR}$ is parallel to $\overline{AC}$. $\overline{BC}$ is perpendicular to $\overline{AC}$, and $\overline{QP}$ is perpendicular to $\overline{QR}$. To the nearest hundredth find the length of $\overline{BC}$.



Ans. ____________________

5 pts 3. The three squares have the indicated lengths in the diagram. What is the area of the shaded quadrilateral?



Ans. ____________________

School ______________________________

Name ______________________________

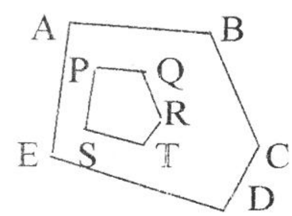
Number ______________________________

6 Team Oct 2012 (You may use Calculators)

3 pts 1. One bag contains twice as much mulch as a second bag. What fraction of mulch in the first bag should be moved to the second so that both bags have the same amount of mulch?

(1) **Ans.** ____________ **3 pts**

3 pts 2. The cardboard ABCDE at left has a piece of the same shape PQRST cut out of it. If AB = 42, PQ = 12, and the area of ABCDE before the cut out was 735, what is the area of the cardboard after the piece PQRST has been cut out?



(2) **Ans.** ________________ **3 pts**

3 pts 3. Given A(2, 12) and B(5, 0), find the coordinates of P such that P separates $\overline{AB}$ into two parts with lengths in a ratio of 2 to 1.

(3) **Ans.** ____________________ **3 pts**

4 pts 4. A girl goes up a ski lift at 4 mph and comes down the ski slope at 24 mph. If the ski slope is the same as the length of the ski lift, and we ignore any time spent at the top or bottom and she goes straight down the slope, what is her average speed for the round trip?

(4) **Ans.** ________________ **4 pts**

4 pts 5. Find all the possible numbers *N*, such that $0 < N < 1000$ such that when divided by 7 have a remainder of 6, when divided by 6 have a remainder of 5, and when divided by 5 have a remainder of 4.

(5) **Ans.** __________________________________ **4 pts**

4 pts 6. Let *N* be the natural number with 2012 digits, all of which are 8's. Find the remainder when *N* is divided by 9.

(6) **Ans.** _____________ **4 pts**

5 pts 7. Let a, b, c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Find the value of $a^2 - b^2 + c^2$.

(7) **Ans.** ______________________ **5 pts**

5 pts 8. If $2 + \sqrt{3}$ is a critical point for the inequality $x^4 - 10x^3 + 32x^2 - 34x + 7 < 0$, find all values of x which satisfy the inequality.

(8) **Ans.** ____ **5 pts**

5 pts 9. Find the greatest prime number that is a factor of either $(3^{16} - 2^{16})$ or $(3^{14} - 2^{14})$.

(9) **Ans.** ________________ **5 pts**

Solutions

Arithmetic with "*" Operations

1. 2 & 5 = 2^5 - (3)(5)(2) = 32 – 30 = 2. 2 & 3 = 2^3 - (3)(3)(2) = 8 - 18 = -10 **Ans. -10**

2. 1234, 1243, 1324, 1342, 1423, 1432 are the first 6 in order. Then the next 6 begin with 2. and the next two are 3124 and 3142, the 14th term. **Ans. 14th**

3. If you recognize the two perfect squares in the numerator, $(x-1)^2 - (y-2)^2$. These make $[(x-1)+(y-2)][(x-1)-(y-2)] = (x+y-3)(x-y+1)$. The denominator cancels to leave x + y – 3. So 3 $ 2 = 3 + 2 – 3 = 2. Then 5 $ 2 = 5 + 2 – 3 = 4. **Ans. 4**

Inequalities and Absolute Values

1. Critical points are at (1) x – 3 = 2 and (2) x – 3 = -2 In (1) x = 5 and in (2) x = 1. Plugging numbers in the respective intervals you will find: **Ans. 1 < x < 5**

2. Since $(1/8)^{-1} = 8$, then the only critical point is where m – 4 = 2. So m = 6. Plugging in 7 for m works, so m > 6. **Ans. m > 6**

3. Since $|2x-5|$ is always positive except at 2½. We need to find where the denominator is positive. Critical points for $x^2 - 4$ are at 2 and -2. x is positive when x > 2 and x < -2. **Ans. x < -2 or 2 < x < 2½ or x > 2½**

Matrices, Determinants, and Systems of Equations

1. (1) 9x + 7y = -31 and (2) 3x + 5y = -29. Mult. (2) by -3: -9x – 15y = 87. Adding this to (1): -8y = 56, so y = -7. In (2) 3x + 5(-7) = -29, so x = 2. **Ans. (2, -7)**

2. If you add all three equations you get 6x + 6y + 6z = 72. So x + y + z = 12. **Ans. 12**

3. AB = $\begin{bmatrix} -6 & 11 \\ 6 & 2 \end{bmatrix}$. $\begin{bmatrix} -6 & 11 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 22 \end{bmatrix}$. Thus -6x + 11y = 4 and 6x + 2y = 22. Adding these yields 13y = 26, the y = 2. Subbing back in x = 3. **Ans.** $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Number Theory

1. 777 ÷ 7 = 111 R 0; 111 ÷ 7 = 15 R 6; 15 ÷ 7 = 2 R 1; 2 ÷ 7 = 0 R 2. **Ans. 2160_7**

2. $(5^{2012})(2^{2021}) = (5^{2012})(2^{2012})(2^9) = 512 \times 10^{2012}$ in scientific notation. This is 3 digits followed by 2012 digits, which makes 2015 digits. **Ans. 2015**

3. Subtracting 4a + 5b + 4c from 5a + 3b + 5c yields a – 2b + c = 0. So 2b = a + c. Substituting this into the first equation: N = 5(a + c) + 3b = 13b, so N must be a multiple of 13. Since 143 is then only multiple of 13 between 131 and 150, N = 143 and b = 11. Since a + c = 2b, a + c = 11, so a + b + c = 11 + 22 + 33. **Ans. 33**

Geometric Similarities

1. 2x + 3 = 6 – x, so 3x = 3 and x = 1. 4/3 y + 1 = 2y, so 1 = 2/3 y. **Ans. x = 1, y = 3/2**

2. Since the triangles are similar $\frac{PQ}{QR} = \frac{BC}{AC} = \frac{20}{100} = \frac{1}{5}$. Let BC = x, then $x^2 + 25x^2 = 300^2$.

Thus x = $300/\sqrt{26}$ = 58.834. **Ans. 58.83**

3. The base of the 3 rectangles is 10. The largest triangle has legs in a ratio of 1:2. So the right hand side of the shaded trapezoid is 2½ and the left side is 1. The area of the trapezoid is ½ (3)(1 + 2½) = $\frac{3}{2}\cdot\frac{7}{2} = \frac{21}{4}$. **Ans. 5.25**

Team

1. Let 2x and x = the amount of mulch in the two bags and let F = the fraction of the mulch in the first bag that must be removed to achieve equality. Then:
(1 – F)(2x) = x + F(2x) ➔ 2x – 2Fx = x + 2Fx ➔ x = 4Fx ➔ F = ¼. **Ans. 1/4**

2. $\frac{PQ}{AB} = \frac{12}{42} = \frac{2}{7}$. So the ratio of the smaller pentagon to larger is $\frac{4}{49}$. $\frac{4}{49} = \frac{x}{735}$.
x = $\frac{4\cdot 735}{49}$ = 60, the area of the small pentagon. 735 – 60 = 675. **Ans. 675**

3. The x-coordinates differ by 3. The y-coordinates differ by 12. From one end of the segment the x value is 1 more or 3 and the y value is 4 less or 8. Thus (3, 8). From the other side (4, 4). **Ans. (3, 8) or (4, 4)**

4. Uf u = time going up and t = time down, then 4u = 24t or u = 6t. To find the average speed, you divide total distance by total time: (4u + 24t)/(u + t) = (24t + 24t)/(6t + t) = 48t/7t = 48/7. **Ans. 48/7**

5. Let N = the first of these numbers. Then N = 7a + 6, N = 6b + 5, and N = 5c + 4. Thus 7a + 6 = 6b + 5 or 7a + 1 = 6b. a = 5 in order for b to be whole. Thus N = 7(5) + 6 = 41. Now we use N = 42d + 41 and N = 5c + 4 to find N. Thus 42d + 41 = 5c + 4 or 42d + 37 = 5c. d = 4 to make c whole number. 42d + 41 = 42(4) + 41 = 209. 209 + multiples of 5(42) = 210 will give the others. **Ans. 209, 419, 629, 839**

6. We can do this by looking to see the pattern. If just 8, the remainder is 8. If 88, the remainder is 7. For 888, the remainder is 6, and for 8888, the remainder is 5. So for every set or 9 8's it repeats this sequence. $2012 \div 9$ leaves 5 8's left over which should be a remainder of 4. Another way to accomplish this is to find the product of the remainders of 8 and 2012, which is 8 and 5 = 40, which when divided by 9 gives a remainder 4. **Ans. 4**

7. Rewrite the two equations as follows: $a + 8c = 7b + 4$ and $8a - c = 7 - 4b$. Now square both sides of each: $a^2 + 16ac + 64c^2 = 49b^2 + 56b + 16$ is one and $64a^2 - 16ac + c^2 = 16b^2 - 56b + 49$ is the second. Adding these: $65a^2 + 65c^2 = 65b^2 + 65$. Thus $a^2 - b^2 + c^2 = 1$. **Ans. 1**

8. If $2 + \sqrt{3}$ is a critical point, then $2 - \sqrt{3}$ is also. Thus one of the quadratic equations for the inequality is $x^2 - 4x + 1$. Dividing this into the inequality gives the other quadratic:

$$\begin{array}{r} x^2 - 6x + 7 \\ x^2 - 4x + 1 \enclose{longdiv}{x^4 - 10x^3 + 32x^2 - 34x + 7} \\ \underline{x^4 - 4x^3 + x^2} \\ -6x^3 + 31x^2 - 34x \\ \underline{-6x^3 + 24x^2 - 36x} \\ 7x^2 - 28x + 7 \end{array}$$

$x^2 - 6x + 7 = 0$ for critical points you need the quad. Form ; $(6 \pm \sqrt{36-28})/2 = 3 \pm 2$ Using the number line and approximations for the roots .3, 1.6, 3.7 and 4.4. We'll plug whole numbers into the intervals: 0 ➔ 7 < 0 no. The intervals alternate.

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<-----o========o---------o=======o----->
 2 - √3   3 - √2   2 + √3   3 + √2
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Ans. $2 - \sqrt{3} < x < 3 - \sqrt{2}$ or $2 + \sqrt{3} < x < 3 + \sqrt{2}$

9. $3^{16} - 2^{16} = (3^8 + 2^8)(3^4 + 2^4)(3^2 + 2^2)(3^2 - 2^2) =$
$(6817)(97)(13)(5) = (401)(97)(17(13)(5)$
$3^{14} - 2^{14} = (3^7 + 2^7)(3^7 - 2^7) = (2315)(2059) = (5)(463)(29)(71)$ **Ans. 463**

Answer Sheet Oct 2012

Arithmetic with * Operations
1. -10
2. 14th
3. 4

Inequalities and Absolute Values
1. $1 < x < 5$
2. $m > 6$
3. $x < -2$ or $2 < x < 2\frac{1}{2}$ or $x > 2\frac{1}{2}$

Matrices, Determinants, and Systems of Equations
1. (2, -7)
2. 12
3. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Number Theory
1. 2160_7 or 2160 base seven
2. 2015
3. 33

Geometric Similarities
1. $x = 1$, $y = 3/2$
2. 58.83
3. 5.25

Team
1. 1/4
2. 675
3. (3, 8) or (4, 4)
4. 48/7 mph
5. 209, 419, 629, 839
6. 4
7. 1
8. $2 - \sqrt{3} < x < 3 - \sqrt{2}$ or $2 + \sqrt{3} < x < 3 + \sqrt{2}$
9. 463