Meet #2

Solutions – Arithmetic with Ratio and Proportion Nov 08

1. Brent has 3/10 of the money. 3/10 of \$5.00 is \$1.50. **Ans. \$1.50**

2. $8.70 - x = \frac{15}{17}(5.70 + x)$ ➔ $17(8.70 - x) = 15(5.70 + x)$ $147.90 - 17x = 85.50 + 15x$
$62.40 = 32x$. Thus $x = 1.95$. **Ans. \$1.95**

3. The proportion is $\frac{mp^2}{h^3}$. Thus $\frac{45 \cdot 20^2}{5^3} = \frac{54 \cdot 24^2}{h^3}$. $h^3 = \frac{54 \cdot 24^2 \cdot 5^3}{45 \cdot 20^2} = \frac{2 \cdot 27 \cdot 3 \cdot 8 \cdot 3 \cdot 8 \cdot 5^3}{3 \cdot 3 \cdot 5 \cdot 4 \cdot 5 \cdot 4 \cdot 5} =$
$27 \cdot 2 \cdot 2 \cdot 2$. Thus $h = 6$. **Ans. 6**

Sequence and Series

1. Since two common differences were added from the 57th to the 59th term, and the difference between them is 16, 8 is the common difference. 60th term = 265. **Ans. 265**

2. 2nd term: $ar = 324$. 5th term: $ar^4 = 96$. $\frac{ar^4}{ar} = \frac{96}{324}$ ➔ $r^3 = \frac{8}{27}$ ➔ $r = 2/3$.
$324(3/2) = 162(3) = 486$. **Ans. 486**

3. Every third term starting with the first is a multiple of 7. Each term succeeding each of these is 2 more than a multiple of seven, and the term succeeding each of these is 1 less than a multiple of 7. The 77th multiple of 7 is 539. 77/3 yields a remainder of 2. The second term in the sequence is 2 more than the multiple of 7, thus 541. **Ans. 541**

Counting Principles and Binomial Theorem

1. $\binom{6}{3}\left(\frac{1}{3}x\right)^3 (9y)^3 = 20\left(\frac{x^3}{27}\right)(9^3)y^3 = 540x^3y^3$ **Ans. $540x^3y^3$**

2. The first girl has 8 boys to choose from. The second has only 7. Thus 8! **Ans. 40,320**

3. ${}_5P_1 + {}_5P_2 + {}_5P_3 + {}_5P_4 + {}_5P_5 = 5 + 20 + 60 + 120 + 120 = 325$. **Ans. 325**

Polynomials

1. The terms containing an x^2 of $(3x^2 - 7x + 5)(4x^2 + 8x - 6)$ would come from $-6(3x^2) + (-7x)(8x) + 5(4x^2) = -54x^2$. Coefficient = - 54. **Ans. -54**

2. $R(x) = \frac{15x^2+5x-13}{5x-3} - \frac{4x-7}{5x-3} = \frac{15x^2+x-6}{5x-3} = \frac{(5x-3)(3x+2)}{5x-3} = 3x+2$ **Ans. 3x + 2**

3. Synthetically:

$$\begin{array}{r|rrrrr} -2 & 6 & -1 & -84 & -61 & 30 \\ & & -12 & 46 & 76 & -30 \\ \hline & 6 & -23 & -38 & 15 & 0 \end{array} \qquad \begin{array}{r|rrrr} 5 & 6 & -23 & -38 & 15 \\ & & 30 & 35 & -15 \\ \hline & 6 & 7 & -3 & 0 \end{array}$$

The integral roots are -2 and 5. The fractional roots are the solution to the equation:
$6x^2 + 7x - 3 = 0$ ➔ $(3x-1)(2x+3) = 0$ ➔ 1/3 and -1½. **Ans. 1/3, -1½**

Areas and Volumes

1. One of the smallest triangles has vertices (0, 0), (0, 2) and (2, 0). Area is ½ (2)(2) = 2. The largest triangle has vertices (0, 0), (4, 0) and (0, 5). Area is ½ (4)(5) = 10. **Ans. 8**

2. Volume of earth $= 4/3\ \pi(4000)^3$. ½ the volume is $2/3\ \pi 4000^3 = 4/3\ \pi r^3$. $r = \frac{4000}{\sqrt[3]{2}}$.
r = 3174.8021, rounded to nearest mile: 3175. **Ans. 3175**

3. Let s = side length. Then area $= \frac{\sqrt{3}}{4}s^2$ and perimeter is $3s$.

$\frac{\frac{\sqrt{3}}{4}(s+1)^2}{3(s+1)} = \frac{11}{10}\frac{\frac{\sqrt{3}}{4}s^2}{3s}$ ➔ $s + 1 = \frac{11}{10}s$ ➔ $s = 10$. Area $= \frac{\sqrt{3}}{4}\cdot 10^2 = 25\sqrt{3}$. **Ans. 25**

Team

1. $\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$. These added to $\frac{1}{2}+\frac{1}{4}=1$. $\frac{1}{8}$ and $\frac{1}{10}$ must be removed. **Ans. $\frac{1}{8}$ and $\frac{1}{10}$**

2. $\pi r^2 = 25\pi$, so r = 5. $90\pi = 5\pi(5 + \sqrt{25+h^2})$ ➔ $18 = 5 + \sqrt{25+h^2}$ **Ans. h = 12**

3. g meters = f jumps ➔ 1 meter $= \frac{f jumps}{g}$. D jumps = e hops ➔ 1 jump $= \frac{e hops}{d}$. Thus
1 meter $= \frac{ef hops}{gd}$. B hops = c skips ➔ 1 hop $= \frac{c skips}{b}$. Thus 1 meter $= \frac{cef skips}{bgd}$. **Ans. $\frac{cef}{bdg}$**

4. $\frac{10!}{4!3!2!} = 12{,}600$ **Ans. 12,600**

5. Using the calculator and the y^x button, where y = .95 and a value of x that produces at least .5, x = 12 => .54+ ; x = 13 => .51+; x = 14 => .48+. **Ans. 13**

6. We are painting a surface area. Since the heights are in a ratio of 6 to 1, their surface areas are in a ratio of 36 to 1. 1/36 of a pint would cover 1 copy. $\frac{1}{36}\cdot 540 = 15$. **Ans. 15**

7. Because p(x) is a factor of $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, it is also a factor of $3(x^4 + 6x^2 + 25) - 3x^4 + 4x^2 + 28x + 5$, which equals $14x^2 - 28x + 70$, or $14(x^2 - 2x + 5)$. Therefore, $p(x) = x^2 - 2x + 5$, and $p(1) = 4$. **Ans. 4**

8. Let R = the number of completed rounds. Then: Total distance east = 10R. The total distance south = $\frac{(R-1)R}{2}$. So, $10R = \frac{(R-1)R}{2}$ → $20R = R^2 - R$ → $R^2 - 21R = 0$. **Ans. 21**

9. If all are placed in one mailbox there are **8 ways**. If placed in two mailboxes: ${}_8C_2$ times the number of ways the 4 different pieces could end up in the two boxes, which is 4 + 4, if 1 was in one mailbox and 3 in another or visa versa, which makes 28(8) = **224 ways**; or if two were placed in one mailbox and two in another which is ${}_4C_2 = 6$, thus making 28(6) = **168 ways**. If placed in three mailboxes: ${}_8C_3$ times the number of ways that the three mailboxes could get 2 in one of them and 1 in each of the other two = ${}_4C_2\cdot 2\cdot 3$, which is 36, thus making 56(36) = **2016 ways**; or if each piece were placed in one mailbox: ${}_8C_4$ multiplied by the number of different ways that the 4 pieces could end up in the 4 mailboxes = 4!, thus making 70(24) = **1680 ways**. (This last one could also be accomplished by ${}_8P_4$). The sum is 4096 ways.

Alternate Solution: The mailman has 8 possible choices to put the first piece of junk mail into. He then has 8 possible choices to put the next piece into, etc. Thus $8\cdot 8\cdot 8\cdot 8 = 4096$.

Ans. 4096

Answer Sheet Nov 07

Arithmetic with Ratio and Proportion

1. \$1.50 or 1.50
2. \$1.95 or 1.95
3. 6 or h = 6

Series and Sequences

1. 265
2. 486
3. 541

Counting Principles and Binomial Theorem

1. $540x^3y^3$
2. 40, 320
3. 325

Polynomials

1. -54
2. $3x + 2$
3. 1/3, -1½

Areas and Volumes

1. 8
2. 3175 or 3175 mi.
3. 25

Team

1. $\frac{1}{8}$ and $\frac{1}{10}$
2. 12 or h = 12
3. $\frac{cef}{bdg}$ or $\frac{cef}{bdg}$ skips
4. 12,600
5. 13 or 13 bounces
6. 15 or 15 pints
7. 4
8. 21 or 21 rounds
9. 6112 or 6112 ways