

1. Arithmetic with * Operations

December 1991

1. A family budgets \$420 every four weeks for rent and \$126 every 3 months for gas and electricity. How much is the family ahead or behind at the end of the year, if the family has a monthly rent of \$420 and an average monthly gas and electric bill of \$48.32? Specify in your answer ahead or behind.

Ans. _____

2. Find all value(s) of x such that: $x * 5 + 5 * x = 37$, if $a * b = a^2 - 4b$

Ans. _____

3. The fifth power of a certain natural number is 1,419,857. Find that number.

Ans. _____

1. Arithmetic with * Operations

December 1992

1. In a distant galaxy the Malamala tribe has this kind of math operation:
 $a * b, c = a + b - ac$. Evaluate $4 * (2 * 3, 5)$, $(3 * 2, 5)$.

Ans. _____

2. A store owner bought an item for \$40. He marked it to make a 30% profit. The item wouldn't sell, so he sale priced at a certain discount rate, wanting to make a profit of 10.5%, based on the original cost. Determine the discount rate as a percent.

Ans. _____

3. How many positive integers less than 50 have an odd number of positive integer divisors?

Ans. _____

1. Arithmetic with * Operations

October 1993

1. If $a * b$ is defined as “half the average of a and b ”, determine the positive difference between: $(3 * 5) * 6$ and $3 * (5 * 6)$. Express answer as a fraction in simplest form or as an exact decimal.

Ans. _____

2. x varies directly as the square of y and inversely as the square root of z . When x is 2 and y is a , then $z = b$. When x is b and y is a then $z = 1$. Find all integral values of a and b , such that the constant of proportionality is $4/9$ and a and b are relatively prime. Express answers in ordered pair form (a, b) .

Ans. _____

3. In the village of Marthos, the square root of M varies directly as the square of P . $M = a * b$, where $a * b$ is defined as $3a + 7b$. $P = c \# d$, where $c \# d$ is defined as $3c - 2d$. When a is 3 and b is 1 and c is 5, then $d = 6$. Find the value(s) of d when a is 20, b is 28 and c is 8.

Ans. _____

1. Arithmetic with * Operations

October 1994

1. Find the value of $\left(2\frac{1}{3} * \frac{7}{8}\right) - \left(\frac{5}{6} * 1\frac{2}{5}\right)$ if $x * y = (x + y)/(x - y)$. Express answer as a mixed number in simplest form.

Ans. _____

2. How many watches purchased at 3 for \$20 and sold at 4 for \$30 are required to make a profit of \$40?

Ans. _____

3. What two positive numbers less than 40 have the greatest difference between their LCM and their GCF?

Ans. _____

1. Arithmetic with * Operations

October 1995

1. If $A * B = \frac{A - B}{A + B}$ and $C \Delta D = \frac{2D - C}{C - D}$, find the value of $4 * (3 \Delta 2)$.

Ans. _____

2. In New Hampshire there is no sales tax. Mrs. Parks used a 20 dollar bill to buy some pens and notebooks for her high school children and received no change back. Notebooks were on sale at 3 for \$5, and pens at 5 for \$3. How many pens and notebooks did she buy?

Ans. _____

3. In 1994 the population of Hannaford, MO increased by 20%. In 1995 it increased 16% from 1994. The population is now 22,272. Was there a larger increase in people in 1994 or 1995, and by how much?

Ans. _____

1. Arithmetic with * Operations

October 1996

1. If $x \otimes y = x + 2y$ find all values of z such that $(z \otimes z) \otimes z = 35$.

Ans. _____

2. $a \wedge b = a^2 + b^2$ and $a * b = 2ab$. Find x if $\sqrt{(x \wedge 73) + (x * 73)} = 200$.

Ans. _____

3. 25% of all wangdoodles are wipdizzles, and all wipdizzles are wangdoodles. 30% of all wangdoodles are wokfinkles, and all wokfinkles are wangdoodles. 60% of all wangdoodles are neither wipdizzles nor wokfinkles. What percentage of all wipdizzles are also wokfinkles?

Ans. _____

1. Arithmetic with * Operations

October 1997

1. Express the answer to the following as a base 5 number: $201_5 - 44_5$

Ans. _____

2. A store is going out of business. Each Monday the price of each item is reduced by 20% from the previous Monday's price. The original prices are reduced on the first Monday, July 14. What is the date of the first Monday that the new prices represent a total reduction of more than $\frac{2}{3}$ of the original prices?

Ans. _____

3. The NAI School System has an "*" Game for students to help them understand math operations. In the "*" Game, students are to form a mathematical expression using only four 2's to make a number. For instance to produce the number 3, they could have done this: $(2 + 2 + 2)/2$ or $\sqrt{2} \cdot 2 + 2/2$ Produce an expression that makes 13 using the four 2's of the "*" Game.

Ans. _____

1. Arithmetic with * Operations

October 1998

1. If $a*b$ is defined to equal $a\sqrt{7+b}$, find $(2*2)*(3*2)$.

Ans. _____

2. Let $\langle x \rangle$ equal the sum of all the positive odd integers less than or equal to x , for positive, even integer x . Example: $\langle 8 \rangle = 1+3+5+7 = 16$. Find $\langle \langle 12 \rangle + \langle 16 \rangle \rangle$.

Ans. _____

3. Define the process of "averaging" a and b to be calculating $\frac{a+b}{2}$. Suppose 1 and 2 are averaged, then the result is averaged with 3, then that result is averaged with 4, and so on until the number 10 is included. What is the final "average"? Express your answer as an improper common fraction in the form a/b where a and b are relatively prime.

Ans. _____

1. Arithmetic with * Operations

October 1999

1. \textcircled{X} Means the smallest prime number greater than X. Find the value of

$$\textcircled{53 + 29}$$

Ans. _____

2. A car dealer gets cars from the distributor and marks the tag to make a 40% profit. One certain vehicle was sale tagged 20% discount because it would not sell. The vehicle was still not selling, so the dealer offered another 10% discount of this tagged price. The vehicle sold and the dealer made \$128 profit. What price does the car dealer pay the distributor for the car?

Ans. _____

3. $X*Y$ means to find the sum of all the odd factors of the sum of $X+Y$. $X\forall Y$ means to find the sum of all the even factors of the sum of $X+Y$. Find the value of $(29*31)\forall(43*11)$

Ans. _____

2. Inequalities and Absolute Value**October 1988**

1. Solve for x when $|-2x - 1| < 3$.

Ans. _____

2. When the Barthley family moved to a new city, they checked the local telephone rates. They found that there were two options, measured service and unmeasured service. The monthly rates were as follows: For measured service; a base rate of \$14.95 with no charge for the first 30 calls and 9 cents for each additional call. For unmeasured service unlimited calls for \$18.25. Find the least number of local calls for which unmeasured service is cheaper than measured service.

Ans. _____

3. Solve for x : $|x^2 - 2x - 4| > 4$.

Ans. _____

2. Inequalities and Absolute Value**October 1989**

1. Find all value(s) of x so that $|2x - 6| = |4 - 5x|$.

Ans. _____

2. Find all the integer solutions for $5 \leq \frac{6}{x} + \frac{14}{x+5}$.

Ans. _____

3. If $|x^2 - 4| < N$ for all x such that $|x - 2| < 0.01$, find the smallest value possible for N .

Ans. _____

2. Inequalities and Absolute Value

October 1991

1. Find all values of h such that $\frac{1}{4}(h-3) - \frac{3h-1}{6} < -1\frac{1}{3}$

Ans. _____

2. Find all values for x , such that $\frac{x-3}{x} \geq 0$

Ans. _____

3. Find all solutions for x , if $x^2 - 6x - 7 \leq 0$ and $|2x - 1| \geq x + 2$

Ans. _____

2. Inequalities and Absolute Value

October 1992

1. How many integral values of x make the following statement true?
 $|2x - 1| \geq |3x + 12|$

Ans. _____

2. Find all values of x which satisfy the inequality $\frac{x+2}{\sqrt{x^2-3x-4}} \geq 0$.

Ans. _____

3. Solve for x : $\frac{2x+3}{(x+1)(x-5)} < \frac{-1}{x+1}$

Ans. _____

2. Inequalities and Absolute Value

October 1993

1. Find all value(s) of x , such that $|x + 9| = 3x + 5$.

Ans. _____

2. Find the solutions for: $x^2(x-1)^2(x-3) < 0$.

Ans. _____

3. Find the solution set for x , if $|x^2 - 5x| < 6$.

Ans. _____

2. Inequalities and Absolute Value

October 1994

1. Solve: $|w^2 + 2w| = w + 2$.

Ans. _____

2. Solve: $\frac{x}{x-3} > \frac{4}{x+2} - 2$.

Ans. _____

3. On a number line the coordinate of point A is $4x-5$ and the coordinate of point B is $8-2x$. Find all positive values of x so that the distance between A and B is at least 6.

Ans. _____

2. Inequalities and Absolute Value

October 1995

1. Find the smallest integer which does not satisfy the inequality $-37x - 32 \geq -3x + 95$.

Ans. _____

2. $A, B, C, D,$ and E have coordinates 1, 2, 3, 4, and 5 on a number line, but not necessarily in this order. If $B < E, D > E, A > C,$ and $AC = 3,$ find the value of $AB + BD + DC$.

Ans. _____

3. Find the values of $x,$ such that $|x + 5| \geq 3x - 2$ and $\frac{2}{x + 3} < 1$

Ans. _____

2. Inequalities and Absolute Value

October 1996

1. Solve for $x:$ $3x + 10 > 10x + 3.$

Ans. _____

2. Find all integer values of x such that $|x - 3| < 2.$

Ans. _____

3. Find all values of x such that $(x + 3)(x^2 - 4) > -12.$

Ans. _____

2. Inequalities and Absolute Value

October 1997

1. Find the solution for $3 - 4x \geq 10$.

Ans. _____

2. Find the positive solutions for the following: $\frac{|x - 5/6|}{x} \leq \frac{x}{6}$.

Ans. _____

3. The solution to a 3rd degree inequality, $ax^3 + bx^2 + cx + d < 0$ is as follows: $x < 1$ or $1 < x < 2$. If $d = -6$, find the values of a , b and c .

Ans. _____

2. Inequalities and Absolute Value

October 1998

1. Solve: $2 - x < x - 2$.

Ans. _____

2. How many integers satisfy the inequality $|x + 3| > |5 - 2x|$?

Ans. _____

3. Find all the values of x which satisfy the equation $|1 - |1 - |1 - |1 - x||| = 0$.

Ans. _____

2. Inequalities and Absolute Value

October 1999

1. If $0 < x < 5$ and $4 < y < 5$, then express $y^2 + 4x$ as an inequality.

Ans. _____

2. Find the solutions for the following inequality: $x^4 - 4x^3 + 16x - 16 \geq 0$.

Ans. _____

3. Find all the values of x , such that $\frac{x+3}{x} - \frac{x+5}{x+2} \leq \frac{3x+1}{x^2+2x}$.

Ans. _____

3. Matrices, Determinants, and Systems of Equations

October 1988

1. Find the existing product of the following matrices $\begin{bmatrix} 2 & 4 & -1 \\ 3 & 6 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 6 & 1 \\ 1 & 7 \\ 0 & 9 \end{bmatrix}$.

Ans. _____

2. Find the value(s) of x in the following 3 x 3 determinant if $\begin{vmatrix} 2 & -1 & 5 \\ x & 3 & -4 \\ -2 & x & -3 \end{vmatrix} = 14$.

Ans. _____

3. If $A = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$ and $A^{-1} \cdot \begin{bmatrix} 5 \\ n \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ find the value(s) of n .

Ans. _____

3. Matrices, Determinants, and Systems of Equations

October 1989

1. Find the value(s) of x if $\begin{vmatrix} 2x & -1 \\ x & x \end{vmatrix} = 3$.

Ans. _____

2. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$, find $A^T \cdot A$.

Ans. _____

3. Find the value of $\begin{vmatrix} 4 & 4 & -3 & 2 \\ -3 & -2 & 2 & 3 \\ 2 & 5 & 4 & -2 \\ 3 & -3 & -2 & 0 \end{vmatrix}$.

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1991

1. If $2\begin{bmatrix} a & 3 & 7 \\ 3b & d & -6 \end{bmatrix} + 3\begin{bmatrix} 12 & c & 5 \\ 2b & 4 & -3 \end{bmatrix} = \begin{bmatrix} 11a & c & 29 \\ 36 & 3d & -21 \end{bmatrix}$ determine $a+b+c+d$.

Ans. _____

2. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 31$, find $\begin{vmatrix} a_1 + a_3 & 3b_1 + 3b_3 & c_1 + c_3 \\ a_2 & 3b_2 & c_2 \\ 2a_3 - a_2 & 6b_3 - 3b_2 & 2c_3 - c_2 \end{vmatrix}$

Ans. _____

3. Determine the area of the quadrilateral whose vertices are (1,3), (6,5), (9,1) and (3,-3).

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1992

1. Find all value(s) of x for which the given matrix has no inverse. $\begin{bmatrix} x+3 & 1 \\ 4x+5 & x-1 \end{bmatrix}$

Ans. _____

2. Given $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3$, find $\begin{vmatrix} a_1 + 4c_1 & a_3 + 4c_3 & a_2 + 4c_2 \\ b_1 - 3a_1 + c_1 & b_3 - 3a_3 + c_3 & b_2 - 3a_2 + c_2 \\ 5c_1 & 5c_3 & 5c_2 \end{vmatrix}$

Ans. _____

3. Find the value of the determinant: $\begin{vmatrix} 38 & 20 & 30 & 11 \\ 3 & 36 & -11 & -2 \\ 9 & 0 & 6 & 3 \\ 22 & 17 & 19 & 6 \end{vmatrix}$

Ans. _____

3. Matrices, Determinants, and Systems of Equations

October 1993

1. Solve for A , if $\begin{bmatrix} 4 & 0 & 3 \\ 2 & 1 & 1 \end{bmatrix} - 2A = A$.

Ans. _____

2. Find all value(s) of x , such that $\begin{vmatrix} 2 & x \\ 3 & 1 \end{vmatrix} \cdot \begin{vmatrix} 4 & x \\ 1 & 4 \end{vmatrix} = 232$.

Ans. _____

3. Evaluate $\begin{vmatrix} 3 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{vmatrix}$

Ans. _____

3. Matrices, Determinants, and Systems of Equations

December 1993

1. If 3 small bags and 4 large bags contain 84 apples, while 5 small bags and 3 large bags contain 85 apples, and each small bag as well as each large bag contains the same number of apples, how many are in 2 small bags and 5 large bags?

Ans. _____

2. An airplane flies at 560 mph in still air. With certain wind speed, it can go 260 miles further in $6\frac{1}{2}$ hrs. than it can go against the same wind in 7 hrs. Determine the speed of the wind and the distance covered when flying with the wind.

Ans. _____

3. If the graph of the quadratic function $f(x) = ax^2 + bx + c$ contains the points (1,2), (2,-3) and (-1,0), find the ordered triple (a,b,c) of the function.

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1994

1. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix}$ find a single matrix for $AB - BA$.

Ans. _____

2. Solve the following system of equations for x and y . Write your answer in ordered pair form.

$$\begin{cases} \frac{3}{x} + \frac{2}{y} = -4 \\ -\frac{4}{x} - \frac{3}{y} = 7 \end{cases}$$

Ans. _____

3. Find the determinant of A if: $\begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix} + A \begin{bmatrix} -2 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 31 \\ 0 & 1 \end{bmatrix}$.

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1995

1. Determine the product $\begin{bmatrix} 6 & 3 & 2 \\ 5 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 \\ 2 & 5 \\ 1 & 2 \end{bmatrix}$

Ans. _____

2. Farmer Smith sent 2(two) animals to two different slaughter houses which paid different prices per pound. The animals weighed 2400 and 2800 pounds. When he received his money of \$3308, he found that he was \$40 short. After some computations he realized that the animals got sent to the wrong places. What price was he paid per pound for his larger animal?

Ans. _____

3. Determine all real values of x such that $\begin{vmatrix} x & 5 & -1 \\ 2 & -2 & x \\ -1 & x+3 & -x \end{vmatrix} = -31$

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1996

1. Find x if $\begin{vmatrix} x & 3 & 2 \\ 4 & 1 & 1 \\ 5 & 2 & 0 \end{vmatrix} = -11$

Ans. _____

2. If $w + x = 17$, $x + y = 24$ and $y + z = 29$, find the value of $w + z$.

Ans. _____

3. Find the quotient A/B if $A = \begin{bmatrix} 4 & 2 \\ 8 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 \\ 3 & 6 \end{bmatrix}$ A/B means $A \times B^{-1}$

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1997

1. Express the solution for y in determinant form using Kramer's rule for the system $x + 2y = 5$ and $2x - y = 6$.

Ans. _____

2. Let $A = \begin{bmatrix} 4 & 3 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ Compute the product of the two matrices A and B that will produce a 3 by 3 matrix.

Ans. _____

3. Find the sum of the solutions for w , x , y and z based on the following system:

$$\begin{aligned} x + 2y + 3z - w &= 6 \\ x - y - z + w &= -2 \\ x + y + z &= 3 \\ 2x - y - w &= 1 \end{aligned}$$

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1998

1. If $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & x & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 8 \\ 3 \end{bmatrix}$ find x .

Ans. _____

2. Two apples and a banana cost 75 cents. Five bananas and one apple cost \$1.14. Angus purchases some apples and some bananas. He pays the clerk two dollars and receives four cents change. There is no sales tax. How many total pieces of fruit did Angus buy?

Ans. _____

3. Determine all real values for q for which $\begin{vmatrix} q & 1 & q \\ 1 & q & 1 \\ 1 & 1 & q \end{vmatrix} = 0$

Ans. _____

3. *Matrices, Determinants, and Systems of Equations*

October 1999

1. If $\begin{bmatrix} a & -a \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ -a & 3 \end{bmatrix} = \begin{bmatrix} 3a & 2a \\ 1 & 18 \end{bmatrix}$ find a .

Ans. _____

2. Mr. Smith went to visit his daughter. The trip took 7 hours. On the return trip he went by a scenic route which was 20 minutes longer. He drove 5 mph slower and it took him 8 hours. How far did Mr. Smith drive?

Ans. _____

3. Find all positive real values of x , such that $\begin{vmatrix} x+2 & 2 & -x \\ x & -1 & 2 \\ -1 & x+1 & -3 \end{vmatrix} = -44$

Ans. _____

4. Number Theory

October 1988

1. Find $d > 0$ such that d divides 18, d does not divide 12 and $36/d$ does not divide 10.

Ans. _____

2. Find the sum of the three smallest positive integers which have exactly five positive integral factors.

Ans. _____

3. There are an infinite number of integers that, when divided by all integers k where $3 \leq k \leq 11$, has a remainder of 2. Find the difference between the two smallest such integers.

Ans. _____

4. Number Theory

October 1989

1. How many multiples of 15 are there in the sequence 10, 20, 30, 40, ..., 560?

Ans. _____

2. Find all the two-digit prime numbers with the product of its digits equal to a prime number.

Ans. _____

3. Find the smallest positive integer that if it is divided by 7, 9, and 11 the remainders are 1, 2, and 3 respectively.

Ans. _____

4. Number Theory

October 1991

1. The sum of the first n prime numbers is a perfect square. Find the least value for n .

Ans. _____

2. Determine the remainder in base 7 when $650,103_7$ is divided by 325_7 .

Ans. _____

3. Find three different positive integers a , b , and c , such that the sum of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is an integer.

Ans. _____

4. Number Theory

October 1992

1. Event A happens every 18 minutes and event B happens every 15 minutes. If both events occur for the first time at 4:37 p.m., when is the fourth time that they will both happen?

Ans. _____

2. If $\text{GCF}(a, b) = 20$, and if $\text{GCF}(a^4, b^3) = k$, find the largest possible value of k .

Ans. _____

3. Find the smallest integer n such that $14n$ is a perfect square and $21n$ is a perfect cube. Leave your answer in simplest factor form with powers.

Ans. _____

4. Number Theory

October 1993

1. How many natural numbers less than 20 are relatively prime to 20?

Ans. _____

2. Find the maximum value of the digit k , such that $.68k_{10} < .321_5$.

Ans. _____

3. A printer used 1992 digits to number the pages of a book. How many pages has the volume?

Ans. _____

4. Number Theory

October 1994

1. Find the positive difference between the largest and the smallest 3-digit numbers each of whose digits is a perfect square.

Ans. _____

2. How many multiples of 6 are there between 1,111 and 4,444?

Ans. _____

3. Find the largest 2-digit number that is 2 more than a multiple of 3, 1 more than a multiple of 4, and 2 less than a multiple of 5.

Ans. _____

4. Number Theory

October 1995

1. Two positive integers differ by 6 and their squares differ by 72. By how much do their cubes differ?

Ans. _____

2. An orchard owner has 7 rows of trees with the same number of trees in each row. He averages 7.40 bushels of apples per tree and \$8.40 per bushel. If he received a profit of 40%, which amounted to \$2610.72, how many trees are there in each row?

Ans. _____

3. The sum of the factors of 3 natural numbers A , B , and C is 14, where A , B , and C are each greater than 1. Determine the smallest value of $A + B + C$.

Ans. _____

4. Number Theory

October 1996

1. Find the exact value in base ten of $33_{\text{six}} + 22_{\text{six}}$

Ans. _____

2. If Whosits have 96 legs and Whatsits have 28 legs, what is the smallest flock of Whosits and Whatsits Farmer Joe could have which has the same number of Whosit legs as Whatsit legs? Assume that his flock has at least one of each and give the total number of creatures in the flock.

Ans. _____

3. Find the smallest positive integer x , such that $x = 1 \pmod{6}$, $x = 1 \pmod{7}$, and $x = 4 \pmod{5}$.

Ans. _____

4. Number Theory

October 1997

1. Find the sum of the prime factors of 1147.

Ans. _____

2. Find the smallest three-digit prime number that meets the following criteria:
a) Each digit of the number is prime
b) All the digits are different

Ans. _____

3. How many whole number divisors does the number A have, if $A = (150)(60)(90)$.

Ans. _____

4. Number Theory

October 1998

1. Three primes p , q , and r satisfy $p + q = r$ and $1 < p < q$. Find the value of p .

Ans. _____

2. In this addition problem in base 17, t and y are the only unknowns. Find the value of $2t + 3y$ in base 17.

34B

7t5

35A

2yt

Gt0

Ans. _____

3. Find the value of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}}$

Ans. _____

4. Number Theory

October 1999

1. An abundant number is a number whose positive divisors, other than itself, have a sum greater than itself. Find the smallest abundant number.

Ans. _____

2. One is not prime, but whole numbers greater than one may be. Find the smallest prime number k such that the sum of the primes up to and including k is a perfect square.

Ans. _____

3. When the product of 91, 67, and 528 is divided by 15 the remainder is k . Find k .

Ans. _____

5. *Geometric Similarities*

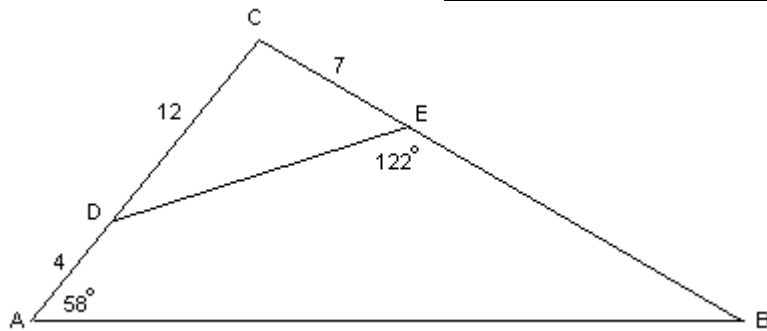
October 1991

1. The measures of the angles of a triangle are in the ratio of 5:7:12. The sides in ascending order are m , n and p . Find the mathematical equality that exists between m , n , and p .

Ans. _____

2. In the figure, $AD = 4$, $DC = 12$, $CE = 7$, $m\angle DEB = 122^\circ$, and $m\angle CAB = 58^\circ$. Find the length of BE .

Ans. _____



- 3.

Given:

$$TV \perp PQ$$

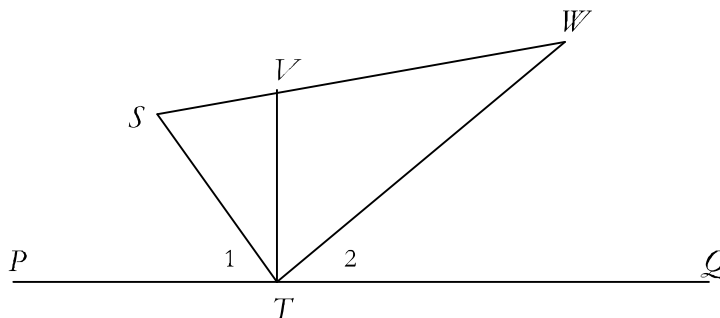
$$\angle 1 \cong \angle 2$$

$$ST = 9$$

$$WT = 15$$

$$SW = 16$$

Find: SV



Ans. _____

5. *Geometric Similarities*

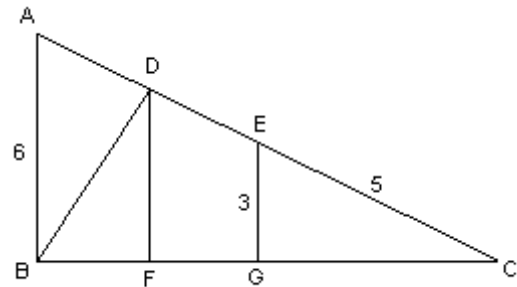
October 1993

1. The height of an equilateral triangle is 3 cm. The side measure of another equilateral triangle is 3 cm. Find the ratio of the areas of the larger to the smaller triangle. Give exact answer.

Ans. _____

2. *Given: $AB = 6$, $EG = 3$, and $EC = 5$. Find DE/FG .*

Ans. _____



3. Two similar triangles have areas of 36 and 108. One pair of corresponding sides differ by 10. Find the length of the smaller of these sides. Express answer correct to 4 significant digits.

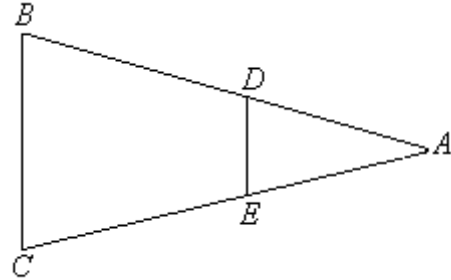
Ans. _____

5. *Geometric Similarities*

October 1994

1. If $\overline{DE} \parallel \overline{BC}$, $AD = 2$, $BD = 3$, $CE = 5\frac{1}{3}$, find AC .

Ans. _____



2. A person 5 feet 5 inches tall weighs 125 pounds. How much does a person 6 feet 6 inches tall weigh, having the same build?

Ans. _____

3. The leg and hypotenuse of a right triangle are 5 and 13. Find the length of the angle bisector to the longer leg.

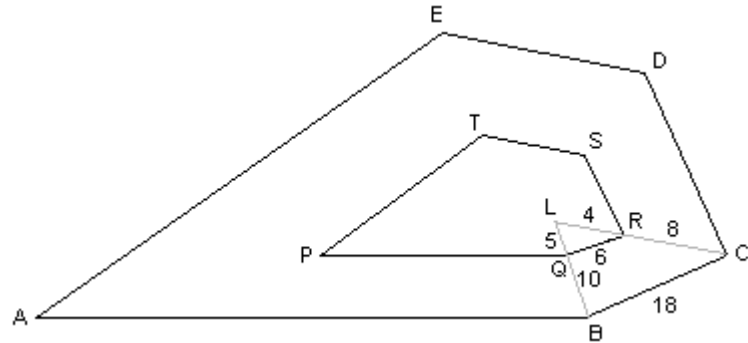
Ans. _____

5. *Geometric Similarities*

October 1995

1. If the area of the pentagon $PQRST$ is 84, what is the area of the region bounded by the two similar polygons?

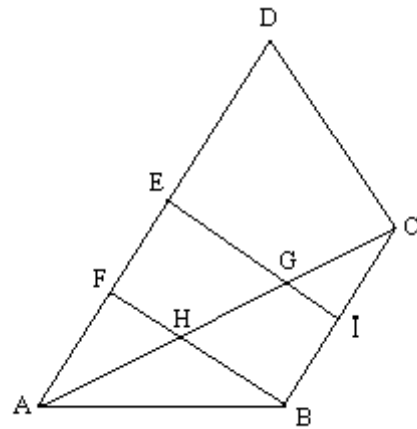
Ans. _____



2. Given:

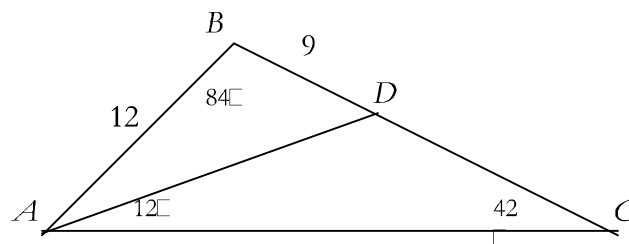
$AF = 20, GH = 4, CG = 6, BC = 12, BF \parallel EI \parallel CD$, and $AD \parallel BC$, find the length of \overline{AC}
Give exact answer or round to hundredths place.

Ans. _____



3. In the figure, the measure of angle B is 84° , of angle C is 42° and of angle DAC is 12° .
 BD is 9 and AB is 12. Find DC .

Ans. _____



5. Geometric Similarities

October 1996

1. If $\triangle ABC \sim \triangle DEF$, $AB = 7$, $BC = 10$, and $DE = 6$, find the length of \overline{EF} .

Ans. _____

2. Triangles CAT , DOG , PIG , and COW are all similar.

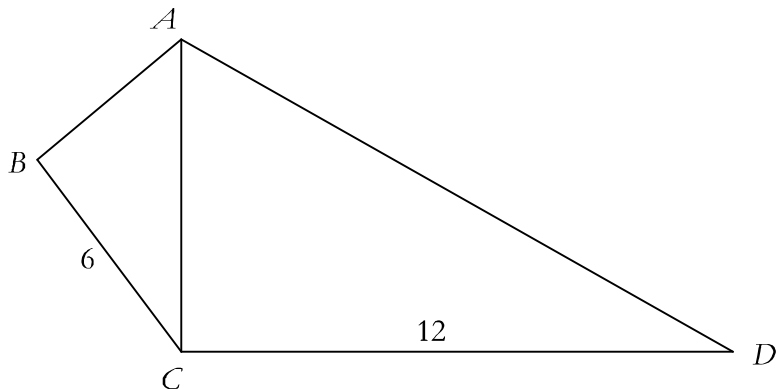
$$\frac{CA}{DO} = \frac{2}{3}, \frac{DO}{CO} = \frac{2}{7}, \text{ and } \frac{CA}{PI} = \frac{2}{5}$$

If the area of PIG is 100, what is the area of COW ?

Ans. _____

3. $\triangle ABC \sim \triangle ACD$. Find the exact perimeter of the quadrilateral $ABCD$.

Ans. _____

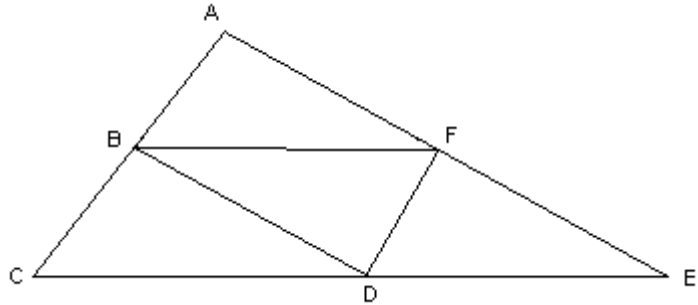


5. *Geometric Similarities*

October 1997

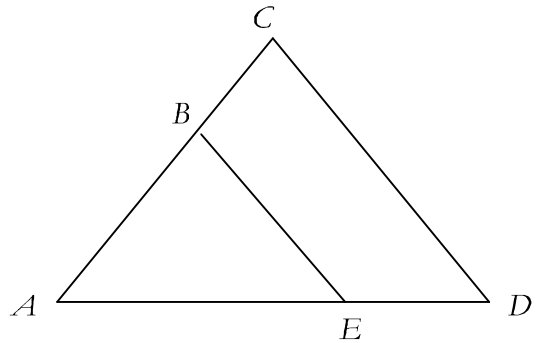
1. B , D and F are midpoints. If the area of $\triangle ACE$ is 96 and $CE = 16$, find the area of quadrilateral $ADFB$.

Ans. _____



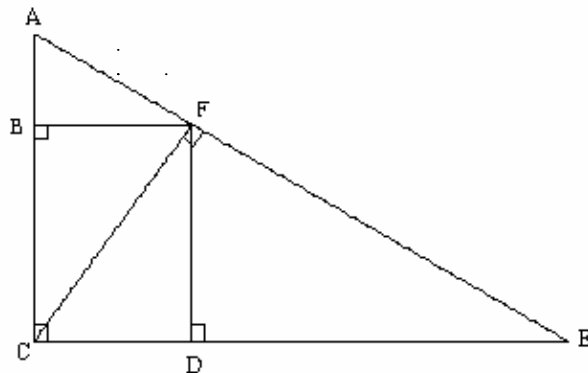
2. $BE \parallel CD$ and $1.5AB = AC$.
If the area of $\triangle ABE = 12$ square units,
Find the area of quadrilateral $BCDE$.

Ans. _____



3. In the figure with right angles as indicated, find the perimeter of rectangle $BCDF$, if $CF = 15$, and $DE = 16$

Ans. _____

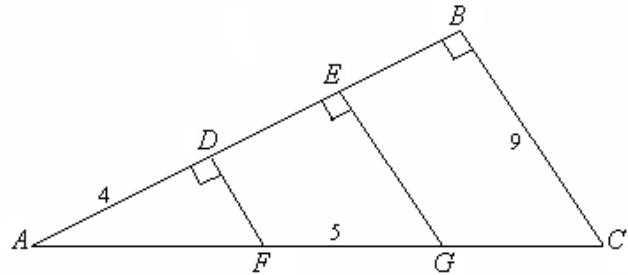


5. *Geometric Similarities*

October 1998

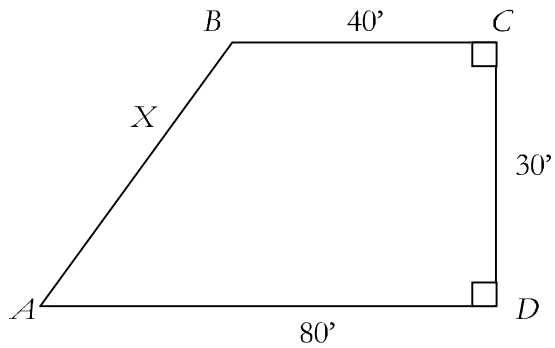
1. Points D and E trisect side \overline{AB} in $\triangle ABC$. Lengths are as marked. Find the length of segment \overline{EG} .

Ans. _____



2. Point X on \overline{AB} is 48 units from segment \overline{CD} . Find the length of \overline{AX} .

Ans. _____



3. There are three segments that can be drawn inside a 3-4-5 triangle parallel to one of the sides such that each will divide the area of the triangle in half. Find the exact length of the shortest of these segments. Express answer in exact and correct form.

Ans. _____

5. *Geometric Similarities*

October 1999

1. On a map two towns are $2\frac{1}{2}$ inches apart. They are actually 30 miles apart. How far apart are two landmarks, if on the map they are $3\frac{5}{16}$ inches apart. Round answer to nearest mile.

Ans. _____

2. One of two similar-shaped pentagonal pyramids has the largest base edge of 28 cm. The corresponding base edge of the other is 21 cm. The total surface area of the larger is 128 cm^2 What is the total surface area of the smaller?

Ans. _____

3. Given:
 $AB = BC$
 $AC = AD = BD$
find $m\angle ADB$

Ans. _____

