1 Arithmetic with "*" Operations Oct 2013 (No Calculators)

3 pts 1. If $x * y = \frac{x+y}{y}$, determine the value of 2* (3 * 4). Express your answer as a mixed number.

Ans. ____________

4 pts 2. Find the value of the following in simplest form:

$$\left[\left(\frac{2}{5}\right)^{-1} + \left(\frac{3}{8}\right)^{-1}\right]^{-1}$$

Ans. ____________

5 pts 3. If $x * y = x^2 - y^2$ and $x \# y = y^2 - x^2$. Find (a * b) # (a * c) in factored form.

Ans. ____________________________

---

2 Inequalities and Absolute Values Oct 2013 (No Calculators)

3 pts 1. Find the solutions to the following:

$$|5 - x| = |2x|$$

Ans. ____________________

4 pts 2. x is an even integer. Find the largest such x for which:

$$2x^2 - 5 \leq -9x$$

Ans. ______________

5 pts 3. Find the values of x such that:

$$|x^2 - 1| - |x| > |x + 1|$$

Ans. ____________________________

3 Matrices, Determinants and Systems of Equations Oct 2013 (No Calculators)

3 pts 1. Find the ordered pair (x, y), such that $4x + 3y = 5$ and $7y - 3x = 24$.

Ans. ______________

4 pts 2. Perform as indicated $\begin{bmatrix} 4 & -1 & 3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & 5 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ -23 & 5 \end{bmatrix}$

Ans. ______________

5 pts 3. Find the value of the following:

$$\begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

Ans. ____________________

4 Number Theory Oct 2013 (No Calculators)

3 pts 1. Find the sum of the prime factors of 2184. Each distinct factor may be used on ly once in the sum.

Ans. ________________

4 pts 2. Find the value of the sum $x + y + z$, if

1. x, y, and z are relatively prime
2. x and y are not relatively prime
3. x and z are not relatively prime
4. y and z are not relatively prime
5. x, y and z are the smallest such natural number triplet.

Ans. ____________

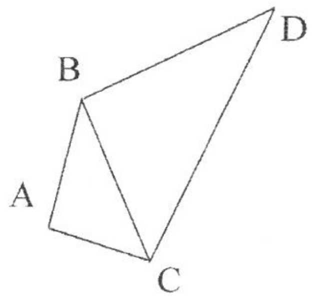
5 pts 3. Find the x for the following base equation: $212_x + 264_{2x} = 166_{3x}$.

Ans. ________________

5 Geometric Similarities Oct 2013 (You may use Calculators)

3 pts 1. ΔABC is similar to ΔBCD, AB = 5 cm, BC = 7 cm, AC = 6 cm. Find the cm length of line segment BD in decimal form.

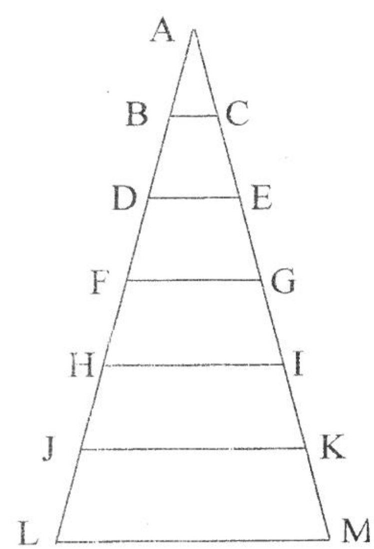
Ans. ______________________



4 pts 2. The area of ΔABC is 1008 cm^2. The area of ΔDEF is 3087 cm^2. Triangle ABC is similar to triangle DEF and AB = 28 cm, find the cm length of segment DE.

Ans. ______________________

5 pts 3. Triangle ALM with vertex angle A is isosceles. All the bases are parallel and are 5 units apart. The distance from A to base BC is also 5. Using the vertices identified, find the trapezoid whose area is 2/9 of the area of triangle ALM.



Ans. ______________________

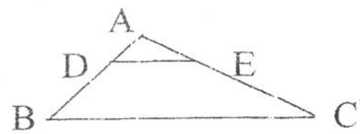
School ______________________________

Name ______________________________

Number ______________________________

6 Team Oct 2013 (No Calculators)

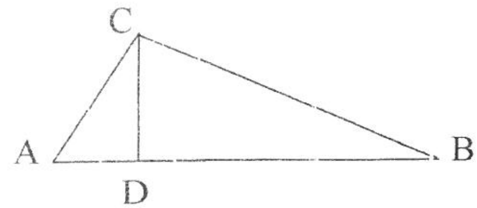
3 pts 1. $\overline{DE} \parallel \overline{BC}$, AD = 4, BD = 8 and DE = 12.
Find BC. (1) **Ans.** ________ **3pts**



3 pts 2. Find all values of x, such that: $\frac{1}{x} < \frac{1}{3}$.

(2) **Ans.** ______________________________ **3pts**

3 pts 3. $\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$, AD = 5 and BC = $10\sqrt{5}$.
Find BD.
(3) **Ans.** ________________ **4pts**



4 pts 4. Find the values of x, such that
$\frac{|x-1|}{|x+1|} - 1 \geq 0$. (4) **Ans.** ______________________________ **4pts**

4 pts 5. Define the operation ∇ as follows: a ∇ b = GCF (a, b) + LCM (a, b).
Find 6∇ (8 ∇ 2) – (6 ∇ 8) ∇ 2.
(5) **Ans.** ______________ **4pts**

4 pts 6. Find the area of pentagon ABCDE, whose vertices are A (-5, 2), B (-3, -3), C (2, -5), D (6, 2) and E (0, 5).
(6) **Ans.** ______________ **4 pts**

5 pts 7. Find the sum x + y + z + w for the following system:

$$\begin{aligned} x + y + z - w &= -2 \\ x - y - z + 2w &= 6 \\ 2x - 3y + z - w &= -1 \\ x - 5y + 2z + w &= 1 \end{aligned}$$

(7) **Ans.** ______________ **5pts**

5 pts 8. Find the sum of the positive integers less than 1000, which when divided by 7 have a remainder of 4, when divided by 6 have a remainder of 3 and when divided by 5 have a remainder of 2.
(8) **Ans.** ______________ **5pts**

5 pts 9. Solve the following system: $x^2 + y^2 = 208$
$xy = 96$

(9) **Ans.** ______________________________ **5pts**

Answer Sheet – Arithmetic with "*" Operations

1. $3 * 4 = \frac{3+4}{4} = \frac{7}{4}$. $2 * \frac{7}{4} = \frac{2+7/4}{7/4} = \frac{8+7}{7} = \frac{15}{7}$. **Ans. $2\frac{1}{7}$**

2. $\left(\frac{5}{2}+\frac{8}{3}\right)^{-1} = \left(\frac{31}{6}\right)^{-1} = \frac{6}{31}$. **Ans. 6/31**

3. $(a^2-b^2)\#(a^2-c^2) = (a^2-c^2)^2 - (a^2-b^2)^2 = [(a^2-c^2)-(a^2-b^2)][(a^2-c^2)+(a^2+b^2)] =$
$[a^2-c^2-a^2+b^2][a^2-c^2+a^2-b^2] = (b^2-c^2)(2a^2-b^2-c^2)$ **Ans. $(b-c)(b+c)(2a^2-b^2-c^2)$**

Inequalities and Absolute Values

1. (1) $5 - x = 2x$ or (2) $5 - x = -2x$. In (1): $5 = 3x$ or $x = 5/3$. In (2): $x = -5$. **Ans. 5/3, -5**

2. $2x^2 - 5 \le -9x$ ➔ $2x^2 + 9x - 5 \le 0$ ➔ $(2x-1)(x+5) \le 0$ ➔ $-5 \le x \le ½$. **Ans. 0**

3. CP: $x = 1$, $x = 0$, $x = -1$. For $x \ge 1$: $x^2 - 1 - x > x + 1$ ➔ $x^2 - 2x - 2 > 0$ ➔
CP: $x = 1 \pm \sqrt{3}$, $x > 1 + \sqrt{3}$. For $0 \le x < 1$: $-x^2 + 1 - x > x + 1$ ➔ $-x^2 - 2x > 0$ ➔
$x(x+2) < 0$ ➔ $-2 < x < 0$. No solution. For $-1 \le x < 0$: $-x^2 + 1 + x > x + 1$ ➔ $-x^2 > 0$,
$x^2 < 0$. No solution. For $x < -1$: $x^2 - 1 + x > -x - 1$ ➔ $x^2 + 2x > 0$ ➔ $x(x+2) > 0$,
$x < -2$. **Ans. $x > 1 + \sqrt{3}$ or $x < -2$**

Matrices, Determinants and Systems of Equations

1. (1) $3(4x + 3y = 5) = 12x + 9y = 15$, (2) $4(7y - 3x = 24) = 28y - 12x = 96$. Adding:
$9y + 28y = 15 + 96$ ➔ $37y = 111$, $y = 3$. In (1): $4x + 3(3) = 5$, $x = -1$. **Ans. (-1, 3)**

2. $$\begin{bmatrix}4 & -1 & 3\\ -5 & 2 & 1\end{bmatrix}\begin{bmatrix}-3 & 2\\ 2 & 5\\ 4 & -3\end{bmatrix}+\begin{bmatrix}5 & 7\\ -23 & 5\end{bmatrix}=\begin{bmatrix}-12-2+12 & 8-5-9\\ 15+4+4 & -10+10-3\end{bmatrix}=\begin{bmatrix}-2 & -6\\ 23 & -3\end{bmatrix}+\begin{bmatrix}5 & 7\\ -23 & 5\end{bmatrix}$$

Ans. $\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$

3. $$\begin{vmatrix}1 & 2 & 3 & 0\\ 2 & 1 & 0 & 0\\ 2 & 0 & 1 & 5\\ 1 & 1 & 1 & 1\end{vmatrix} = -2C_2 + C_1 = \begin{vmatrix}-3 & 2 & 3 & 0\\ 0 & 1 & 0 & 0\\ 2 & 0 & 1 & 5\\ -1 & 1 & 1 & 1\end{vmatrix} = \begin{vmatrix}-3 & 3 & 0\\ 2 & 1 & 5\\ -1 & 1 & 1\end{vmatrix} = C_2 + C_1 = \begin{vmatrix}0 & 3 & 0\\ 3 & 1 & 5\\ 0 & 1 & 1\end{vmatrix} = -3\begin{vmatrix}3 & 5\\ 0 & 1\end{vmatrix} =$$

$-3(3) = -9$. **Ans. -9**

Number Theory

1. $2184 = 21(104) = 3(7)(4)(26) = 3(7)(4)(2)(13)$. $2 + 3 + 7 + 13 = 25$. **Ans. 25**

2. Using the smallest prime factors: 2(3), 2(5), 3(5): $6 + 10 + 15 = 31$. **Ans. 31**

3. $2x^2 + x + 2 + 8x^2 + 12x + 4 = 9x^2 + 18x + 6$ ➔ $x^2 - 5x = 0$. So $x = 5$. **Ans. 5**

Geometric Similarities

1. $\frac{AB}{AC} = \frac{BC}{BD}$ ➔ $\frac{5}{6} = \frac{7}{BD}$ 5 BD = 42, BD = 42/5 = 8.4. **Ans. 8.4 cm**

2. $\frac{1008 \div 9}{3087 \div 9} = \frac{112 \div 7}{343 \div 7} = \frac{16}{49}$ = ratio of areas. $\frac{4}{7}$ = ratio of sides. $\frac{4}{7} = \frac{28}{DE}$, DE = 49. **Ans. 49**

3. Since the triangles are all similar, the bases have the same ratio as there corresponding altitudes. In triangles ABC and DEF the ratio is 1 to 2, so if base BC is x, base DE is 2x, and thusly base FG is 3x, HI is 4x, base JK is 5x and base LM is 6x. aΔALM = .5(30)(6x) = 90x. The area of the trapezoidal region has to be 20x. That area has to be .5(B + b)ht. If the height is 5, then $\frac{5}{2}$(B + b) has to equal 20x or two consecutive bases have to add to 8x, but none do. If the height is 10, then two non-consecutive bases that make one trapezoid have to add to 4x. x and 3x do, which are in Trapezoid BCFG. Using more than two non-consecutive bases makes area too high. **Ans. BCGF**

Team

1. $\frac{AD}{AB} = \frac{DE}{BC}$ ➔ $\frac{4}{12} = \frac{12}{x}$ ➔ x = 36. **Ans. 36**

2. CP: x = 0 and x = 3. -1 => -1 > 1/3, yes. 1 => 1 < 3, no. 4 => ¼ < 1/3, yes.
Ans. x < 0 or x > 3

3. $\frac{BD}{BC} = \frac{BC}{AB}$ ➔ $\frac{BD}{10\sqrt{5}} = \frac{10\sqrt{5}}{5 + BD}$ ➔ $BD^2 + 5BD - 500 = 0$ ➔ (BD–20)(BD+25) = 0. **Ans. 20**

4. $\frac{|x-1|}{|x+1|} - 1 \geq 0$ ➔ $\frac{|x-1|}{|x+1|} \geq 1$. CP: x = -1, and where x -1 = ± (x + 1), which is at x = 0.
-2 => $\frac{3}{1} - 1 \geq 0$, yes; -1/2 => $\frac{3/2}{1/2} - 1 \geq 0$, yes; 1 => $\frac{0}{2} - 1 \geq 0$, no. 0 works.
Ans. x < -1 or -1 < x ≤ 0

5. 6∇ (8 ∇ 2) => 6∇(2 + 8) => 6∇10 => 2 + 30 = 32.
(6 ∇ 8) ∇ 2 => (2 + 24) ∇2 => 26 ∇ 2 = 2 + 26 = 28. 32 – 28 = 4. **Ans. 4**

6. We triangulate the pentagon into ΔABE, ΔBCE, and ΔCDE and find the area of the sum: aΔABC = $\frac{1}{2}\left\|\begin{matrix} 0 & 5 & 1 \\ -5 & 2 & 1 \\ -3 & -3 & 1 \end{matrix}\right\| = \frac{1}{2}[0 + 15 - 15 + 6 + 0 + 25] = 15\frac{1}{2}$.

$a\triangle BCE = \frac{1}{2}\begin{Vmatrix} 0 & 5 & 1 \\ -3 & -3 & 1 \\ 2 & -5 & 1 \end{Vmatrix} = \frac{1}{2}[0 + 10 + 15 + 6 + 0 + 15] = 23$. $a\triangle CDE = \frac{1}{2}\begin{Vmatrix} 0 & 5 & 1 \\ 2 & -5 & 1 \\ 6 & 2 & 1 \end{Vmatrix} =$

$\frac{1}{2}[\,0 + 30 + 4 + 30 + 0 - 10\,] = 27$. $15\frac{1}{2} + 23 + 27 = 65\frac{1}{2}$. **Ans. $65\frac{1}{2}$**

7. (1) $x + y + z - w = -2$ (3) + (4): $3x - 8y + 3z = 0$ (5) (5) – (7): $x - 4y = 1$ (8)
(2) $x - y - z + 2w = 6$ 2(3) + (2): $5x - 7y + z = 4$ (6) -3(6) + (5): $-12x + 13y = -12$
(3) $2x - 3y + z - w = -1$ (1) + (4): $2x - 4y + 3z = -1$ (7) 12(8): $\underline{12x - 48y = 12}$
(4) $x - 5y + 2z + w = 1$ $y = 0$

Using (8): $x - 4(0) = 1 \Rightarrow x = 1$. Using (6): $5(1) - 7(0) + z = 4 \Rightarrow z = -1$
Using (4): $1 - 5(0) + 2(-1) + w = 1 \Rightarrow w = 2$. Thus $1 + 0 + -1 + 2 = 2$. **Ans. 2**

8. Let N be the smallest number to satisfy the conditions. Then N = 7a +4, N = 6b + 3 and N = 5c + 2. 7a + 4 = 6b + 3 ➔ 7a = 6b -1. Plugging in the numbers 0 through 0 for b, to find a multiple of 7 for 6b -1, b = 6 produces 35. Therefore N = 7(5) + 4 or 39. Now from these two equations we get N = 42d + 39, and thus 42d + 39 = 5c + 2 or 42d + 37 = 5c. When d = 4, 42(4) + 37 is a multiple of 5. Thus 42d + 39 = N = 42(4) + 39 = 207, the smallest positive integer that satisfies all the conditions. The other numbers less than 1000 are those which are additions of the LCM of 7,6 and 5 which is 210. The others are 417, 627 and 837. There sum is 2088. **Ans. 2088**

9. $x^2 + y^2 = 208$, $xy = 96$. $2xy = 192$. Thus $x^2 + 2xy + y^2 = 400$ or $(x + y)^2 = 400$ or $x + y = \pm 20$. We need a factor pair that adds up to 20 or – 20, so any combination of 8 with 12 or -8 with -12. **Ans. (8, 12), (12, 8), (-8, -12), (-12, -8)**

Answer Sheet – Oct 2013

Arithmetic with "*" Operations

1. $2\frac{1}{7}$
2. 6/31
3. $(b - c)(b + c)(2a^2 - b^2 - c^2)$

Inequalities and Absolute Values

1. 5/3 or -5 or $1\frac{2}{3}$ or -5
2. 0
3. $x < -2$ or $x > 1 + \sqrt{3}$

Matrices, Determinants and Systems of Equations

1. (-1, 3)
2. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$
3. -9

Number Theory

1. 25
2. 31
3. 5

Geometric Similarities

1. 8.4 cm or 8.4
2. 49 cm or 49
3. Trapezoid BCGF or BCGF

Team

1. 36
2. $x < 0$ or $x > 3$
3. 20
4. $x < -1$ or $-1 < x \leq 0$
5. 4
6. $65\frac{1}{2}$
7. 2
8. 2088
9. (8, 12), (12, 8), (-8, -12) or (-12, -8)