Meet #2

Solutions – Arithmetic with Ratio and Proportion Nov 09

1. $\frac{girls}{boys} = \frac{3}{4} = \frac{x}{28-x}$ ➔ $84 - 3x = 4x$ ➔ $84 = 7x$, thus $x = 12$. 12 girls, 16 boys. **Ans. 4**

2. Numbers left: 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49. **Ans. 14**

3. Hunted: $3020(.90) = 2718$. Natural causes: $29000(.3) = 8700$. Adding: **Ans. 11, 418**

Series and Sequences

1. $a_{17} = -3 + (-3/2)16 = -3 - 24 = -27$. **Ans. -27**

2. $(1)(3/4)^4 = 81/256$. **Ans. 81/256**

3. There are 3 sequences here: 1, 2, 4, 8, … 1, 3, 9, … and 2, 5, 8, … . The 17th term is the 6th term of the second sequence: 243. **Ans. 243**

Counting Principles and Binomial Theorem

1. There 8 letters, 3 s's, and 2 e's. Thus $\frac{8!}{3!2!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{2\cdot 3!} = 4\cdot 7\cdot 6\cdot 5\cdot 4$ **Ans. 3360**

2. $\binom{7}{4}(2x)^3\left(\frac{1}{4}y\right)^4 = 35(8)x^3(\frac{1}{4})^4y^4 = \frac{35}{32}x^3y^4$. **Ans.** $\frac{35}{32}x^3y^4$

3. ${}_4C_3\cdot{}_4C_2\cdot{}_{13}P_2 = 4(6)(13)(12) = 144(26) = 3744$. **Ans. 3,744**

Polynomials

1. Multiplying $(x^3+4x^2-1)(x^2+5x-6)$ synthetically:

1	4	0	-1		
		1	5	-6	
	-6	-24	0	6	
5	20	0	-5		
1	4	0	-1		
1	9	14	-25	-5	6

Ans. $x^5+9x^4+14x^3-25x^2-5x+6$

2. Dividing synthetically:

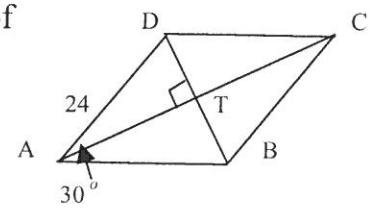
-1 |	1	3	-1	0	0	1
		-1	-2	3	-3	3
	1	2	-3	3	-3	4

Ans. 4

3. In $f(x) = x^6 - 5x - 1$, there is only 1 change of signs, so there is only 1 positive real zero for f. $f(-x) = x^6 + 5x - 1$, there is only 1 change of signs, so there is only 1 negative real zero for f. Since the polynomial is a positive x^6, then it has a parabolic shape and it only has one interval that it crosses the x-axis. In that interval f(x) has to be negative. Since f(0) = -1, then the zeros have to be around 0. Dividing 1 0 0 0 0 5 -1 synthetically, by 1 makes -5, by 2 makes 53. So there is a solution between 1 and 2. Synthetically -1 makes 5. So the function crosses the axis between 0 and -1. **Ans. a = -1,b = 2**

Areas and Volumes

1. $m\angle ADC = 120$, $\overline{BD}$ and $\overline{AC}$ are perpendicular bisectors of each other. $m\angle DAT = 30$. Since the perimeter is 96, then AD = 24, DT = 12 and $AT = 12\sqrt{3}$. The area of ΔADT is $\frac{1}{2}(12)(12\sqrt{3}) = 72\sqrt{3}$. Therefore the area of the rhombus is $4(72\sqrt{3}) = 288\sqrt{3} = 498.8306$.



Ans. $288\sqrt{3}$ or 498.8306

2. Area of base = 12(9) = 108. Area of cross-section $\left(\frac{5}{15}\right)^2(108) = 12$. **Ans. 12**

3. By symmetry, the four regions lying outside the circle and inside the square are equal to the four regions lying outside the square and inside the circle. Therefore the square and circle have the same area. $s^2 = \pi r^2 \rightarrow \frac{s^2}{r^2} = \frac{\pi}{1}$. Thus $\frac{s}{r} = \frac{\sqrt{\pi}}{1}$. **Ans. $\sqrt{\pi}$:1**

Team

1. $\binom{9}{3}\left(\frac{2}{x^2}\right)^6\left(\frac{-x^4}{4}\right)^3 = -84$ **Ans. -84**

2. All possible members: ${}_9C_3 = 84$. All men: ${}_5C_3 = 10$. All women: ${}_4C_3 = 4$. **Ans. 70**

3. $LW = 2L + 2W$. $W = \frac{2L}{L-2}$ or $L = \frac{2W}{W-2}$. Therefore $W > 2$ and $L > 2$. Simple substitution yields 3 by 6 and 4 by 4. 5 doesn't work 6 repeats, and the rest also do not work. **Ans. 3 by 6 and 4 by 4**

4. Let L = # of Latin exams, G = # of Greek exams. $88(L + G) = 80G + 92L$.
$88L + 88G = 80G + 92L \rightarrow 8G = 4L \rightarrow \frac{G}{L} = \frac{4}{8}$. **Ans. 1:2**

5. Let r = the radius of the inscribed sphere and x = the radius of the circumscribed sphere. A right triangle is made by r, and x and ½ the diagonal of a face of the cube. The diagonal $= r\sqrt{2}$, and x is the hypotenuse of the right triangle. Thus $x^2 = r^2 + (r\sqrt{2})^2 = 3r^2$ and $x = r\sqrt{3}$. The ratio of their volumes is $\frac{\frac{4}{3}\pi\left(r\sqrt{3}\right)^3}{\frac{4}{3}\pi r^3} = \frac{3\sqrt{3}}{1}$. **Ans: $3\sqrt{3}$:1**

6. Each row ends with a perfect square of the row number. So row 44 ends with 1936 and row 45 ends with 2025 leaving 2009 in row 45. **Ans. 45th row**

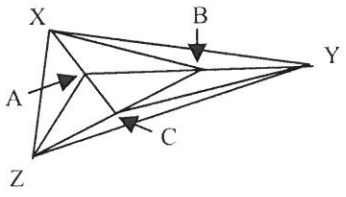
7. There are 5! ways to order the digits 1, 2, 3, 4, 5. So there are 5! 5-digit numbers. Then there are 5 other digits to make a 6-digit number 5(6!), but 0 cannot be the first digit, if it were, there would be 5! numbers with 0 as the first digit. Therefore the total number of numbers is 5! + 5(6!) – 5! = 5(6!) = 5(720) = 3600. **Ans. 3600**

8. Since f(x) is a polynomial and $f(x^2+1)$ has a degree 4, then f(x) has degree 2. That is $f(x) = ax^2 + bx + c$ for some constants a, b and c.

$$f(x^2+1) = x^4 + 5x^2 + 3 = a(x^2+1)^2 + b(x^2+1) + c$$
$$= a(x^4 + 2x^2 + 1) + bx^2 + b + c$$
$$= ax^4 + (2a+b)x^2 + a + b + c$$

Thus a = 1. 2a + b = 5, so b = 3. a + b + c = 3, so c = -1. $f(x) = x^2 + 3x - 1$.
$F(x^2 - 1) = (x^2 - 1)^2 + 3(x^2 - 1) - 1 = x^4 - 2x^2 + 1 + 3x^2 - 3 - 1$. **Ans. $x^4 + x^2 - 3$**

9. Draw segments BX, AZ, and CY as shown.
ΔABX's area = area of ΔABC since they have the same height and equal length bases. Likewise triangles ABX and BYX have equal areas. All 3 triangles have same area. Using th same thought process around the figure, all 7 small triangles have the same area. The area of ΔXYZ is 7 cm^2.



Ans. 7 cm^2

Answer Sheet

Arithmetic with Ratio and Proportion

1. 4
2. 14
3. 11,418 or 11,418 moose

Series and Sequences

1. -27
2. 81/256
3. 243

Counting Principles and Binomial Theorem

1. 3360
2. $\frac{35}{32}x^3y^4$
3. 3744

Polynomials

1. $x^5+9x^4+14x^3-25x^2-5x+6$
2. 4
3. $a = -1, b = 2$

Areas and Volumes

1. $288\sqrt{3}$ or 498.8306
2. 12 or 12 in^2
3. $\sqrt{\pi}$:1 or 1.7725:1

Team

1. -84
2. 70
3. 3 by 6 and 4 by 4
4. 1:2
5. $3\sqrt{3}$:1 or 5.1962:1
6. 45^{th} or 45^{th} row
7. 3600
8. x^4+x^2-3
9. 7 or 7cm^2

NOTES:

1. Polynomials # 3 – answer can also be given as an ordered pair, as long as the value for "a" is first. Thus, (-1, 2) is acceptable.
2. Ratios: Both Areas and Volumes # 3 and Team round # 5 have a ratio of the form "a : 1". The answer MUST include the one. An answer that is not a ratio MUST be marked wrong.
3. Units: Remember that units are NOT required, but if they are used, they must be correct.
4. Remember that in calculator rounds, students who give a decimal must give it to 4 decimal places, correctly rounded. Acceptable answers are listed above.