Meet # 2 11/5/03 2003

Solutions – Arithmetic with Ratio and Proportion

1. $2[8+3(8\cdot 6\div 4\cdot 12)] = 2[8+3(12\cdot 12)] = 2[8+432] = 880$ Ans. 880

2. $\frac{5ft10in}{7ft6in} = \frac{x}{41ft3in} \Rightarrow \frac{70}{90} = \frac{x}{495} \Rightarrow 9x = 7(495) \Rightarrow x = 7(55) = 385$ Ans. 32 ft. 1 in.

3. $k = \frac{men \cdot hr}{feet} \Rightarrow \frac{18\cdot 6}{720} = \frac{24\cdot 8}{x} \Rightarrow 18\cdot 6x = 720\cdot 24\cdot 8 \Rightarrow x = 40\cdot 4\cdot 8 = 1280$ Ans. 1280 ft.

Series and Sequences

1. $84 = 35 + (8-1)d \Rightarrow 49 = 7d$, $d = 7$. $35 + 7 = 42$ Ans. 42

2. $S = \frac{52(5+5(51)1)}{2} = 26(61) = 1586$ Ans. 1586

3. $a\cdot ar\cdot ar^2 = 8000 \Rightarrow a^3r^3 = 8000$ or $ar = 20$ or $r = \frac{20}{a}$

$a + ar + ar^2 = 124$. Thus $a + a(\frac{20}{a}) + a\left(\frac{400}{a^2}\right) = 124$ or $a + 20 + \frac{400}{a} = 124$

Thus the quadratic equation: $a^2 + 20a + 400 = 124a \Rightarrow a^2 - 104a + 400 = 0$ or $(a-100)(a-4) = 0$. If $a = 100$, then $r = 1/5$ and thus the sequence 100, 20 , 4. If $a = 4$, then $r = 5$ and thus the sequence 4, 20, 100. Ans. 4, 20,100 or 100, 20, 4

Counting Principles and Binomial Theorem

1. ${}_{10}P_3 = 10\cdot 9\cdot 8 = 720$ Ans. 720

2. $\binom{9}{6}(4x^2)^3\left(\frac{1}{2x}\right)^6 = 84(4^3x^6)\left(\frac{1}{2^6x^6}\right) = 84$ Ans. 84

3. 3 separate mailboxes: $8\cdot 7\cdot 6 = 336$
   2 in one mailbox, one in another: $({}_8C_2)\cdot 2 = (28)2 = 56$
   3 in one mailbox: 8 Ans. 400

Polynomials

1. Synthetically:

2	3	-2	-5	-3	4
		6	8	6	6
	3	4	3	3	10

Ans. 10

2. $$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\qquad 3x^2 - x + 3 \\ x^3 - 3x^2 + 2x - 5 \overline{)\, 3x^5 - 10x^4 + 12x^3 - 26x^2 + ax + b} \\ \qquad\qquad\qquad\quad \underline{-3x^5 + 9x^4 - 6x^3 + 15x^2} \\ \qquad\qquad\qquad\qquad\quad - x^4 + 6x^3 - 11x^2 \\ \qquad\qquad\qquad\qquad\quad \underline{+ x^4 - 3x^3 - 2x^2 - 5x} \\ \qquad\qquad\qquad\qquad\qquad\quad 3x^3 - 9x^2 + (a-5)x + b \\ \qquad\qquad\qquad\qquad\qquad\quad \underline{- 3x^3 + 9x^2 - 6x + 15} \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad (a-11)x + (b+15) \end{array}$$

Ans. a = 11, b = -15

3. By synthetic division:

5	1	-7	3	43	-28	-60
		5	-10	-35	40	60
	1	-2	-7	8	12	0

We now need to solve the equation: $x^4 - 2x^3 - 7x^2 + 8x + 12 = 0$. Rearranging and factoring:
$x^4 - 7x^2 + 12 - 2x^3 + 8x = 0$ ➔ $(x^2 - 3)(x^2 - 4) - 2x(x^2 - 4) = 0$ ➔ $(x^2 - 4)(x^2 - 2x - 3) = 0$
Thus $(x - 2)(x + 2)(x - 3)(x + 1) = 0$. Ans. $-1, 3, \pm 2$

Areas and Volumes

1. V = 1/3 Ah: 12 = 1/3 A(2) ➔ a = 18 Ans. 18

2. $V = 4/3\ \pi r^3 = \frac{\sqrt{3}}{2}\pi$ ➔ $r^3 = \frac{3\sqrt{3}}{8}$, thus $r = \frac{\sqrt{3}}{8}$.

$SA = 4\pi r^2 = 4\pi\left(\frac{3}{4}\right) = 3\pi = 9.42477$ Ans. 3π or 9.425

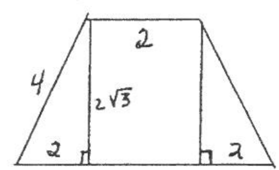
3. If you extend the congruent sides of the trapezoid to meet, they meet at a $60°$ angle. Thus the measure of the arc is $120°$, since the exterior angle and its intercepted arc are supplementary.

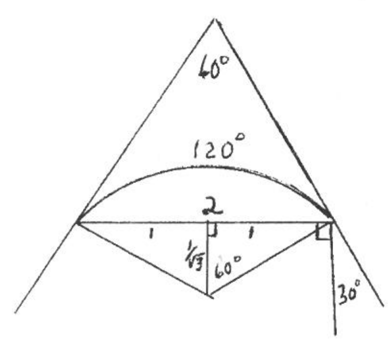
The area of this sector is $\frac{120}{360}\cdot\pi\left(\frac{4}{3}\right) = \frac{4}{9}\pi$. Use figure.

The area of the two triangles is $\frac{\sqrt{3}}{3}$. Area of segment $\frac{4}{9}\pi - \frac{\sqrt{3}}{3}$

The area of the trapezoid is $\left(\frac{2+6}{2}\right)(2\sqrt{3}) = 8\sqrt{3}$.

Thus the area of the figure is: Ans. $\frac{4}{9}\pi + \frac{23\sqrt{3}}{3}$.





Team

1. 2/5 n = 12 ➔ n = 12(5/2) = 30. Ans. 30

2. Multiplying (x + 1)(x + 2)(x + 3)(x + 4)(x + 5) sinthetically:

$$\begin{array}{rrrrr}
 & & 1 & 3 & 2 \\
 & & 1 & 7 & 12 \\
\cline{3-5}
 & & 12 & 36 & 24 \\
 & 7 & 21 & 14 & \\
1 & 3 & 2 & & \\
\hline
1 & 10 & 35 & 50 & 24
\end{array}
\qquad\qquad
\begin{array}{rrrrrr}
 & 1 & 10 & 35 & 50 & 24 \\
 & & & & 1 & 5 \\
\cline{2-6}
 & 5 & 50 & 175 & 250 & 120 \\
1 & 10 & 35 & 50 & 24 & \\
\hline
 & & & 225 & &
\end{array}$$

Ans. 225

3.
$$\begin{array}{r}
ax + (a+b) \\
x^2 - x - 1\;\overline{)\;ax^3 + bx^2 + 0x + 1} \\
\underline{-ax^3 + ax^2 + ax} \\
(a+b)x^2 + ax + 1 \\
\underline{-(a+b)x^2 + (a+b)x + (a+b)} \\
(2a+b)x + (a+b+1)
\end{array}$$

Now: (1) $2a + b = 0$ and (2) $a + b + 1 = 0$
Subtracting (2) from (1): $a - 1 = 0$
Thus $a = 1$. In (2): $2(1) + b = 0$
or $b = -2$.

Ans. –2

4. $x^2 + x^2 + x^2 = 5^2$. $3x^2 = 25$, $x = \frac{5}{\sqrt{3}}$ the side of the square. The surface area is

$6e^2$ where e is the edge. Thus: $6\left(\frac{5}{\sqrt{3}}\right)^2 = 6(25/3) = 50$.

Ans. 50

5. If the letters are all different: ${}_4P_3 = 24$

If two letters are the same: for 2 of the letters A: AAB, AAC, AAD, but each of these can be switched around in 3 different ways to produce a different "word" thus 3(3) = 9. For each of the 4 letters to form these "words" 4(9) = 36. Total 24 + 36 = 60.

Ans. 60

6. Because $0 < x < 1$, the sequence is an infinite decreasing series with the first term being x and the common ratio between terms is $x^{1/8}$. The sum is $\frac{x}{1 - x^{1/8}}$.

Ans. $A = 1 - x^{1/8}$

7. Each of the inner diagonals is $\sqrt{3}$ times the side. So the 8th term in the sequence is $1(\sqrt{3})^7 = (\sqrt{3})^6(\sqrt{3}) = 27\sqrt{3}$ which is the side of the 8th cube made in the fashion. The volume is $(27\sqrt{3})^3 = 59{,}049\sqrt{3}$.

Ans. $59{,}049\sqrt{3}$

8. $\binom{-\frac{1}{2}}{100} = \frac{(-\frac{1}{2})(-1\frac{1}{2})(-2\frac{1}{2})\cdots(-99\frac{1}{2})}{100!}$ $\qquad$ $\binom{\frac{1}{2}}{100} = \frac{(\frac{1}{2})(-\frac{1}{2})(-1\frac{1}{2})\cdots(-98\frac{1}{2})}{100!}$

Thus $\binom{-\frac{1}{2}}{100} \div \binom{\frac{1}{2}}{100} = \frac{(-\frac{1}{2})(-1\frac{1}{2})(-2\frac{1}{2})\cdots(-99\frac{1}{2})}{(\frac{1}{2})(-\frac{1}{2})(-1\frac{1}{2})\cdots(-98\frac{1}{2})} = \frac{-99\frac{1}{2}}{\frac{1}{2}} = -199$.

Ans. –199

9. The base of the triangle made along the ground is $15\sqrt{3}$. All the horizontal stay wires are parallel to this and thus make similar triangles, and a progression which is:

$15\sqrt{3}\left(\frac{29}{30} + \frac{28}{30} + \frac{27}{30} + \cdots + \frac{1}{30}\right) = 15\sqrt{3}\left(\frac{1}{30}\right)\left(\frac{29(30)}{2}\right) = \frac{435}{2}\sqrt{3}$

Ans. $\frac{435\sqrt{3}}{2}$ or 376.72

Answer Sheet

Arithmetic with Ratio and Proportion

1. 880
2. 32 ft. 1 in.
3. 1280 ft.

Series and Sequences

1. 42
2. $1586
3. 4, 20, 100 or 100, 20, 4

   (Both answers must be given)

Counting Principles and Binomial Theorem

1. 720
2. 84
3. 400

Polynomials

1. 10
2. a = 11, b = -15
3. –1, 3, 2, -2

Areas and Volumes

1. 18
2. 9.425
3. $\frac{4}{9}\pi + \frac{23\sqrt{3}}{3}$ or 14.68

Team

1. 30 or 30 students
2. 225
3. -2
4. 50
5. 60
6. $1 - x^{1/8}$ or $1 - \sqrt[8]{x}$
7. $59049\sqrt{3}$
8. -199
9. $\frac{435\sqrt{3}}{2}$ or 376.72