

1 Arithmetic with Ratio and Proportion Nov 2012 (No Calculators)

3 pts 1. A large cylindrical tube is accurately marked from bottom to top in 8ths. When it was $\frac{1}{8}$ full, 15 gallons of liquid were added, making it now $\frac{3}{4}$ full. How many more gallons are needed to fill the tank?

Ans. _____

4 pts 2. Suppose that $\frac{x}{y} = \frac{4}{7}$ and that $\frac{y}{z} = \frac{14}{3}$. What is the numerical value of $\frac{x+y}{z}$?

Express your answer as a fraction in the form $\frac{a}{b}$.

Ans. _____

5 pts 3. In a class of 100 students, 38 students took calculus, 46 took physics and 55 took US History. 22 took physics and history, 16 took calculus and history, 9 took calculus and physics, and 4 students took all three courses. How many took none of the three subjects?

Ans. _____

2 Series and Sequences Nov 2012 (No Calculators)

3 pts 1. Find the sum of the series $(-300) + (-297) + (-294) + \dots + (306) + (309)$.

Ans. _____

4 pts 2. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

Ans. _____

5 pts 3. w , x , y , and z are consecutive terms of an arithmetic sequence with common difference of 2012. Find the following: $\frac{z^2 - w^2}{y^2 - x^2}$.

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2012 (No Calculators)

3 pts 1. How many distinguishable automobile license plates of 6 digits can be made, if the first digit (on the left) cannot be 0 (zero)?

Ans. _____

4 pts 2. Find the coefficient of the term in the expansion of $\left(4x^2 - \frac{1}{2x^3}\right)^8$ that contains x to a power of 1.

Ans. _____

5 pts 3. Three fair, standard six-faced dice of different colors are rolled. In how many ways can the dice show a sum of 10 on the top faces?

Ans. _____

4 Polynomials Nov 2012 (No Calculators)

3 pts 1. Find the solutions of $2x^2 + 3x - 5 = 0$.

Ans. _____

4 pts 2. Let $P(x) = kx^3 + 2k^2x^2 + k^3$. Find the sum of all real numbers k for which $x - 2$ is a factor of $P(x)$.

Ans. _____

5 pts 3. If $P(x) = 3x^3 + x^2 - 62x + 40$, find all values of x such that $P(x) = 0$.

Ans. _____

5 Areas and Volumes Nov 2012 (You may use calculators)

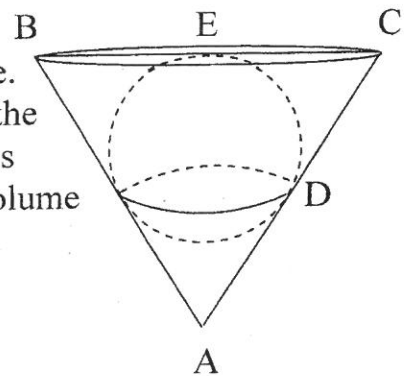
3 pts 1. A slanted pentagonal pyramid has a base area of 32 cm^2 . If it stands 3 cm high, what is its volume?

Ans. _____

4 pts 2. The area of a cross-section of a pyramid is 36 cm^2 . The cross-section is 4 cm from the base and the height of the pyramid is 16 cm. What is the volume of the pyramid?

Ans. _____

5 pts 3. \overline{BC} is the diameter of the base of the right circular cone. The measure of angle BAC is 60° . A metallic ball is placed in the cone tangent to the lateral surface such that $AD = 20$. The ball is also tangent to the base of the cone at point E . Find the exact volume of the cone or round the volume to the nearest 100^{th} .



Ans. _____

6 Team Nov 2012 (You may use calculators)

3 pts 1. An ancient manuscript contained this statement “ $x^2 + bx + 30$ has two integral roots”. Unfortunately, it is impossible to read the positive integer b . How many possibilities are there for b ?

(1)Ans. _____ 3pts

3 pts 2. The 11th term of an arithmetic sequence is 111. The 23rd term is 195. What is the sum of the first 60 terms of the sequence?

(2)Ans. _____ 3pts

3 pts 3. It takes Mandy 56 puffs to increase the diameter of a spherical beach ball from 2 to 4 feet. How many puffs will it take her to increase the diameter from 4 to 5 feet? Assume the density of air inside the ball is constant.

(3)Ans. _____ 3pts

4 pts 4. 6 different Spiderman comic books, 5 different Archie comic books and 4 different Garfield comic books are placed in a single pile. All the Spiderman books are together, all the Archie books are together and all the Garfield books are together. In how many distinguishable ways can this be done?

(4)Ans. _____ 4pts

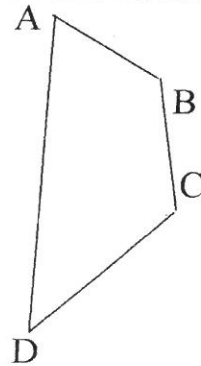
4 pt 5. Given twenty couples, how many different three-member committees can be formed that do not contain both members of any of the couples?

(5)Ans. _____ 4pts

4 pts 6. For which positive integers n is $n^3 - 8n^2 + 20n - 13$ a prime number?

(6)Ans. _____ 4pts

5 pts 7. In the quadrilateral ABCD, the measure of angles B and C is 120° , $AB = 3$, $BC = 4$ and $CD = 5$. Find the area of ABCD.



(7)Ans. _____ 5pts

5 pts 8. A full cylindrical glass of limeade has inside diameter of 8 cm and a height of 20cm. At what rate in cm/sec must the limeade travel through a straw with inside diameter of 8 mm so that the entire glass is emptied in 40 seconds?

(8)Ans. _____ 5pts

5 pts 9. If the sequence $\{a_n\}$ is defined by $a_1 = 2$, $a_{n+1} = a_n + 2n$ ($n > 1$), find the value of a_{101} .

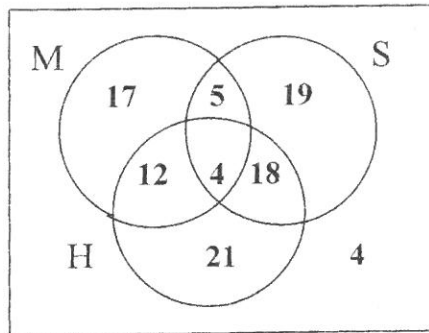
(9)Ans. _____ 5pts

Solutions – Arithmetic with Ratio and Proportion

1. Change $\frac{3}{4}$ to $\frac{6}{8}$. So 5 parts were filled with 15 gal. $\frac{5}{15} = \frac{2}{x}$, $x = 6$. **Ans. 6 gal**

2. $\frac{x+y}{z} = \frac{x}{z} + \frac{y}{z}$. $x = \frac{4}{7}y$ and $y = \frac{14}{3}z$, so $x = \frac{4}{7} \cdot \frac{14}{3}z = \frac{8}{3}z$ or $\frac{x}{z} = \frac{8}{3}$. $\frac{8}{3} + \frac{14}{3} = \mathbf{Ans. 22/3}$

3. Making a Venn Diagram:



Ans. 4

Series and Sequences

1. The last three terms are not cancelled: $303 + 306 + 309 = 918$. **Ans. 918**

2. $24 = 2(8 + x)$, x being the 8th term. $x = 4$. $y = 2(24 + 4)$, y being the 10th term. $y = 56$.
 $z = 2(24 + 56)$, z being the 11th term. $z = 160$. **Ans. 160**

3. The common difference is going to be eliminated. By factoring the numerator, you get $(z + w)(z - w)$ and since $z = w + 3d$, this yields $(2w + 3d)(3d)$. By factoring the denominator, you get $(y + x)(y - x)$ and since $y = w + 2d$ and $x = w + d$, this yields factors of $(2w + 3d)(d)$. Dividing the numerator by the denominator gives 3. **Ans. 3**

Counting Principles and Binomial Theorem

1. The first digit has 9 possibilities. Thus $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$. **Ans. 900,000**

2. $\binom{8}{3} (4x^2)^5 \left(-\frac{1}{2x^3}\right)^3 = -56(4^5)x^{10} \left(\frac{1}{8x^9}\right) = -56(2^7)x = -7168x$. **Ans. -7,168**

3. To get a sum of 10, you could get $1 + 3 + 6$ (six ways) or $1 + 4 + 5$ (six ways) or $2 + 3 + 5$ (six ways) or $2 + 2 + 6$ (three ways) or $3 + 3 + 4$ (three ways) or $2 + 4 + 4$ (3 ways). Total is 27 ways. **Ans. 27 ways**

Polynomials

1. $2x^2 + 3x - 5 = 0 \rightarrow (x - 1)(2x + 5) = 0$. $x = 1$ or $-5/2$. **Ans. 1 or -5/2**

2. In order for $(x - 2)$ to be a factor of $P(x)$, $P(2) = 0 = k(8) + k^2(8) + k^3$. Factoring: $k(k^2 + 8k + 8) = 0$. One value of k is 0. The sum of the other two is -8, the opposite of the coefficient of k . Thus the sum is -8. **Ans. -8**

3. Synthetically:
$$\begin{array}{r|rrrr} -5 & 3 & 1 & -62 & 40 \\ & & -15 & 70 & -40 \\ \hline & 3 & -14 & 8 & 0 \end{array}$$
 $3x^2 - 14x + 8 = 0 \rightarrow (3x - 2)(x - 4) = 0$
Thus $x = 2/3$ or 4. **Ans. -5, 4 or 2/3**

Areas and Volumes

1. Volume = $1/3$ area of base times the height: $1/3 (32) 3 = 32$ **Ans. 32cm^3**

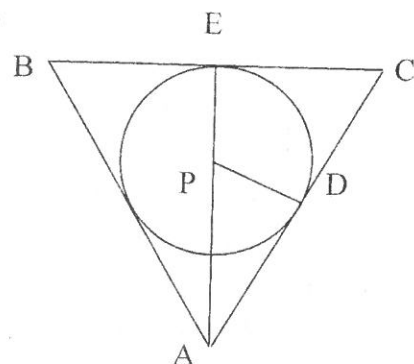
2. $\frac{k^2}{h^2} A_B = A_{AC} \rightarrow \frac{3^2}{4^2} (A_B) = 36 A_B = 36 \left(\frac{16}{9}\right) = 64$. $V = \frac{1}{3} (64)(16) = 1024/3$

Ans. = $1024/3$ or $341 \frac{1}{3}$ or 341.33

3. Draw a perpendicular from D to the center of the sphere meeting at P. $\triangle APD$ is a 30-60-90, so $PD = 20/\sqrt{3}$ and $AP = 40/\sqrt{3}$. This makes $AE = 60/\sqrt{3}$. Since $\triangle AEC$ is also a 30-60-90, then $\frac{AD}{AP} = \frac{AE}{EC}$ or $\frac{20}{20/\sqrt{3}} = \frac{60/\sqrt{3}}{EC}$. Thus

$20EC = \frac{60}{\sqrt{3}} \cdot \frac{20}{\sqrt{3}} = 20(20)$ or $EC = 20$. So the volume of

the cone is $\frac{1}{3} \pi (400) \frac{60}{\sqrt{3}} =$ **Ans. $8000\sqrt{3} \pi/3$**



Team

1. Since 30 has 4 factors: 1, 30; 2, 15; 3, 10; 5, 6 be could any of the 4 sums. **Ans. 4**

2. (1) $111 = a + 10d$. (2) $195 = a + 70$. Subtracting (1) from (2): $84 = 12d$, so $d = 7$. In (1) $111 = a + 10(7)$, so $a = 41$. The 60th term is $41 + 59(7) = 454$. The sum of the 60 terms is $\frac{60(41+454)}{2} = 14,850$. **Ans. 14,850**

3. As a ratio: $\frac{56}{\frac{4}{3}\pi(4^3 - 2^3)} = \frac{x}{\frac{4}{3}\pi(5^3 - 4^3)} = \frac{56}{56} = \frac{x}{61}$. Thus $x = 61$. **Ans. 61**

4. Ordering the Spiderman books: $6! = 720$. Archie books: $5! = 120$. The Garfield books: $4! = 24$ Ordering the three varieties: $3! = 6$. Multiplying $720(120)(24)(6)$ **Ans. 12,441,600**

5. There are ${}_{40}C_3 = 9880$ different committees possible using any combination of the 40 people. Of these, if husband 1 and wife 1 are on one committee, there would be 38 other people possible to be on that committee, so 38 committees with couple #1. There are 20 couples, $38(20) = 760$ committees with a couple. $9880 - 760 = 9120$. **Ans. 9120**
6. For $n^3 - 8n^2 + 20n - 13$ to be prime, one factor must be 1 and the other must prime. Factoring we get $(n - 1)(n^2 - 7n + 13)$. In order for $n - 1$ to be 1, $n = 2$. when $n = 2$, $n^2 - 7n + 13 = 3$, so 2 satisfies the conditions. If $n^2 - 7n + 13 = 1$, then $n^2 - 7n + 12 = 0$ or $(n - 4)(n - 3) = 0$ and $n = 3$ or 4. Substituting 3 and 4 into $n - 1$ yields 2 and 3 respectively (both are prime). So there are three values for n , 2, 3, and 4. **Ans. 2, 3, 4**
7. Extend \overline{AB} through B and \overline{DC} through C to meet at E, making equilateral triangle BEC with sides of length 4. $\triangle BEC$ has an area of $16\sqrt{3}/4$. $\triangle DAE$ has an area equal to $\frac{1}{2}(7)(9)\sin 60^\circ = 63\sqrt{3}/4$. So quadrilateral ABCD has area $47\sqrt{3}/4$. **Ans. $47\sqrt{3}/4$**
8. In cm, the glass has radius 4 and the straw has radius $2/5$. Suppose the straw has length L and that it is long enough to hold the contents of the glass. Then $16\pi(20) = (\frac{4}{25}\pi)L$.
 $L = \frac{16 \cdot 20 \cdot 25}{4} = 2000$ cm. To fill 2000 cm in 40 seconds, the rate would equal $\frac{2000}{40} = 50$ cm/sec. **Ans. 50 cm/sec**
9. $a_1 = 2$, $a_2 = 2 + 2$, $a_3 = 2 + 2 + 4$, $a_4 = 2 + 2 + 4 + 6$, $a_5 = 2 + 2(1 + 2 + 3 + 4)$. The sum of n integers is $\frac{n(n+1)}{2}$. So $a_n = 2 + 2\frac{(n-1)n}{2} = 2 + (n-1)n$.
 Thus $a_{101} = 2 + 100(101) = 10102$. **Ans. 10102**

Answer Sheet – Nov 2012

Arithmetic with Ratio and Proportion

1. 6 or 6 gal
2. $\frac{22}{3}$
3. 4 or 4 students

Series and Sequences

1. 918
2. 160
3. 3

Counting Principles and Binomial Theorem

1. 900,000
2. -7,168
3. 27

Polynomials

1. 1, $-\frac{5}{2}$ or 1, $-2\frac{1}{2}$
2. -8
3. -5, 4, $\frac{2}{3}$

Areas and Volumes

1. 32 or 32 cm^3
2. $\frac{1024}{3}$ or $341\frac{1}{3}$ or 341.3333
in³(optional)
3. $8000\pi\sqrt{3}/3$ or 14510.39

Team

1. 4
2. 14,850
3. 61
4. 12,441,600
5. 9,120
6. 2, 3, 4
7. $47\sqrt{3}/4$ or 20.3516
8. 50 or 50 cm/sec
9. 10102