

1. Arithmetic with \* Operations

October 2000

7 a 2

1. If  $a$ ,  $b$ , and  $c$  are digits for which  $\frac{-48b}{c73}$  find  $a + b + c$ .

Ans. \_\_\_\_\_

2. If  $X * Y = \sqrt{XY}$ , find  $(3 * 48) * 75$ .

Ans. \_\_\_\_\_

3. Given  $a * b = a/b - b/a$  and  $a \# b = a/b + a$  find the value of  $t$  so that  $(t \# 6) \# (4 * 2) = 21$ .

Ans. \_\_\_\_\_

1. Arithmetic with \* Operations

October 2001

1. A farmer told his workers to go out and weed the 4<sup>th</sup> and 5<sup>th</sup> rows of beans. The worker returned and asked, "From which end?" The farmer responded, "It's the same from either end". How many rows of beans are there?

Ans. \_\_\_\_\_

2. Write the simplest form as a mixed number or improper fraction:

Ans. \_\_\_\_\_

$$1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + 5}}}$$

3. If  $a * b = ab - a^2$  and  $a \Delta b = \frac{a}{b - a}$ , then find  $(2137 * 2136) \div (2137 \Delta 2136)$

Ans. \_\_\_\_\_

**1. Arithmetic with \* Operations**

**October 2002**

1. If  $a * b = a - b$ , find  $[(x * y) * x] * (-y)$  in simplest form.

**Ans.** \_\_\_\_\_

2. Find the following product in base 7.  $\begin{array}{r} 345_7 \\ \underline{36_7} \end{array}$

**Ans.** \_\_\_\_\_

3. Let  $x \diamond = \frac{2x+1}{4}$ ,  $x \diamond \diamond = \frac{2\left(\frac{2x+1}{4}\right)+1}{4}$ , and so on. Find  $x \diamond \diamond \diamond \dots$  for all real  $x$ . Your answer will be a numerical constant.

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations**

**October 2003**

1. If  $a * b = (a + b)^2 - (b + a^2)$ , find the value of  $(1 * 2) * 3$ .

**Ans.** \_\_\_\_\_

2. If  $a \nabla b = a^2 + b$ , find all positive value(s) of  $x$  such that  $3(2 \nabla x) = x \nabla 2$ .

**Ans.** \_\_\_\_\_

3. A merchant marked an item to sell giving him 20% profit over cost. He then took 10% off the ticket price. He will now make \$16 profit. What was the original cost of the item to the merchant?

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations**

**October 2004**

1. If  $a * b = a^b - b^a$ , find  $5 * 2$ .

**Ans.** \_\_\_\_\_

2. If  $a \Delta b = ab - 1$  and  $x \nabla y = x^2 - y^2$ , find  $[4 \Delta (2 \nabla 3)] \nabla 1$

**Ans.** \_\_\_\_\_

3. In simplest form, find the value of this expression:

**Ans.** \_\_\_\_\_

$$\frac{\left(1 + \frac{17}{1}\right)\left(1 + \frac{17}{2}\right)\left(1 + \frac{17}{3}\right) \cdots \left(1 + \frac{17}{19}\right)}{\left(1 + \frac{19}{1}\right)\left(1 + \frac{19}{2}\right)\left(1 + \frac{19}{3}\right) \cdots \left(1 + \frac{19}{17}\right)}$$

**1. Arithmetic with \* Operations (No Calculators)**

**October 2005**

1. A CD sells for \$12.95. There is a 5% sales tax that will be added. The CD goes on sale for 20% off. In this certain state, however, the state requires that the sales tax be applied to the original price of the item. What is the total cost including tax to the nearest cent if it is sold at the 20% off sale?

**Ans.** \_\_\_\_\_

2. If  $a * b = b^2 - ab$  and  $m \Delta p = 2mp + m^2$ , find the value of  $(2 * 4) \Delta (4 * 2)$ .

**Ans.** \_\_\_\_\_

3. If  $a * b = \frac{a+b}{2}$ , and  $a \# b = \frac{a - \frac{1}{2}b}{2}$ , for what values of  $a$  and  $b$  will  $(a * b) \# b = 4$ ?

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations (No Calculators)**

**October 2006**

1. If  $x \circ y = x + y + 1$ , find  $[(1 \circ 2) \circ 3] \circ [(4 \circ 5) \circ 6]$ .

**Ans.** \_\_\_\_\_

2. For positive real numbers  $a$  and  $b$ , define  $a \Delta b$  to equal  $ab + \frac{a}{b}$ . If  $a \Delta b = b \Delta a$ , solve for  $a$  in terms of  $b$ .

**Ans.** \_\_\_\_\_

3. Let  $x^* = 1 - 2x$ . For how many integer values of  $x$  will  $(x^*)^*$  be a negative integer greater than or equal to  $-100$ ?

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations**

**October 2007 (No Calculators)**

1. If  $A * B = 3A - 2^B$ , find the value of  $(6 * 4) * 2$ .

**Ans.** \_\_\_\_\_

2. If  $C * D = 3C^2 - 4D$ , find the sum of  $C$  and  $D$ , if  $C * D = D * C$ , where  $D$  and  $C$  are not equal.

**Ans.** \_\_\_\_\_

3. After a discount of 20% of the original price  $A$  of an item and then a 25% discount of that discounted price, a merchant raised the last discounted price by 40% because the item was selling so well. The item then sold for \$37.80. Find  $A$ .

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations**

**September 2008 (No Calculators)**

1. If  $a * b = a^2 - ab + b^2$ , find  $4 * (3 * 2)$ .

**Ans.** \_\_\_\_\_

2. A five-digit number exists such that putting a 1 after it to make it a six-digit number, will make it three times as large as if the 1 were placed in front to make a six-digit number. What is the number?

**Ans.** \_\_\_\_\_

3. Given  $1 * 3 = 5$ ,  $6 * 9 = 21$ ,  $8 * 2 = 18$  and  $3 * 1 = 7$ , find the value of  $11 * 20$ .

**Ans.** \_\_\_\_\_

**1. Arithmetic with \* Operations**

**October 2009 (No Calculators)**

1. Simplify:  $\frac{1386}{3003}$

**Ans.** \_\_\_\_\_

2. Let  $x * y = xy - 2y$ . If  $2a * n = n * 2a$ , find  $a - n$  in terms of  $n$ .

**Ans.** \_\_\_\_\_

3. Let  $x * y = x^2 - 2y$ . Find  $x + y$ , if  $x * y$  is a two-digit perfect square and  $y = x + 5$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2000**

1. Find the sum of the least value and the greatest value of  $y$  that satisfies the following inequality:  $|5 + y| \leq 100$ .

**Ans.** \_\_\_\_\_

2. For what value(s) of  $x$  is  $|3x - 7| > 5$  and  $|7 - x| \leq 5$ .

**Ans.** \_\_\_\_\_

3. Given that  $f(x) = -|x - 4|$  and  $g(x) = 1/2 x^2 - 8$ , for which integral values of  $x$  is  $g(x)$  less than  $f(x)$ ?

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2001**

1. Find all values of  $x$  such that:  $|2x + 3| \leq 4x - 1$ .

**Ans.** \_\_\_\_\_

2. Find the solution for:  $x^2 - 3x < 2x$ .

**Ans.** \_\_\_\_\_

3. Find all possible value(s) of  $x$  such that:  $||x - 1| - |x + 1|| + |x| = 4$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2002**

1. Solve:  $|4 - x| > |2 - x|$ .

**Ans.** \_\_\_\_\_

2. Find the least possible integer value of  $x$  if:  $|x - 5| < 3$  and  $|x - 3| < 5$ .

**Ans.** \_\_\_\_\_

3. Let  $N$  and  $D$  be relatively prime positive integers. Suppose the fraction  $\frac{N}{D} < \frac{7}{10}$ . Suppose also that  $7 + D < 10 + N$ . Find the number of ordered pairs,  $(N, D)$ , that meet these requirements.

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2003**

1. Find the largest whole number value of  $x$  such that  $|3x - 7| < 25$ .

**Ans.** \_\_\_\_\_

2. Determine all values of  $x$  so that  $|3x - 2| \geq 2x + 1$ .

**Ans.** \_\_\_\_\_

3. Find all values of  $x$  such that  $\frac{2}{x-1} - \frac{1}{x-2} > \frac{1}{x^2 - 3x + 2}$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2004**

1. Solve over the set of real numbers:  $|6 - (y - 3)| \leq 9$ .

**Ans.** \_\_\_\_\_

2. Solve for  $m$ , if  $m^2 > 2m$ .

**Ans.** \_\_\_\_\_

3. Solve over the set of reals:  $|x - 2| \leq 3x$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2005 (No Calculators)**

1. Solve for  $x$ , if  $|2x - 1| = |3x + 2|$ .

**Ans.** \_\_\_\_\_

2. Find all values of  $x$ , such that  $x^3 - 2x < x^2$ .

**Ans.** \_\_\_\_\_

3. For what values of  $x$  is  $x|x - 1| < x$ ?

**Ans.** \_\_\_\_\_



**2. Inequalities and Absolute Value**

**October 2006 (No Calculators)**

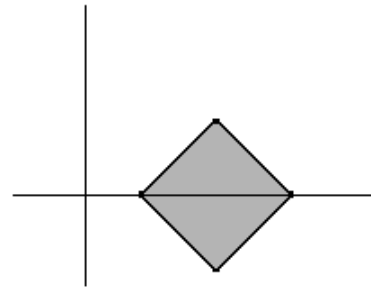
1. For what positive value of  $k$  will  $x = -5$  be a solution of  $|7 - kx| = |3x - 2|$ ?

**Ans.** \_\_\_\_\_

2. Find the average of all values of  $x$  satisfying:  $|2 - |2 - |2 - x|| = 0$ .

**Ans.** \_\_\_\_\_

3. The shaded region shown includes the boundary, which is a square with endpoints  $(1, 0)$ ,  $(2, -1)$ ,  $(3, 0)$  and  $(2, 1)$ . Using a single statement that includes just one symbol selected from the list  $\{\geq, >, \leq, <\}$ , give an equality in  $x$  and  $y$  that defines this region.



**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2007 (No Calculators)**

1. Find the smallest integer  $x$ , for which  $\frac{x+2}{3} - \frac{2}{5}(3x-1) < \frac{4(2x+1)}{15}$ .

**Ans.** \_\_\_\_\_

2. Find all values of  $x$  such that  $|3x - 5| \leq 5x + 8$ .

**Ans.** \_\_\_\_\_

3. Find all values of  $x$ , such that  $x^2 - x - 12 > 0$  and  $6x^2 - 47x + 80 < 0$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**September 2008 (No Calculators)**

1. Solve for  $x$ :  $x^2 - 6x - 7 > 0$ .

**Ans.** \_\_\_\_\_

2. Find all  $x$ , such that  $2 < |3x - 1| < 8$ .

**Ans.** \_\_\_\_\_

3. Find all values of  $x$ , such that:  $|x| + |x - 2| > 5$ .

**Ans.** \_\_\_\_\_

**2. Inequalities and Absolute Value**

**October 2009 (No Calculators)**

1. Find only integers such that  $3x < x + 6 < 3x + 5$ .

**Ans.** \_\_\_\_\_

2. Determine the solution set for  $|x^2 - 2| = x$ .

**Ans.** \_\_\_\_\_

3. Find the values of  $x$  such that  $|x^2 - 5x + 6| < |x^2 + x - 6|$ .

**Ans.** \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations*

October 2000

1. Simplify the following expression  $\begin{vmatrix} 3/7 & 5/7 \\ 4/5 & 2/3 \end{vmatrix} - \begin{vmatrix} 3/5 & 4/5 \\ 5/7 & 2/3 \end{vmatrix}$

Ans. \_\_\_\_\_

2. Find the sum of  $a + b + c + d$ , if  $\begin{bmatrix} 3 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 3 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Ans. \_\_\_\_\_

3. An escalator moves up at a rate of so many steps per second. A commuter walks up the escalator at a rate of one step per second and reaches the top in twenty (20) seconds. The next day the commuter's rate was two steps per second, and he reaches the top in sixteen (16) seconds. How many steps does the escalator have?

Ans. \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations*

October 2001

1. Find the sum of  $x$  and  $y$ , if  $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ x & y \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -2 & -1 \end{bmatrix}$

Ans. \_\_\_\_\_

2. Find the ordered triple  $(x,y,z)$  such that, 
$$\begin{aligned} 2x + y + z &= 8 \\ x - y - z &= 1 \\ x - 2y - 3z &= -1 \end{aligned}$$

Ans. \_\_\_\_\_

3. If  $A = \begin{bmatrix} -1 & 2 \\ 1 & x \end{bmatrix}$ , find all value(s) of  $x$  such that the determinant of  $A^2 + 2A = -3$

Ans. \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations*

October 2002

1. Find  $a + b + c + d$  if:
- $$\begin{aligned} a - b &= 5 \\ b - c &= -7 \\ c - d &= 9 \\ d + a &= -3 \end{aligned}$$

Ans. \_\_\_\_\_

2. Find  $a$  if:
- $$\begin{aligned} 2a + 2b - c &= 1 \\ -3a + 5b + 5c &= -30 \\ 7a - 7b + 6c &= 69 \end{aligned}$$

Ans. \_\_\_\_\_

3. Find the sum of all 4 elements of the 2 by 2 matrix  $B$  if:  $\begin{bmatrix} 2 & -1 \\ -3 & -1 \end{bmatrix} \cdot B = \begin{bmatrix} -5 & 7 \\ 5 & -8 \end{bmatrix}$

Ans. \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations*

October 2003

1. If  $A = \begin{bmatrix} 7 & 2 \\ -3 & 1 \\ 5 & 8 \end{bmatrix}$ ,  $B = \begin{bmatrix} 9 & 3 \\ 4 & -2 \\ 4 & -3 \end{bmatrix}$  and  $3A + 2B = C$ , find the sum of the elements of the matrix  $C$ .

Ans. \_\_\_\_\_

2. Find the area of the triangle whose vertices are  $(3,7)$ ,  $(2,1)$ , and  $(5,-5)$ .

Ans. \_\_\_\_\_

3. Solve the following system of equations for  $a$  and  $b$ :
- $$\begin{aligned} \frac{5}{a+b} - \frac{2}{a-b} &= -6 \\ \frac{15}{a+b} - \frac{4}{a-b} &= -6 \end{aligned}$$

Ans. \_\_\_\_\_

**3. Matrices, Determinants, and Systems of Equations**

**October 2004**

1. Find the ordered pair  $(x, y)$  such that  $\begin{bmatrix} x - y & x + y \end{bmatrix} = \begin{bmatrix} 6 & -2 \end{bmatrix}$ .

**Ans.** \_\_\_\_\_

2. Solve for the ordered pair  $(x, y)$ , if  $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 8 \end{bmatrix}$ .

**Ans.** \_\_\_\_\_

3. If  $A = \begin{bmatrix} -7 & 4 \\ -9 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}$ , find the determinant of  $AB$ .

**Ans.** \_\_\_\_\_

**3. Matrices, Determinants, and Systems of Equations**

**October 2005**

1. Find all values of  $x$ , such that  $x \begin{vmatrix} x & 1 \\ 1 & 1 \end{vmatrix} = 6$ .

**Ans.** \_\_\_\_\_

2. Find the sum of  $x + y + z$  for the following system:

$$2x + y = 8$$

$$y + z = 1$$

$$x + 2z = 6$$

**Ans.** \_\_\_\_\_

3. If  $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & -1 & -1 \end{bmatrix}$  and  $A = \begin{bmatrix} x+1 & x & x+1 \\ x & x+1 & x+1 \\ x-1 & x-1 & x-1 \end{bmatrix}$ , find the numerical determinant of  $A$ .

**Ans.** \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations(No Calculators)*

October 2006

1. Find  $\begin{vmatrix} 1 & 2 \\ 3 & 4 \\ 9 & 10 \\ 11 & 12 \end{vmatrix} \begin{vmatrix} 5 & 6 \\ 7 & 8 \\ 13 & 14 \\ 15 & 16 \end{vmatrix}$

Ans. \_\_\_\_\_

2. Jim is 4 inches taller than Mary. Mary is 2 inches taller than Sue. Amanda's height is the average of Jim's and Mary's. Sue is  $\frac{16}{17}$  times as tall as Amanda. How tall is Jim in inches?

Ans. \_\_\_\_\_

3. If  $\begin{pmatrix} 1 & x & y \\ 3 & 1 & x+1 \\ z & 1 & y \end{pmatrix} \cdot \begin{pmatrix} y & z \\ 1 & x \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 10 & 13 \\ 15 & 18 \end{pmatrix}$ , find the value of  $x + y + z$ .

Ans. \_\_\_\_\_

3. *Matrices, Determinants, and Systems of Equations(No Calculators)*

October 2007

1. Evaluate the following:  $2 \begin{vmatrix} 7 & 9 \\ 5 & 7 \end{vmatrix} - \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 2 \\ -1 & 2 & -3 \end{vmatrix}$ .

Ans. \_\_\_\_\_

2. Simplify the following:  $\begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 3 \\ 5 & -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 2 & 3 \\ 3 & -2 \end{bmatrix}$ .

Ans. \_\_\_\_\_

3. Find the ordered triple  $(x, y, z)$  such that:
- $$\begin{aligned} 5x + 3y - 2z &= -1 \\ 3x + 2y + 3z &= 7 \\ 4x - y - 7z &= 2 \end{aligned}$$

Ans. \_\_\_\_\_

**3. Matrices, Determinants, and Systems of Equations**

**September 2008 (No Calculators)**

1. In a group of pigs, chickens, the number of legs is eighty-four more than twice the number of heads. How many pigs are there?

Ans. \_\_\_\_\_

2. Find all ordered pairs  $(x, y)$  for which the following matrix equation is true:

Ans. \_\_\_\_\_

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} x \\ 4y \end{bmatrix} - 6 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3. Evaluate:

$$\begin{vmatrix} 2 & -4 & 7 & 3 \\ 0 & 5 & 1 & -2 \\ -2 & 1 & 0 & -3 \\ 0 & -6 & 4 & 2 \end{vmatrix}$$

Ans. \_\_\_\_\_

**3. Matrices, Determinants, and Systems of Equations**

**October 2009 (No Calculators)**

1. If  $A = \begin{bmatrix} 3 & 7 & 1 \\ 4 & 5 & -1 \\ 3 & 1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ , find  $BA$ .

Ans. \_\_\_\_\_

2. Find  $x + y + z$ , if

Ans. \_\_\_\_\_

$$3x - 2y + 5z = 14$$

$$-x + y - 3z = 10$$

$$-x + 2y - z = -2$$

3. Find all values of  $x$  such that

Ans. \_\_\_\_\_

$$\begin{vmatrix} \sqrt{x} & 2 & 0 \\ 3 & -1 & \sqrt{x} \\ 1 & -1 & 1 \end{vmatrix} = 14$$

**4. Number Theory**

**October 2000**

1. When Rebecca divides her favorite number by seven, she gets a remainder of 5. What will the remainder be if she multiplies her favorite number by five and divides by seven?

**Ans.** \_\_\_\_\_

2. The product of a set of distinct positive integers is 84. What is the least possible sum of these integers?

**Ans.** \_\_\_\_\_

3. A five-digit positive integer is a “mountain number” if the first three digits are in ascending order and the last three digits are in descending order. For example, 35761 is a “mountain number”, but 32323 and 35655 are not. How many five digit numbers greater than 70,000 are “mountain numbers”?

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2001**

1. If  $x$  and  $y$  are prime numbers and if  $|x - y| = 2$ , then  $x$  and  $y$  are called “twin” primes. Find the sum all “twin” primes  $x$  and  $y$ , such that  $x$  and  $y$  are between 30 and 60.

**Ans.** \_\_\_\_\_

2. Find the positive difference between any two numbers between 300 and 400 that have an odd number of factors.

**Ans.** \_\_\_\_\_

3.  $x$ ,  $y$ , and  $z$  are distinct prime numbers. Determine the total number of factors for the number which is the LCM for  $x^4y^5z^6$ ,  $x^2yz^7$ ,  $xy^6z$ .

**Ans.** \_\_\_\_\_



**4. Number Theory**

**October 2002**

1. A corral contains three times as many people as horses. If there are 80 feet on the ground in the corral (and every creature is standing on all of his or her feet), how many people are in the corral?

**Ans.** \_\_\_\_\_

2. Ms. Z is a kindergarten teacher with 16 students. She wants to take a number of tokens to school so that if any number up to 4 of her students is absent, she can still divide all the tokens evenly among the students present with no tokens left over. What is the least positive number of tokens Ms. Z should take to school? Assume that Ms. Z was trying to figure out how many she needed, not that she actually took tokens to school?

**Ans.** \_\_\_\_\_

3. Find the smallest natural number with exactly 20 positive factors.

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2003**

1. Find the LCM of 48 and 84.

**Ans.** \_\_\_\_\_

2. Find all the numbers between 400 and 500 which when divided by 7 have a remainder of 6 and when divided by 5 have a remainder of 4.

**Ans.** \_\_\_\_\_

3. Find the sum of the two smallest positive whole numbers each of which has twelve factors.

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2004**

1. Find the two-digit number, the sum of whose digits is equal to the square of its cube root.

**Ans.** \_\_\_\_\_

2. How many 3-digit numbers in base 4 are also 3-digit numbers in base 3?

**Ans.** \_\_\_\_\_

3. If you have a balancing scale and a minimum number of weights to weigh any integral number of pounds from 1 to 40 inclusive (using weights on either side of the scale), what weights would you need?

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2005 (No Calculators)**

1. Find the sum of the prime numbers from 80 to 100.

**Ans.** \_\_\_\_\_

2. The GCF of  $x$  and  $y$  is 4. The LCM of  $x$  and  $y$  is 572. Find the smallest value of the sum of  $x$  and  $y$ .

**Ans.** \_\_\_\_\_

3. In the following equation,  $x$  and  $y$  are digits in each of the base numbers. Neither  $x$  nor  $y$  equal zero.

$$(xy)_5 = (y11)_4$$

Find the base 10 value of  $x + y$ .

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2006 (No Calculators)**

1. How many integers divide 48 with a remainder of 0?

**Ans.** \_\_\_\_\_

2. Sam's social security number contains each of the nonzero digits exactly once. By examining the digits from left to right, he also found the 1 divides the first digit, 2 divides the first two digits, 3 divides the first three digits, and so on. If the number begins with 3816, what is his complete social security number?

**Ans.** \_\_\_\_\_

3. Find the greatest positive prime factor of  $2^{20} - 1$ .

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2007 (No Calculators)**

1. Find the sum of all prime numbers between 40 and 65.

**Ans.** \_\_\_\_\_

2. Find the product of the following in base 8:

$$\begin{array}{r} 624_8 \\ \underline{56_8} \end{array}$$

**Ans.** \_\_\_\_\_

3. Find the least and greatest integers between 500 and 1000 which when divided by 6 have a remainder of 5 and when divided by 7 have a remainder of 6.

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2008 (No Calculators)**

1. Which is greater,  $2^{100}$  or  $3^{75}$ ?

**Ans.** \_\_\_\_\_

2. I am the product of four prime numbers (each prime number is not necessarily unique). My three digits are each prime and unique. The sum of my prime factors is 30. Who am I?

**Ans.** \_\_\_\_\_

3.  $(231_4)(120_4) = b_4$ . Find the number of prime factors of the base ten equivalent of  $b_4$ .

**Ans.** \_\_\_\_\_

**4. Number Theory**

**October 2009 (No Calculators)**

1. Find the least common multiple for 1176 and 378.

**Ans.** \_\_\_\_\_

2. Determine the number of factors for the GCF of  $4x^3y^4z^5$  and  $6x^2y^7z^4$ . Assume that  $x$ ,  $y$ , and  $z$  are distinct prime numbers greater than 3.

**Ans.** \_\_\_\_\_

3.  $a$  and  $b$  are prime numbers and  $a < b$ .  $\frac{a+b}{2}$  is a perfect square and a perfect cube which is less than 100. Find all ordered pairs of  $(a, b)$ .

**Ans.** \_\_\_\_\_

5. *Geometric Similarities*

October 2000

1. Right triangles  $ABC$  and  $DEF$  are similar, with  $m\angle A = m\angle D = 90^\circ$ . If  $BC = 25$ ,  $AC = 10$ , and  $EF = 60$ , find the number of units in the length of  $DF$ .

Ans. \_\_\_\_\_

2. The sides of a right triangle have lengths  $x - y$ ,  $x$ ,  $x + y$ , where  $x > y > 0$ . Find the ratio of  $x : y$ .

Ans. \_\_\_\_\_

3.  $\triangle ABC$  is such that  $AC = 5$ ,  $BC = 9$ , and  $AB = 7$ .  $AB$  is extended through  $B$  to  $D$  so that  $BD = 8$ .  $AC$  is extended through  $C$  to  $E$  so that  $CE = 16$ . Find the length of  $DE$ .

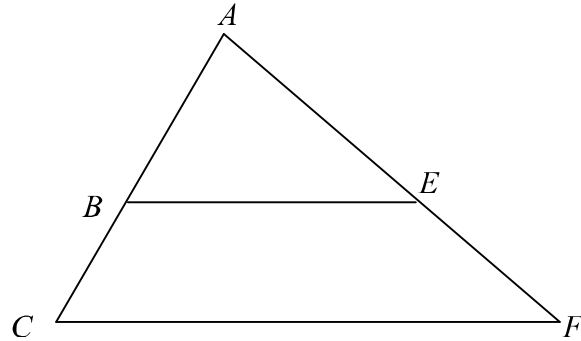
Ans. \_\_\_\_\_

5. *Geometric Similarities*

October 2001

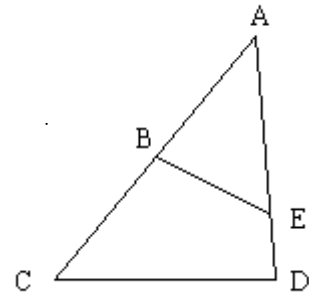
1. If segment  $BE$  is parallel to segment  $CD$ .  $AB = 4$ ,  $BC = 8$  and  $CD = 15$ , find the length of segment  $BE$ .

Ans. \_\_\_\_\_



Use the diagram and information for questions 2 and 3.

$\angle C \cong \angle AEB$ ,  $AE = 1.2n$ ,  $AC = 1.5n$ , and  $CD = n$ .



2. Find the length of segment  $BE$  in decimal form in terms of  $n$ .

Ans. \_\_\_\_\_

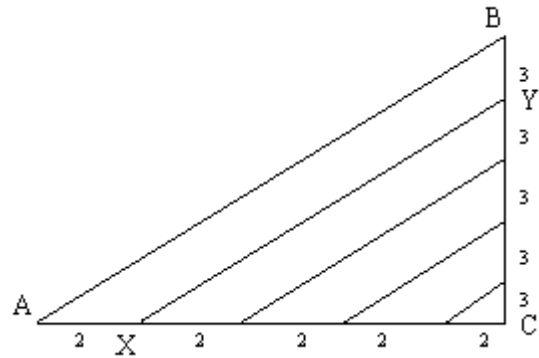
3. Find the square unit measure of the area of quadrilateral  $BEDC$ , if the area of triangle  $ABE$  equals 12 square units.

Ans. \_\_\_\_\_

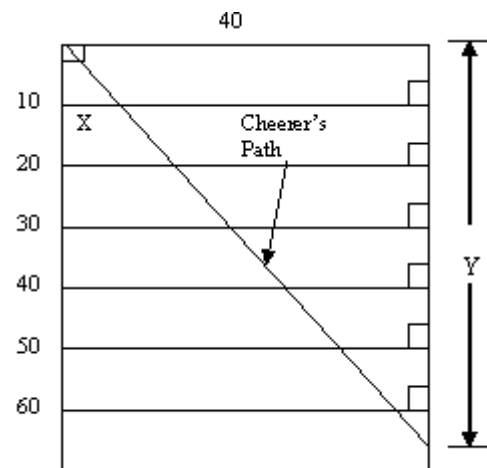
5. *Geometric Similarities*

October 2002

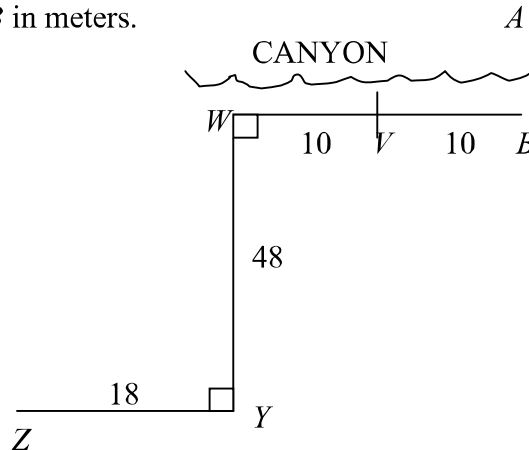
1. Given that  $XY = 16$ , find the perimeter of triangle  $ABC$ .



2. On a rectangular athletic field 40 meters wide, a cheerer walks straight from the 0 meter line on one side toward a point on the other side, crossing the 10 meter line  $X$  meters from the side, as shown. Find the distance  $Y$  from the 0 meter line to the point on the opposite side of the field that the cheerer crosses. Express your answer as a common fraction involving  $X$ , in simplest form.



3. Elmer must measure the distance between electric pylons  $A$  and  $B$  on opposite sides of a canyon. He does the following:
- From  $B$ , he walks 10m parallel to the canyon edge and pounds in a stake  $V$ ;
  - He continues 10 meters further in the same direction (to a point marked  $W$ );
  - He turns  $90^\circ$  left, walks 48m until  $A$  and  $B$  are in line (this is point  $Y$ );
  - He turns  $90^\circ$  right and walks 18m until  $V$  and  $A$  are in line. This is point  $Z$ .
- Find the distance between  $A$  and  $B$  in meters.



5. *Geometric Similarities*

October 2003

1. A person 5 ft. 4 in. tall has a shadow 13 ft. 4 in. long. How tall is a person who cast a shadow 11 ft. 8 in. long at the same time of the day? Assume both are on level ground. Express your answer in feet and inches.

Ans. \_\_\_\_\_

2. A person 5 feet 4 inches tall weighs 142 lbs. How much does a 6 foot 1 inch tall person weigh having the same stature? Round answer to the nearest pound.

Ans. \_\_\_\_\_

3. In  $\triangle ABC$ ,  $AC = 12$ ,  $AB = 15$  and  $BC = 19$ . Segment  $AC$  is extended through  $C$  to  $Q$  such that  $CQ = 23$ . Segment  $AB$  is extended 13 units through  $B$  to  $P$ . To the nearest  $100^{\text{th}}$  of a unit, find the distance from  $P$  to  $Q$ .

Ans. \_\_\_\_\_

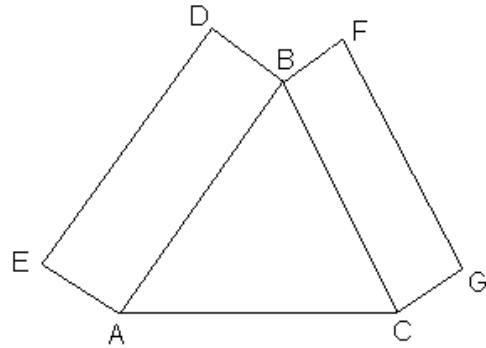


5. *Geometric Similarities*

October 2004

1. In the figure,  $AB = CB$ , quadrilaterals  $ABDE$  and  $CBFG$  are both rectangles and  $m\angle BAC = 70^\circ$ . Find  $m\angle DBF$ .

Ans. \_\_\_\_\_

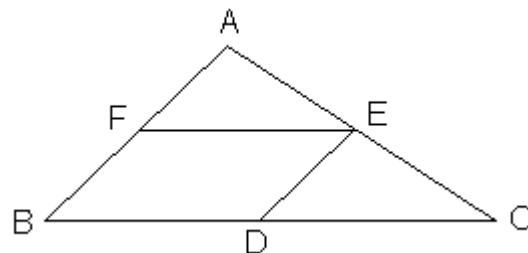


2. Triangle  $A$  has sides of lengths 4 cm, 8 cm and 10 cm. Triangle  $B$  has sides of lengths 6 cm, 12 cm and 15 cm. What is the ratio of the area of  $A$  to  $B$ ? Express your answer in simplest form.

Ans. \_\_\_\_\_

3. Rhombus  $BDEF$  is inscribed in triangle  $ABC$  as shown. If  $AB = 10$  ft. and  $BC = 15$  ft., find the length of segment  $DE$ .

Ans. \_\_\_\_\_



**5. Geometric Similarities**

**October 2005 (You may use Calculators)**

1. Triangle  $ABC$  is magnified by a power of 3 from point  $A(1,2)$  through points  $B(0,3)$  and  $C(3,5)$  so that the new triangle  $ADE$  is similar to triangle  $ABC$ . Find the ordered pair for point  $E$ .

**Ans.** \_\_\_\_\_

2.  $\triangle ABC \sim \triangle DAB$ . If  $AB = 6$  and  $BC = 8$ , find the length of segment  $AD$ .

**Ans.** \_\_\_\_\_

3. A spherical balloon could be painted with exactly 2 cans of paint. The volume of the balloon is tripled. How many cans of paint are needed to cover this larger inflated balloon. Express your answer as a decimal to the nearest 1000<sup>th</sup>.

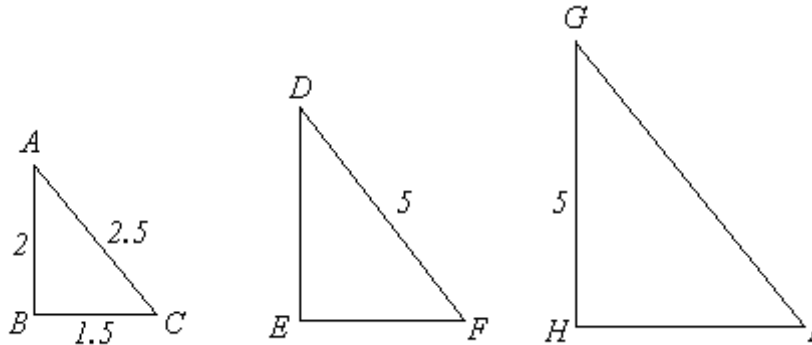
**Ans.** \_\_\_\_\_

5. Geometric Similarities

October 2006 (You may use Calculators)

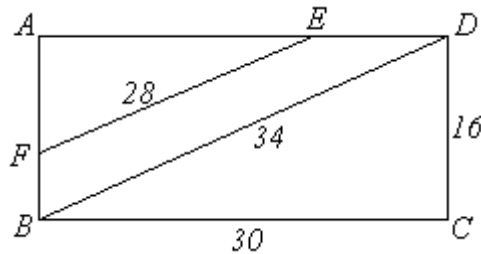
1. Triangles  $ABC$ ,  $DEF$  and  $GHI$  are all similar and have side lengths as marked. Find the sum of the perimeters of all three triangles.

Ans. \_\_\_\_\_



2. In the 16 by 30 rectangle  $ABCD$ ,  $\overline{BD}$  is a diagonal of length 34 and  $\overline{EF}$  is parallel to  $\overline{BD}$ . If  $EF = 28$ , find the perimeter of quadrilateral  $BDEF$ . Express your answer as a mixed number.

Ans. \_\_\_\_\_



3. Tetrahedron  $ABCD$  has four equilateral triangles for faces. Each of the six side lengths is 6, each of the four faces has area  $9\sqrt{3}$ , and the volume is  $18\sqrt{2}$ . Tetrahedron  $EFGH$  is similar to tetrahedron  $ABCD$ . The surface area of each of the faces of  $EFGH$  is  $16\sqrt{3}$ . Find the volume of  $EFGH$ .

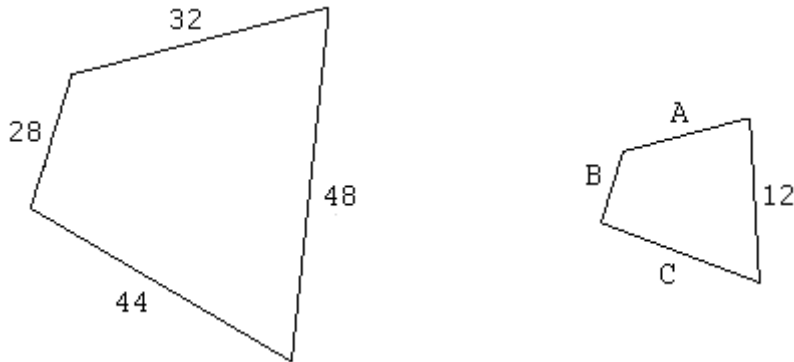
Ans. \_\_\_\_\_

5. *Geometric Similarities*

October 2007 (You may use Calculators)

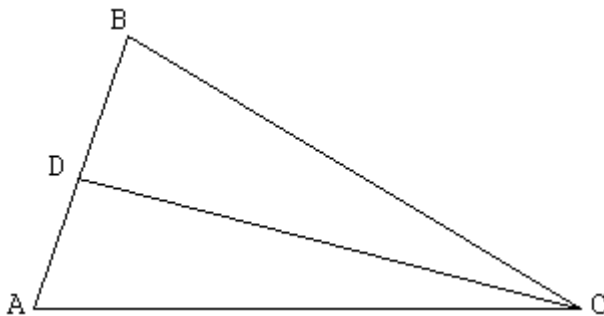
1. The following shapes are similar. Find the sum of  $A$ ,  $B$ , and  $C$ .

Ans. \_\_\_\_\_



2. In triangle  $ABC$ ,  $\overline{DC}$  bisects angle  $ACB$ .  $BC = 36$ ,  $AC = 39$  and  $AB = 25$ . Find the length of  $\overline{AD}$ .

Ans. \_\_\_\_\_



3. The volumes of two similar square pyramids are 1280 and 14,580. The area of the base of the smaller pyramid is 64. Find the area of the base of the larger pyramid.

Ans. \_\_\_\_\_

5. *Geometric Similarities*

*September 2008 (You may use Calculators)*

1. The lengths of the sides of triangle ABC are 6 cm, 4 cm, and 9 cm. Triangle DEF is similar and the length of one of its sides is 36 cm. What is the greatest possible perimeter of triangle DEF?

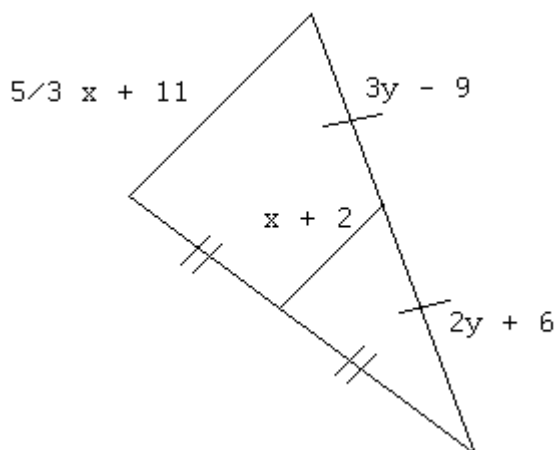
Ans. \_\_\_\_\_

2. Two bottles are similar in shape and have the same type of contents. The larger bottle is three times as high as the smaller. If the contents of the smaller bottle is valued at \$0.60, what is the value of the contents of the larger bottle?

Ans. \_\_\_\_\_

3. Find x and y, if

Ans. \_\_\_\_\_



5. *Geometric Similarities*

*October 2009 (You may use Calculators)*

1. Square A measures  $k$  units on a side. Square B measures  $2.5k$  units on a side. Let  $a:b$  represent the smallest possible ratio of the areas of square A to square B, such that  $a$  and  $b$  are integers. Find  $a + b$ .

Ans. \_\_\_\_\_

2. The surface area of a sphere is  $9\pi$ . Find the volume of the sphere.

Ans. \_\_\_\_\_

3. In the pentagon ABCDE below,  $\angle 1 \cong \angle 2 \cong \angle 3$ .  $m\angle AED \cong m\angle ADC \cong \angle ACB$ , which are each  $90^\circ$ . If  $AB = 10$ ,  $BC = 6$  and  $AC = 8$ , find the exact perimeter of the pentagon as a decimal.

Ans. \_\_\_\_\_

