Meet #2 Nov. 2004

Solutions – Arithmetic with Ratio and Proportion

1. $\frac{\$30}{4m^2} = \frac{x}{3m^2}$ ➔ $90 = 4x$ ➔ $x = 22.5$ Ans. \$22.50

2. $y = ax^2z^3$. Changes: $a(2x)^2\left(\frac{z}{2}\right)^3 = \frac{ax^2z^3}{2}$ Ans. y is halved

3. D = RT ➔ to: D = 60T, back: D = 90(2-T). 60T = 180-90T ➔ 150T = 180. T = 6/5
60(6/5) = 72. Ans. 72

Series and Sequences

1. The sequence 5, 11, 23, 47, etc has 6,12,24 as a difference between terms, thus 47 + 48 = 95, and 95 + 96 = 191. Ans. 191

2. The rows have the sequence 1, 3, 5, so for the 10th row 1 + 9(2) = 19. The sum is $S = 10\left(\frac{1+19}{2}\right) = 5(20) = 100$. Ans. 100

3. The first time it rebounds it covers 81(2/3) both up and down = 54 ft. The next time it does the same. The sum of the down distances is $S = \frac{81}{1-\frac{2}{3}} = 243$. The sum of the up distances is $S = \frac{54}{1-\frac{2}{3}} = 162$. 243 + 162 = 405 Ans. 405 or 405 ft

Counting Principles and Binomial Theorem

1. $\binom{15}{10}(r^2)^{10}(-t^3)^5$ ➔ $-3003r^{20}t^{15}$ Ans. $-3003r^{20}t^{15}$

2. There are 5 possibilities for apples: (0,1,2,3,4). There are 4 for the pears. And 5 for the oranges. So there are 5(4)5 different baskets, but 0,0,0 cannot be one of the possibilities. There are 100 – 1 = 99 different possibilities. Ans. 99

3. If they win in 4 games AAAA = 1. In 5 games : $\binom{4}{1} = 4$. In 6 games : $\frac{5!}{3!2!} = 10$.
In 7 games : $\frac{6!}{3!3!} = 20$. 20 + 10 + 4 + 1 = 35. Ans. 35

Polynomials

1. Dividing Synthetically yields

	1	0	0	0	0	-32
2		2	4	8	16	32
	1	2	4	8	16	0

Ans. $a^4 + 2a^3 + 4a^2 + 8a + 16$

2. $(x + y)^2 - (x^2 + y^2) = 40 \Rightarrow x^2 + 2xy + y^2 - x^2 - y^2 = 40 \Rightarrow 2xy = 20$ Ans. 20

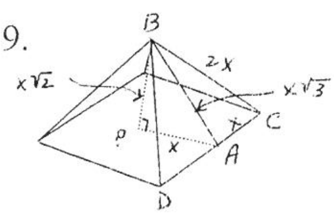
3. The roots are $\frac{1 \pm \sqrt{4 - 4n}}{2} = 1 \pm \sqrt{1 - n}$. So $1 + \sqrt{1-n} = (1 - \sqrt{1-n})^2 = 1 - 2\sqrt{1-n} + 1 - n$
or $3\sqrt{1-n} = 1 - n \Rightarrow$ Squaring: $9(1 - n) = 1 - 2n + n^2 \Rightarrow 0 = n^2 + 7n - 8 = (n+8)(n-1)$
Ans. –8 or 1

Areas and Volumes

1. $x(4x) = 16 \Rightarrow 4x^2 = 16 \Rightarrow x = \pm 2$. Rectangle sides are 2 and 8. Perimerter = Ans. 20

2. V(box) = 8(8)4 = 256. Vol(4 balls) = $4(4/3)\pi(2)^3 = 128/3\ \pi = 121.9587$ Ans. 121.96

3. Δ BDC is equilateral. Its area is $\frac{1}{2}(2x)x\sqrt{3} = 81\sqrt{3}$. Thus x = 9.
In Δ APB, PA = 9. So $PB^2 = (9\sqrt{3})^2 - 9^2 = 243 - 81 = 162$.
So $PB = 9\sqrt{2}$. The volume = $(1/3)(9\sqrt{2})(18)^2 = 972\sqrt{2}$
Ans. $972\sqrt{2}$ or 1374.66



Team

1. The words can be any permutation or all 3 letters the same: $3^3 = 27$ Ans. 27

2. The roots of $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The roots of $cx^2 + bx + a$ are
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$. The ratio of the first to the second is $\frac{1/2a}{1/2c}$ = c/a Ans. c/a

3. Quadrilateral GEFC is ¼ of the square. So ¼ (121) = 30.25 Ans. 30.25

4. Squaring $(1 + 2x - x^2)$ synthetically:

		1	2	-1
		1	2	-1
		-1	-2	1
	2	4	-2	
1	2	-1		
1	4	2	-4	1

Squaring this for the desired coefficient:

1	4	2	-4	1
1	4	2	-4	1
			-4	1
			-4	
			-8	1

Ans. –8

5. Solving the quadratic: $\frac{-2h \pm \sqrt{4h^2 - 4(-3)}}{2}$ = -h $\pm \sqrt{h^2+3}$. The sum of the squares:

$(-h + \sqrt{h^2+3})^2 + (-h - \sqrt{h^2+3})^2 = 10$ ➔

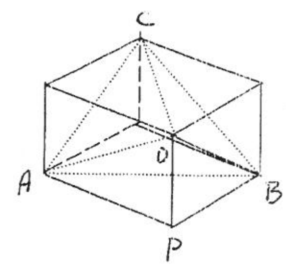
$h^2 - 2h\sqrt{h^2+3} + h^2 + 3 + h^2 + 2h\sqrt{h^2+3} + h^2 + 3 = 10$ ➔ $4h^2 = 4, h = \pm 1$ Ans. ± 1

6. There is much busy work on this one, but this is a team problem, good practice for organization:

3p = .03	n,2d = .25
3n = .15	n,2q = .55
3d = .30	2n,d = .20
3q = .75	2n,q = .35
2p,n = .07	d,2q = .60
2p,d = .12	2d,q = .45
2p,q = .27	n,d,q = .40
p,2n = .11	p,n,d = .16
p,2d = .21	p,n,q = .31
p,2q = .51	p,d,q = .36

Total is $6.15 Ans. $6.15

7. The volume of pyramid APBD = $(1/3)(1/2\ a^2)(a) = a^3/6$
Since there are 4 of these around the tetrahedron their total volume Is $2/3\ a^3$. Since the volume of the cube is a^3, then the volume of the tetrahedron is $1/3\ a^3$. This is the simplest way to get the answer. You can go through the volume of the tetrahedron and produce the same result. Ans. $1/3\ a^3$



8. Let P = The number of people on the work squad. We need $\binom{P}{2} \geq 180$ ➔

$P(P-1) \geq 360$ ➔ $P^2 - P - 360 \geq 0$ ➔ By inspection the minimum P is 20. Ans. 20

9. Arithmetic sequence: a, a+d, a+2d. The sum 3a + 3d = 39 or a + d = 13.
Geometric sequence: a+3, a+d+2, a+2d+21.

$\frac{a+d+2}{a+3} = \frac{a+2d+21}{a+d+2}$ ➔ $(a+d+2)^2 = (a+3)(a+2d+21)$ ➔

$a^2 + 2ad + 4a + 4d + d^2 + 4 = a^2 + 2ad + 24a + 6d + 63$ ➔ $d^2 - 2d - 20a - 59 = 0$

Since a = 13 – d: $d^2 - 2d - 20(13-d) - 59 = 0$ ➔ $d^2 + 18d - 319 = 0$ ➔ $(d+29)(d-11) = 0$

Ans. d = 11 or -29

Answer Sheet

Arithmetic with Ratio and Proportion

1. $22.50
2. y is halved
3. 72

Series and Sequences

1. 191
2. 100
3. 405

Counting Principles and Binomial Theorem

1. $-3003r^{20}t^{15}$
2. 99
3. 35

Polynomials

1. $a^4 + 2a^3 + 4a^2 + 8a + 16$
2. 20
3. -8 or 1

Areas and Volumes

1. 20
2. 121.96
3. $972\sqrt{2}$ or 1374.62

Team

1. 27
2. 30.25
3. -8
4. 9/16
5. ± 1
6. $6.15
7. $a^3/3$
8. 20 or 20 people
9. 11 or -29

Meet # 2 Nov. 2004

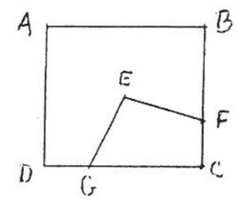
IV Team (You may use Calculators)

3 pts 1. If the alphabet for a certain language had only 3 letters, how many different 3-letter words could be formed?

Ans. ____________________

3 pts 2. The area of square ABCD is 121 cm^2.
E is the center of the square and the $m\angle GEF = 90^\circ$.
Find the area of quadrilateral GEFC.

Ans. ______________



3 pts 3. Find the coefficient of the term containing x^7, when $(1 + 2x - x^2)^4$ is expanded. **Ans.** ____________________

4 pts 4. If an arc of 60° on circle I has the same length as an arc of 45° on circle II, find the ratio of the area of circle I to that of circle II.

Ans. ____________________

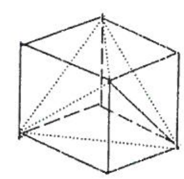
4 pts 5. The sum of the squares of the roots of the equation $x^2 + 2hx = 3$ is 10. Find all values of h.

Ans. ____________________

4 pts 6. Find the total value of all combinations of three coins you can make using only pennies, nickels, dimes, and quarters. (One could be 3 pennies ➔ 3 cents) **Ans.** ____________________

5 pts 7. Four of the vertices of a cube are vertices of a regular tetrahedron. If an edge of the cube is *a*, find the volume of the tetrahedron in terms of *a*. Express in simplest form.

Ans. ____________________



5 pts 8. A work squad must have a two-person safety team designated from the squad members each work day. There are enough people on the squad so that a unique team can be designated each day for the 180 day duration of the project, but if there were one fewer person, it would not be possible. How many people are on the work squad?

Ans. ____________________

5 pts 9. If 3 numbers which are successive terms in an arithmetic progression are increased by 3, 2, and 21, respectively, the resulting numbers are in geometric progression. Find the difference between successive terms of the original numbers, if their sum is equal to 39. **Ans.** ____________________