Meet #2 2001

Solutions – Arithmetic with Ratio and Proportion

1. On the 41 foot side it would need $41/3 = 13^+$ or 15 posts, since one had to be put at the 0 position. Thus 30 posts for both sides. On the 31 foot side it would need $31/3 = 10^+$ or 12 posts. But 2 posts are already in the end positions from the other sides. Thus 20 posts are needed on these sides. The total is 50. Ans. 50 or 50 posts

2. $\frac{1}{2}x+\frac{1}{3}x+\frac{1}{4}x=195$ or $\frac{13}{12}x=195$. Thus $x=15\cdot 12$, $\frac{1}{2}x=90$, $\frac{1}{3}x=60$, $\frac{1}{4}x=45$. Ans. 90,60,45

3. $\frac{Ll}{bd^2}=\frac{200\cdot 6}{2\cdot 4^2}=\frac{L\cdot 15}{4\cdot 6^2}$ or $L=\frac{200\cdot 6}{2\cdot 4^2}\cdot\frac{4\cdot 6^2}{15}=360$ Ans. 360

Series and Sequences

1. (1) a + d = 1 ¾ and (2) a + 4d = 5 ½. Subtracting (1) from (2): 3d = 3 ¾ or d = 1 ¼. Subtracting 1 ¼ from 1 ¾ yields ½. Ans. ½

2. 4^{th} : a + 3d = 62. 11^{th} : a + 10d = 167. Subtracting: 7d = 105 or d = 15. Subbing back in: a + 3(15) = 62. Thus a = 17. The 15^{th}: 17 + (14)15 = 227. The sum of the 1^{st} 15 terms: $s=\frac{15(17+227)}{2}=\frac{15(244)}{2}=15(122)=1830$ Ans. 1830

3. There are two infinite geometric sequences: (1) 81 + 54 + 36 + … and (2) 64 + 48 + 36 + … The first sum is $\frac{81}{1-\frac{2}{3}}=243$. The second is $\frac{64}{1-\frac{3}{4}}=256$ Ans. 499

Counting Principles and Binomial Theorem

1. There are 21 consonants. Thus ${}_{21}C_3=\frac{21\cdot 20\cdot 19}{3\cdot 2\cdot 1}=\frac{7\cdot 10\cdot 19}{1}=1330$ Ans. 1330

2. The ratio of the powers on the terms is 1:3. In order for them to cancel out the ratio would have to be the reciprocal 3:1. 3x + x = 8, thus x = 2 and the powers are 6 and 2. Thus the term: $\binom{8}{6}\left(\frac{x}{6}\right)^6\left(\frac{-8}{x^3}\right)^2=\frac{8\cdot 7}{2}\left(\frac{64}{6^6}\right)=\frac{28}{3^6}=\frac{28}{729}$ Ans. $\frac{28}{729}$

3. We will figure for American League to win (Go Yankee's) and double to get all:
AAAA – **1** NAAAA – **4** (All the letters can be switched around, but the **A** Must be at the end. NNAAAA - $\frac{5!}{2!3!}$ **= 10** NNNAAAA - $\frac{6!}{3!3!}$ **= 20.** Ans. 70

Areas and volumes

1. The area of the trapezoid with bases 3 and 6 and height 8. $\frac{1}{2}(3+9)8 = 48$. Ans. 48

2. The volume of the container = 9(11)(38.5) = 1.1h(9)(11). Thus 38.5 = 1.1h Ans. 35

3. The smallest section is made up of 1 triangle. The next has 3 triangles. The next has 5 triangles, etc. So the 10^{th} section has 19 triangles. Each triangle is congruent to all the rest. Each triangle has an area of 76/19 = 4. There are 1 + 3 + 5 + … 19 = 100 triangles. Thus the total area of the original triangle is 400. Ans. 400

Polynomials

3 pts 1.

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 7 & 8 \\ & & -3 & -6 & -3 \\ \hline & 1 & 2 & 1 & 5 \end{array}$$

Ans. 5

4 pts 2.

$$\begin{array}{r l} & \underline{x^3 - 4x^2 + (a+8)x + (-17-2a)} \\ x+2 \,) & x^4 - 2x^3 + ax^2 \quad - \quad x - 1 \\ & \underline{x^4 + 2x^3} \\ & \quad -4x^3 + ax^2 \\ & \quad \underline{-4x^3 - 8x^2} \\ & \qquad (a+8)x^2 \quad - \quad x \\ & \qquad \underline{(a+8)x^2 + (2a+16)x} \\ & \qquad\qquad -(2a+17)x - 1 \\ & \qquad\qquad \underline{-(2a+17)x - 34 - 4a} \\ & \qquad\qquad\qquad 33 + 4a = 5 \Rightarrow 4a = -28 \end{array}$$

Ans. -7 or a = -7

5 pts 3. Since $x^3 + 2px^2 - px + 10 = 0$ has three roots whose product is –10, and they must be in arithmetic progression, the only choice is –1, 2, 5. Thus:
(x+1)(x-2)(x-5) = $x^3 + 2px^2 - px + 10$ or $x^3 - 6x^2 + 3x + 10 = x^3 + 2px^2 - px + 10$. So 2p = -6

Ans. –3 or p = -3

Team

1. Since there are 4 odd numerals, the first and last digits are 4(3) = 12. That leaves 4(3)(2) for the middle three digits which equals 24. The product is 288. Ans. 288

2. (.8) (July 23) $(.8)^2 = .64$ (July 30) $(.8)^3 = .512$ (August 6) $(.8)^4 = .4096$
This happens on August 13. Ans. August 13

3. The ratio of the area of the upper triangle to the trapezoid is 2/3 to 1/3. Thus the ratio of the areas of the upper triangle to the large triangle is 2/3 to 1. Since the triangles are similar, then the square of the ratio of any two corresponding sides equals the ratio of the area of the triangles: $\frac{2/3}{1} = \frac{x^2}{18^2}$ or $x^2 = \frac{2}{3} \cdot 18^2 = 12 \cdot 18 = 6 \cdot 36$ or $x = 6\sqrt{6}$. Ans. $6\sqrt{6}$

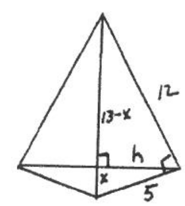
4. Each row of Pascal's Triangle has a sum of coefficients equal to 2^n, where n is the power of the binomial. Thus if $2^n = 64$, then n = 6. Thus the 5[th] term of $(x + y)^6$ equals $\binom{6}{4} x^2 y^4$ has a coefficient of 15. Ans. 15

5. The consecutive odd integers if a is the first are a, a+2, a + 4. Thus: $a^2 - 2b^2 + c^2 = a^2 - 2(a+2)^2 + (a+4)^2 = a^2 - 2(a^2 + 4a + 4) + a^2 + 8a + 16 = 8$ Ans. 8

6. The series is a, ar, ar^2, ar^3, ar^4. Thus (1) $ar^4 - ar^3 = 576$ and (2) $ar - a = 9$
In (1) $ar^3(r-1) = 576$ or $r - 1 = \frac{576}{ar^3}$. In (2) $a(r-1) = 9$ or $r - 1 = \frac{9}{a}$. Thus:

$\frac{576}{ar^3} = \frac{9}{a} \Rightarrow 9r^3 = 576$ or $r^3 = 64$. Thus r = 4 and in (2): a(4) – a = 9, thus a = 3.
Therefore the first 5 terms are 3, 12, 48, 192 , 768 and their sum is 1023. Ans. 1023

7. In the figure at right, to solve for h, which is the radius of the circle which is the base of the two cones, we use the area of the triangle: ½ 12(5) = ½ 13h, thus $h = \frac{60}{13}$. If we let the lower cone have the height x, then the upper cone has a height of 13-x. Thus the volume of the solid is $\frac{1}{3}(\frac{60}{13})^2(13-x) + \frac{1}{3}(\frac{60}{13})^2 x$. Factoring: $\frac{1}{3}(\frac{60}{13})^2(13) = \frac{3600}{39} = \frac{1200}{13} = 92\frac{4}{13}$



Ans. $\frac{1200}{13}$ or $92\frac{4}{13}$

8. The terms are: 3, 3r, $3r^2$, $3r^3$. The sum of the 2[nd], 3[rd], and 4[th] : $3r + 3r^2 + 3r^3 = 171/8$
Thus $r + r^2 + r^3 = 57/8$ or $8r + 8r^2 + 8r^3 - 57 = 0$

$\frac{3}{2}$	8	8	8	-57
		12	30	57
	8	20	38	0

Since r = 3/2 and the first term is 3, then the 4[th] term is:

$3(\frac{3}{2})^3 = \frac{81}{8}$ or $10\frac{1}{8}$ Ans. $\frac{81}{8}$ or $10\frac{1}{8}$

9. (1) p(x) = (x – 1776)a + 1776
(2) p(x) = (x + 1776)b – 1776
(3) $p(x) = (x^2 - 1776^2)c + (gx + h)$

Since $p(1776) = 1776$ and $p(-1776) = -1776$, then using (3):
$1776 = 1776g + h$ and $-1776 = -1776g + h$. Solving simultaneously, $h = 0$, and $g = 1$.
So in (3), $p(x) = (x^2 - 1776^2)c + x$, which implies that x is the remainder. Ans. x

Answer Sheet

Arithmetic with Ratio and Proportion
1. 50
2. 90, 60, 45
3. 360

Series and Sequences
1. $\frac{1}{2}$
2. 1830
3. 499

Counting Principles and Binomial Theorem
1. 1330
2. $\frac{28}{729}$
3. 70

Areas and Volumes
1. 48
2. 35 or 35cm
3. 400

Polynomials
1. 5
2. –7
3. –3

Team
1. 288
2. August 13
3. $6\sqrt{6}$
4. 15
5. 8
6. 1023
7. $\frac{1200\pi}{13}$ or $92\frac{4}{13}\pi$
8. $\frac{81}{8}$ or $10\frac{1}{8}$
9. x