1. Arithmetic with * Operations ***October 2010 (No Calculators)***

1. Let a * b = $\big||a|-|b|\big|$. If x * (-4) = 2, how many values could x have?

Ans. _____

2. If a * b = $\dfrac{a^b - b^a}{a-b}$, then find the value of (2 * 4) * (2 * 3).

Ans. _____

3. Use the pattern below to calculate $994^2 + 989^2$.

Ans. _____

$999^2 = 1000(998) + 1^2$

$998^2 = 1000(996) + 2^2$

$997^2 = 1000(994) + 3^2$

1. Arithmetic with "*" Operations ***Oct 2011 (No Calculators)***

1. If $a * b = 2a - 2b$ and $c \nabla d = 3c + 3d$, determine the value of $(3 * 2) \nabla (2 * 3)$.

Ans. _____

2. A man sold one third of his crop of bales of hay to his neighbor. He then sold one fifth of what was left to a second neighbor. He finally sold on seventh of what was then left to a third neighbor, leaving him with 336 bales of hay to feed his horses for the winter. How many bales of hay were in his total crop?

Ans. _____

3. If $a * b = a^2 + b$ and $c * d = d * c$, find d in terms of c, where c > 0 and d > 0.

Ans. _____

1. Arithmetic with "*" Operations ***Oct 2012 (No Calculators)***

1. If $a \,\&\, b = a^b - 3ba$, find (2 & 5) & 3.

Ans. _____

2. The digits 1, 2, 3, and 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are listed in order from smallest to largest, in what position is 3142?

Ans. _____

3. If $x * y = \dfrac{\left(x^2 - 2x + 1\right) - \left(y^2 - 4y + 4\right)}{x - y + 1}$, find 5 * (3 * 2).

Ans. _____

2. Inequalities and Absolute Value ***October 2010 (No Calculators)***

1. There is only one point with integer values for x and y that satisfies x > -2, y > x, and y < |x|. Find the point.

Ans. _____

2. Solve: $|4x-7| \le |5x-6|$.

Ans. _____

3. How many distinct vertices appear in the graph of $1-|y| = |1-|x||$?

Ans. _____

2. Inequalities and Absolute Values ***October 2011 (No Calculators)***

1. Find all values of x, such that $5(2x-3)+8 \ge 6(x-5)+7$

Ans. _____

2. Find all real value solutions for $\frac{3}{2x-3} \le \frac{2}{3x+2}$.

Ans. _____

3. Solve the following for all possible real values of x:
$\left|\frac{4}{x-2}\right| < x+1$

Ans. _____

2. Inequalities and Absolute Values ***Oct 2012 (No Calculators)***

1. Solve for x: $|x-3|<2$.

Ans. _____

2. Solve for m: $(m-4)^3 > (1/8)^{-1}$

Ans. _____

3. Solve for x: $\dfrac{|2x-5|}{x^2-4} > 0$.

Ans. _____

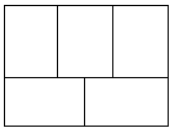
3. Matrices, Determinants, and Systems of Equations ***October 2010 (No Calculators)***

1. Find the following product: $\begin{bmatrix} 4 & 2 & -2 & -1 \\ -2 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 3 & -1 \\ 2 & -2 \\ -4 & 0 \end{bmatrix}$

Ans. _____

2. A rectangle with perimeter 176 is divided into 5 congruent rectangles as shown in the diagram. What is the perimeter of one of the congruent rectangles?

Ans. _____



3. Find all value(s) of k such that $\begin{vmatrix} 2 & k & 1 \\ 3 & -3 & -k \\ -4 & 2 & k \end{vmatrix} = 9.$

Ans. _____

3. Matrices, Determinants and Systems of Equations ***October 2011 (No Calculators)***

1. Solve the following system: $\begin{aligned} 3x + 2y &= 3 \\ 2x - 3y &= 28 \end{aligned}$

Ans. _____

2. Find the product of $\begin{bmatrix} 3 & 5 & 2 \\ -1 & 6 & 7 \end{bmatrix} \begin{bmatrix} 9 & 4 & -8 \\ 2 & -5 & 6 \\ -3 & 7 & 4 \end{bmatrix}$

Ans. _____

3. Find all real numbers for x, such that:
$\begin{vmatrix} x & -x & 3 \\ 1 & x+1 & x \\ -x & 1 & 4 \end{vmatrix} = 57$

Ans. _____

3. Matrices, Determinants and Systems of Equations ***Oct 2012 (No Calculators)***

1. Given that $\begin{vmatrix} x & -7 \\ y & 9 \end{vmatrix} = -31$ and $\begin{vmatrix} 5 & x \\ -3 & y \end{vmatrix} = -29$, find (x, y).

Ans. _____

2. Given the equations $3x + y = 17$, $5y + z = 14$, and $3x + 5z = 41$, what is the value of the sum $x + y + z$?

Ans. _____

3. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ 0 & 4 \\ 3 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 22 \end{bmatrix}$, and $D = \begin{bmatrix} x \\ y \end{bmatrix}$, then find D, if ABD = C.

Ans. _____

4. Number Theory ***October 2010 (No Calculators)***

1. What is the least positive integer that must be added to 999,999,999,999,999 so that the sum will be divisible by 33?

Ans. _____

2. If all the positive factors of 400 are arranged in order from smallest to largest, what is the one in the middle?

Ans. _____

3. The product of any two of the numbers 30, 72 and z is divisible by the third. What is the smallest possible integer z with this property?

Ans. _____

4. Number Theory ***October 2011 (No Calculators)***

1. Find the larger of two positive numbers whose product is 50 and whose difference is 23.

Ans. _____

2. Find the sum of the positive factors of 180.

Ans. _____

3. Find the remainder when 23^{15} multiplied by 47^{17} is divided by 5.

Ans. _____

4. Number Theory ***Oct 2012 (No Calculators)***

1. Change the base ten number, 777, to a base seven number.

Ans. _____

2. Find the number of digits in the number that results from the expansion of the following product $\left(5^{2012}\right)\left(2^{2021}\right)$.

Ans. _____

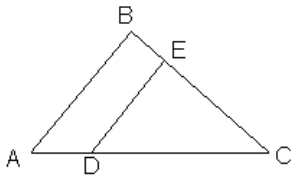
3. Four distinct, positive integers a, b, c, and N exist, such that $N = 5a + 3b + 5c$. Also, $N = 4a + 5b + 4c$, and N is between 131 and 150. What is the numerical value of $a + b + c$?

Ans. _____

5. Geometric Similarities ***October 2010 (You may use Calculators)***

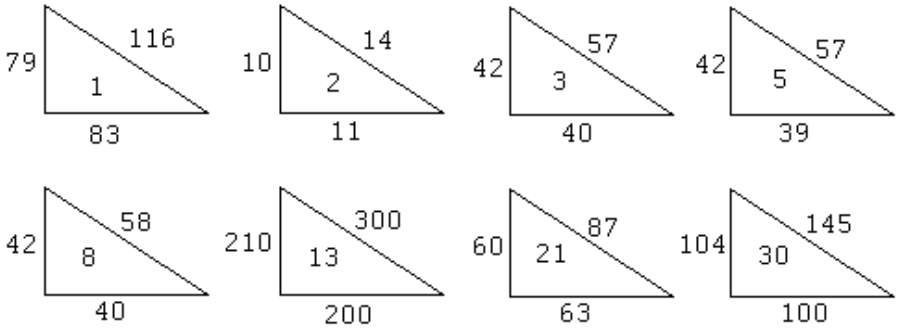
1. In 5-6-7 ΔABC. AB = 5, BC = 6, AC = 7, point D is on $\overline{AC}$, point e is on $\overline{BC}$, AD = 1 , and $\overline{DE} \parallel \overline{AB}$. Find DE.

Ans. _____



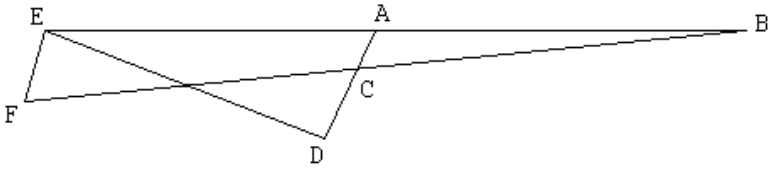
2. Each of the 8 triangles below (not drawn to scale) is marked with all three side lengths and an identifying number (1, 2, 3, 5, 8, 13, 21, 30). Two of these triangles are similar. Find the sum of the identifying numbers of the two similar triangles.

Ans. _____



3. A surveyor standing at point A follows these directions to estimate the distance between points A and B:
   - Face point B
   - Turn right between 100° and 170°
   - Walk 5 meters and set your hat on the ground (point C).
   - Walk 27 meters further in the same direction to point D
   - Turn right 90° and walk until the $\overline{AB}$ line is reached (point E), measuring the distance (60 meters).
   - Turn left 90° and walk until the $\overline{CB}$ line is reached (point F), measuring the distance (7 meters).
   - What is the surgeyor's estimate of the length of $\overline{AB}$ in meters?

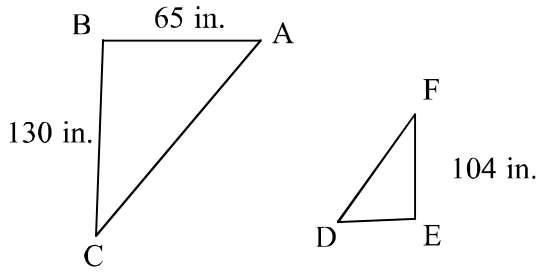
Ans. _____



5. Geometric Similarities ***October 2011 (You may use Calculators)***

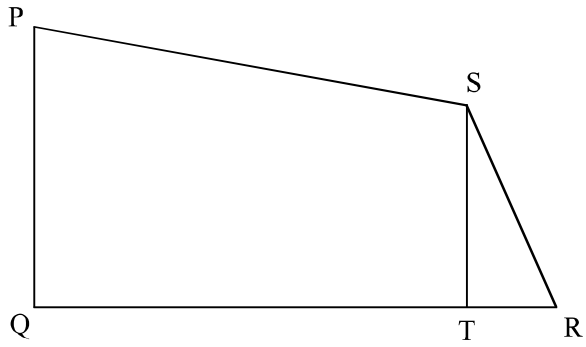
1. Given $\Delta ABC \sim \Delta DEF$ with side lengths as given, find the length of side DE. Express your answer in feet and inches.

Ans. _____



2. In the figure, $PQ \perp QR$, $ST \perp QR$, $\angle P \cong \angle R$, PQ = 15, QT = 24 and ST = 8. Find the perimeter of quadrilateral PQRS.

Ans. _____

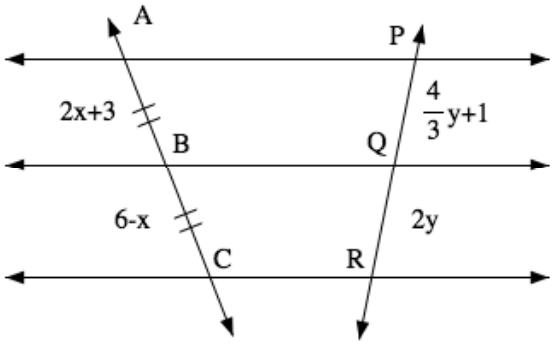


3. Triangle ABC is a right triangle with right angle at B. AB = 12 and BC = 9. A segment is drawn from B perpendicular to side AC at D and extends through D to a point E, such that $\overline{DE} \cong \overline{BD}$. A segment is then drawn from E perpendicular to $\overline{BC}$ (extended), meeting at point F. Find the length of $\overline{CF}$. Express answer as a decimal.

Ans. _____

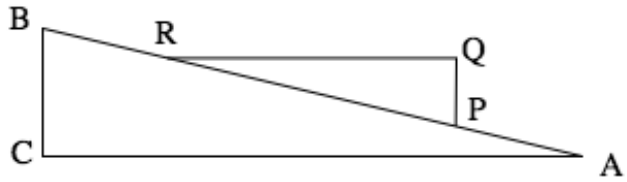
5. Geometric Similarities ***Oct 2012 (You may use calculators)***

1. Find x and y for the segments AB, BC, PQ, and QR in the following diagram. The horizontal lines are parallel.



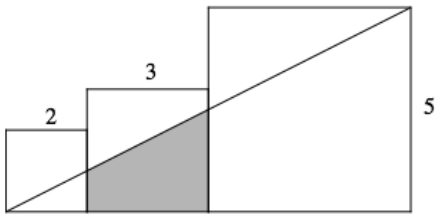
Ans. _____

2. In the diagram, AB = 300, PQ = 20, and QR = 100. $\overline{QR}$ is parallel to $\overline{AC}$. $\overline{BC}$ is perpendicular to $\overline{AC}$, and $\overline{QP}$ is perpendicular to $\overline{QR}$. To the nearest hundredth, find the length of $\overline{BC}$.



Ans. _____

3. The three squares have the indicated lengths in the diagram. What is the area of the shaded quadrilateral?



Ans. _____