Meet #2

Solutions – Arithmetic with Ratio and Proportion Nov 06~~A~~07

1.

	A	5	3
	x	2	3
	(3A + 1)	5	9
(2A + 1)	0	6	
	(3A + 2)	1	9

3A + 2 has unit's digit of 3, and A ≤ 9.

Only 3A + 2 = 23 is possible, and A = 7.

Ans. 7

2. $\frac{3x+2y}{11x-4y}=\frac{3}{4}$ ➔ 12x + 8y = 33x - 12y ➔ 20y = 21x ➔ $\frac{x}{y}=\frac{20}{21}$ **Ans.** $\frac{20}{21}$

3. The constant k as a result of the linked proportions is $\frac{(SL)L}{b\cdot d^2}=k$. Filling in:

$\frac{1800\cdot 8}{6\cdot 4^2}=\frac{(SL)\cdot 15}{3\cdot 8^2}$ ➔ (SL) = $\frac{1800\cdot 8}{6\cdot 4^2}\cdot\frac{3\cdot 8^2}{15}$ = (SL) = 1920. **Ans. 1920**

Series and Sequences

1. 7(1 + 2 + 3 +...+ 10) = 7(55) = 385. **Ans. 385**

2. S = $\frac{N(a+l)}{2}$, l = 40,000 + 600(19) = 51,400. Thus the sum is:

S = $\frac{20(40{,}000+51{,}400)}{2}$ = 91,400. **Ans. $914,000**

3. $\frac{x+2}{x}=\frac{x+5}{x+2}$ ➔ $x^2+4x+4=x^2+5x$ ➔ x = 4. Thus $\frac{60x+3}{x+4}=\frac{243}{8}$, and the common ratio between terms is $\frac{x+2}{x}=\frac{6}{4}=\frac{3}{2}$. $4(\frac{3}{2})^{n-1}=\frac{243}{8}$ ➔ $(\frac{3}{2})^{n-1}=\frac{243}{32}=(\frac{3}{2})^5$. **Ans. 6**

Counting Principles and Binomial Theorem

1. Possible persons from first to last: $4\cdot 4\cdot 3\cdot 2\cdot 1=96$ **Ans. 96**

2. Need to find the minimum n such that $\binom{n}{2}\geq 180$ ➔ $\frac{n(n-1)}{2}\geq 180$

n(n – 1) ≥360. 20(19) = 380, 19(18) = 342. Clearly 20 is minimum. **Ans. 20**

3. $(a+b+c)^5=[(a+b)+c]^5$. The third term in the expansion contains all the ultimate terms where c^2 is a factor: $\binom{5}{2}(a+b)^3c^2=c^2(10)(a^3+3a^2b+3ab^2+b^3)=10(8)$ **Ans. 80**

Polynomials

1. Since the leading coefficient is 1 and all other coefficients and the constant are integers, all solutions must be factors of the constant -48. Greatest possible is 48. **Ans. 48**

2. $-144x^4 + 145x^2 - 36 = 0 \Rightarrow 144x^4 - 145x^2 + 36 = 0 \Rightarrow (16x^2 - 9)(9x^2 - 4) = 0 \Rightarrow$ $(4x-3)(4x+3)(3x-2)(3x+2) = 0$. $x = \pm\frac{3}{4}, \pm\frac{2}{3}$ **Ans.** $\pm\frac{3}{4}$ **or** $\pm\frac{2}{3}$

3. Substituting in points are (-1,0), (1,2) and (3,8). Equations from quad. Equation:

$a - b + c = 0$ From the first two, b = 1. From all three a = c = ½.
$a + b + c = 2$ abc = (1)(1/2)(1/2) = ¼.
$9a + 3b + c = 8$

Ans. 1/4

Areas and Volumes

1. $\pi r^2 + 2\pi r^2 = 3\pi r^2 = 18.75\pi$. $r^2 = 6.24$, so r = 2.5. **Ans. 2.5**

2. The square has area 36. The triangle has area $3(3\sqrt{3}) = 9\sqrt{3}$. The area of the semicircle is $(½)\pi(9) = 4½\pi$. Adding the three: 65.7256. **Ans. 65.73**

3. Since the two pyramids are similar, the ratio of their volumes is the cube of the ratio of their sides. The sides are in a ratio of 1 to 2, so their volumes are 1 to 8. Let the side of the base of the large pyramid be x, then the diagonal of the base is $x\sqrt{2}$. The altitude of the pyramid hits the center of the base, and the right triangle formed by the lateral edge has legs of 6 and $x/\sqrt{2}$ and the hypotenuse is x. Thus: $x^2 = ½x^2 + 36 \Rightarrow x^2 = 72$ or the sides of the square are $6\sqrt{2}$. The volume of the large pyramid = $\frac{1}{3}(6\sqrt{2})^2 6 = 144$. The area of the small pyramid is $\frac{1}{8}(144) = 18$. **Ans. 18**

Team

1. Continuing the pattern, 16/81 + 32/243 **Ans.** $\frac{32}{243}$

2. When $b^2 - 4ac = 0$, there are two equal roots. $9 - 4(2)k = 0 \Rightarrow k = \frac{9}{8}$. **Ans.** $\frac{9}{8}$

3. $10^3 + 3(10)^2(\frac{1}{8}) = 1000 + 300/8 = 1037.5$. **Ans. 1037.5**

4. The number of arrangements is $\frac{11!}{4!4!3!} = 11{,}550$ **Ans. 11,550**

5. Using RT = D: 7.5mi/hr(.25)sec = D $\Rightarrow$ D = $\frac{75mi}{10hr} \cdot \frac{1\sec}{4} \cdot \frac{1\min}{60\sec} \cdot \frac{1hr}{60\min} \cdot \frac{5280ft}{1mi} \cdot \frac{12in}{1ft}$

This equals 33 inches.

Ans. 33 in.

6. To insert m numbers creates m + 1 differences between 5 and 45, so 5 + (m + 1)d = 45.

$(m + 1)d = 40 \Rightarrow d = \frac{40}{m+1}$. **Ans. d =** $\frac{40}{m+1}$

7. Let n = the row number. Then $\binom{n}{2} = 136 \Rightarrow n^2 - n - 272 = 0 \Rightarrow (n - 17)(n + 16) = 0$.

n = 17, so th sum of the coefficients on that row is $2^{17} = 131{,}072$. **Ans. 131,072**

8. $A + C = 344$, and $A + B = \frac{7}{8}C = \frac{7}{8}(344 - A) = 301 - \frac{7}{8}A$ or (1) $B = 301 - \frac{15}{8}A$ and A has o be a multiple of 8. Using (1): if A = 80, B = 151, If A = 160, B = 1. A has to be

closer to 80. If A = 96, B = 121; if A = 104, B = 106; if A = 112, B = 91. **Ans. 2**

9. The series is of the form $\frac{1}{(a)(a+2)} = \frac{4}{(2a)(2a+4)} = \frac{(2a+4) - 2a}{(2a)(2a+4)} = \frac{1}{2a} - \frac{1}{2a+4}$.

So, the series equals $\left(\frac{1}{2} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{14}\right) + \ldots + \left(\frac{1}{34} - \frac{1}{38}\right) + \left(\frac{1}{38} - \frac{1}{42}\right) = \frac{20}{42}$ **Ans.** $\frac{10}{21}$

Answer Sheet

Arithmetic with Ratio and Proportion

1. 7 or $A = 7$
2. $\frac{20}{21}$
3. 1920

Series and Sequences

1. 385
2. $914,000
3. 6

Counting Principles and Binomial Theorem

1. 96
2. 20
3. 80

Polynomials

1. 48
2. $\pm\frac{3}{4}$ or $\pm\frac{2}{3}$
3. $\frac{1}{4}$

Areas and Volumes

1. 2.5
2. 65.73
3. 18

Team

1. $\frac{32}{243}$
2. $\frac{9}{8}$ or $1\frac{1}{8}$
3. 1037.5
4. 11,550
5. 33 or 33 inches
6. $\frac{40}{m+1}$
7. 131,072
8. 2
9. $\frac{10}{21}$