

**Solutions – Arithmetic with Ratio and Percent**

1.  $\frac{5}{3} = \frac{x}{72} \rightarrow 3x = 5(72) \rightarrow x = 5(24) = 120.$

**Ans. \$120**

2.  $10(136 - x) + 25x = 2440 \rightarrow 1360 - 10x + 25x = 2440 \rightarrow 15x = 1080$  thus  $x = 72.$

The number of dimes is  $136 - 72 = 64.$  value - \$6.40.

**Ans. \$6.40**

3.  $k = \frac{pq^3}{m^2}$ , thus  $\frac{4 \cdot 16^3}{8^2} = \frac{16 \cdot 4^3}{m^2} \rightarrow m^2 = \frac{16 \cdot 4^3 \cdot 8^2}{4 \cdot 16^3} \rightarrow m^2 = \frac{4^2 \cdot 8 \cdot 8}{16 \cdot 16} = \frac{4^2}{2 \cdot 2} = 4$

**Ans. m = 2**

**Series and Sequences**

1.  $13 + 10(7) = 83.$

**Ans. 83**

2.  $S = \frac{40(5 + 5 + 39 \cdot 5)}{2} = 20(5 \cdot 41) = 4100.$

**Ans. \$41.00**

3. Multiples of 6  $\rightarrow$  204 is the first, 300 is the last.  $\rightarrow 300 = 204 + (n - 1)6 \rightarrow 96 = (n - 1)6$   
 $n - 1 = 16$ , so 17 terms.  $Sum = \frac{17(204 + 300)}{2} = \frac{17(504)}{2} = 17(252) = 4284.$

Those which are multiples of 9 are multiples of 18, the LCM. The first is 216 and the last is 288.  $288 = 216 + (n - 1)18 \rightarrow 72 = (n - 1)18 \rightarrow 4 = n - 1$ , so  $n = 5.$

$Sum = \frac{5(216 + 288)}{2} = \frac{5(504)}{2} = 5(252) = 1260.$   $4284 - 1260 = 3024.$

**Ans. 3024**

**Counting Principles and Binomial Theorem**

1. Each time the M's are side-by-side (which are 3 places), the A and L can be switched, thus making 6. Like wise when the M's are not side-by side (3 places) there are 6 more.

Otherwise if students have prepared for the category:  $\frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2}{2} = 12.$

**Ans. 12**

2.  ${}_{10}C_3 = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120.$

**Ans. 120**

3. Setting up exponents for sum:  $(x^2)^5(y^3)^0 \Rightarrow 10$ ;  $(x^2)^4(y^3)^1 \Rightarrow 11$ ; it appears that the sum is increasing and the highest sums should be 14 and 15 resulting from the last two

terms:  $\binom{5}{4}(3x^2)^1(-5y^3)^4$  and  $\binom{5}{5}(3x^2)^0(-5y^3)^5$ . The 5<sup>th</sup> term's coefficient is  $5(3)(-5)^4 =$

$15(625) = 9375.$  The 6<sup>th</sup> term's coefficient is  $(-5)^5 = -3125.$

**Ans. 6250**

**Polynomials**

1.  $(x+2y)(2x-y) = 2x^2 + 3xy - 2y^2.$   $(2x^2 + 3xy - 2y^2)(x+y) =$   
 $2x^3 + 3x^2y - 2xy^2 + 2x^2y + 3xy^2 - 2y^3$

**Ans.  $2x^3 + 5x^2y + xy^2 - 2y^3$**

2.  $3x^3 + x^2 - 62x + 40 = 0$ . Solving by synthetic division:  $-5 \left| \begin{array}{cccc} 3 & 1 & -62 & 40 \\ & -15 & 70 & -40 \\ \hline 3 & -14 & 8 & 0 \end{array} \right.$

$3x^2 - 14x + 8 = 0 \rightarrow (3x - 2)(x - 4) = 0$

**Ans. -5, 4, 2/3**

3.  $7x^3 - 5x^2 + 3x + 4 \div \frac{3x^2 - 5x + 8}{21x^5 - 50x^4 + 90x^3 - 43x^2 + kx + p}$

$$\begin{array}{r} 21x^5 - 50x^4 + 90x^3 - 43x^2 + kx + p \\ \underline{21x^5 - 15x^4 + 9x^3 + 12x^2} \\ -35x^4 + 81x^3 - 55x^2 + kx \\ \underline{-35x^4 + 25x^3 - 15x^2 - 20x} \\ 56x^3 - 40x^2 + (k+20)x + p \\ \underline{56x^3 - 40x^2 + 24x + 32} \\ (k-4)x + p - 32 \end{array}$$

**Ans.  $k = 4, p = 32$**

### Areas and Volumes

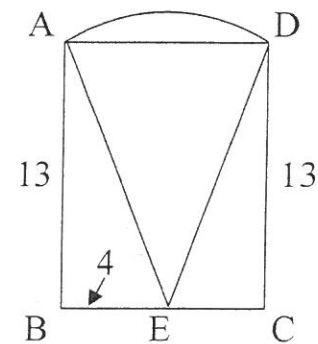
1. It would take 4 cubes to fill the bottom of one side, 16 cubes to fill one row 4 cubes high, and it will take 64 to fill all four rows of the base.

**Ans. 64**

2.  $\frac{375}{648} = \frac{125}{216} = \frac{5^3}{6^3}$ . The weight of the bears is in three dimensions. The height is in one dimension. So  $\frac{5}{6} = \frac{x}{42}$  or  $x = 35$ .

**Ans. 35 inches**

3. Let E be the midpoint of  $\overline{BC}$ . Draw  $\overline{AE}$  and  $\overline{DE}$ . Then  $m\angle AEB = m\angle DEC = \tan^{-1}(13/4)$  and  $m\angle AED = 180 - 2\tan^{-1}(13/4)$ . Sector AED then has radius  $\sqrt{4^2 + 13^2} = \sqrt{185}$  and area  $\frac{180 - 2\tan^{-1}(13/4)}{360}(185\pi)$  and the area of the figure is  $(2)(1/2)(4)(13) + \frac{180 - 2\tan^{-1}(13/4)}{360}(185\pi)$ .



Using calculator: 107.2223

**Ans. 107.2223**

### Team

1.  $37\frac{1}{2}\% = \frac{3}{8}$ .  $\frac{3}{8}x = 297 \rightarrow 3x = 8(297) \rightarrow x = 8(99) = 792$ .

**Ans. 792**

2.  $\frac{3}{28} + 37\frac{1}{2}\% + .42\% \rightarrow \frac{3}{28} + \frac{3}{8} + \frac{3}{7} = \frac{6}{56} + \frac{21}{56} + \frac{24}{56} = \frac{51}{56}$ .

**Ans. 51/56**

3. The number of circular arrangements of n people is  $(n - 1)!$ .  $5! = 120$ .

**Ans. 120**