1 Arithmetic with Ratio and Proportion Nov 2013 (No Calculators)

3 pts 1. Five line segments are drawn such that:

length of segment 2 = 1/2 length of segment 1
length of segment 3 = 2/3 length of segment 2
length of segment 4 = 3/4 length of segment 3
length of segment 5 = 4/5 length of segment 4

Find the ratio of the length of segment 5 to the length of segment 1. **Ans.** _____

4 pts 2. A carpet installation company offers a sale such that if the largest room is carpeted at full price, then it will include the installation in 2 smaller rooms free of charge. A homeowner takes advantage of the offer. The two smaller rooms each need 1/3 of the carpet as the homeowner's largest room. What fractional part of the cost of installing the carpet at full price in all three rooms did the customer pay?

Ans. _____

5 pts 3. Find the fractional equivalent of $.819\overline{4}$ in simplest form.

Ans. _____

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2 Series and Sequences Nov 2013 (No Calculators)

3 pts 1. An arithmetic sequence is defined as $A_{N+1} = A_N + 3/4$. If $A_1 = 1/2$, find the value of A_9.

Ans. _____

4 pts 2. One-fourth of the air in a tank is removed with each stroke of a vacuum pump. What fractional part of the air remains in the tank after 4 strokes?

Ans. _____

5 pts 3. The 9th term of an arithmetic sequence is 16. The 42nd term is 214. What is the sum of the first 50 terms of the sequence?

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2013 (No Calculators)

3 pts 1. How many distinguishable ways can ten A's and three B's be arranged in a row?

Ans. _____

4 pts 2. Find the third term in the expansion of $\left((2x)^3 - \frac{1}{x^2}\right)^5$.

Ans. _____

5 pts 3. A committee of three people is randomly selected from a club consisting of 11 females and 10 males. If A is the number of committees comprised of a dominance of females, and B is the number of committees comprised of dominance of males. How many more committees are in A than there are in B?

Ans. _____

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4 Polynomials Nov 2013 (No Calculators)

3 pts 1. Let $A = 3x^2y - xy^2 - 4x^3y + 2x^2y^2$ and $B = x^2y^2 - xy^2 - x^2y + x^3y$. Find the polynomial created by $2(A + B) - 3(A - B)$.

Ans. _____

4 pts 2. Find the solutions for the equation: $x^3 + 19x = 8x^2 + 12$.

Ans. _____

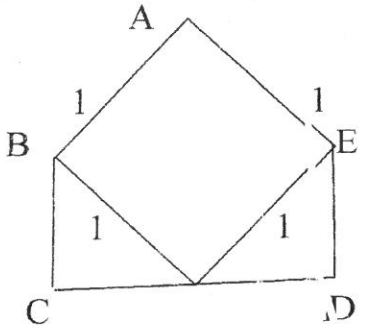
5 pts 3. Determine a and b such that 2 is a double zero for the polynomial:

$$x^4 + (a-2)x^3 + bx^2 + (a+b)x + 4$$

Ans. _____

5 Areas and Volumes Nov 2013 (You may use calculators)

3 pts 1. Home plate used in baseball is in the shape of the pentagonal region ABCDE at right. The square with 1 foot sides has two right isosceles triangles attached as shown in the figure. What is the area of home plate?



Ans. _____

4 pts 2. A hemisphere and an inscribed right cone have the same base. What is the ratio of the volume of the cone to the volume of the hemisphere?

Ans. _____

5 pts 3. Two circles are concentric. A chord C units long of the larger circle is tangent to the smaller circle. Express the area between the two circles, in terms of C.

Ans. _____

6 Team Nov 2013 (Calculators allowed)

3 pts 1. Find the sum of the multiples of 7 between 0 and 200.

(1) _____ **3 pts**

3 pts 2. Find the 8^{th} term of an arithmetic sequence having a common difference of 2/3 and a fifth term of 3.

(2) _____ **3 pts**

3 pts 3. Find the sum of the coefficients in the expansion of $(x + y)^9$.

(3) _____ **3 pts**

4 pts 4. Find the zeros of $18x^4 - 33x^3 - 117x^2 + 22x + 70$.

(4) _____ **4 pts**

4 pts 5. Find two numbers such that their difference, sum and product are in the ratio of 1: 4: 15, respectively.

(5) _____ **4 pts**

4 pts 6. Simplify the following: $({}_{x+1}C_2)^2 - ({}_xC_2)^2$ (6) _____ **4 pts**

5 pts 7. One of the terms in the expansion of $(ax + by)^c$ is $4320x^3y^3$. If a, b and c are integers and $c > 0$, find all possible binomials $ax + by$.

(7) _____ **5 pts**

5 pts 8. A cube is contained in a sphere such that all the vertices of the cube are points on the surface of the sphere. Find the ratio of the surface area of the sphere to the surface area of the cube.

(8) _____ **5 pts**

5 pts 9. A line intersects two sides of an equilateral triangle and is parallel to the third side. If this line divides the triangular region into a trapezoid and a smaller triangle having equal perimeters, what is the ratio of the area of the smaller triangle to that of the trapezoid?

(9) _____ **5 pts**

Solutions – Arithmetic with Ratio and Proportion

1. If line segment 1 = 1, then line segment $5 = \left(1 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}\right) = \frac{1}{5}$. **Ans. 1 :5**

2. 1 + 1/3 + 1/3 = 5/3 total carpets. $\frac{1}{5/3} = 3/5$. **Ans. 3/5**

3. Let $x = .819\overline{4}$, then $10{,}000x = 8194.\overline{4}$ and $1{,}000x = 819.\overline{4}$. So $9{,}000x = 7375$.
$x = \frac{7375}{9000} = \frac{1475}{1800} = \frac{295}{360} = \frac{59}{72}$. **Ans. 59/72**

Series and Sequences

1. $a_9 = 1/2 + 8(3/4) = 1/2 + 6 = 6\ 1/2$. **Ans. 6 1/2**

2. $a_4 = \frac{3}{4}\left(\frac{3}{4}\right)^3 = \frac{3^4}{4^4} = \frac{81}{256}$. **Ans. 81/256**

2. 9th term: (1) $16 = a_1 + 8d$; 42nd term: (2) $214 = a_1 + 41d$. (2) – (1): $198 = 33d$, $d = 6$.
In (1): $16 = a_1 + 8(6)$, $a = -32$. The 50th term is $-32 + 49(6) = 262$. Sum $= \frac{50}{2}(-32 + 262) = 25(230) = 5750$ **Ans. 5750**

Counting Principles and Binomial Theorem

1. $\left(\frac{13!}{3!10!}\right) = \frac{13 \cdot 12 \cdot 11}{3 \cdot 2} = \frac{13 \cdot 2 \cdot 11}{1} = 143(2) = 286.$ **Ans. 286**

2. $\binom{5}{2}(8x^3)\left(-\frac{1}{x^2}\right) = (10)(512x^9)\left(\frac{1}{x^4}\right) = 5120x^5$ **Ans. $5120x^5$**

3. A is all females or 2 females and one male: ${}_{11}C_3 + {}_{11}C_2 \cdot {}_{10}C_1 = \left(\frac{11!}{8!3!}\right) + \left(\frac{11!}{9!2!}\right) \cdot 10 =$
$\left(\frac{11 \cdot 10 \cdot 9 \cdot 8!}{3 \cdot 2 \cdot 8!}\right) + \left(\frac{11 \cdot 10}{2}\right) \cdot 10 = 11 \cdot 5 \cdot 3 + 11 \cdot 5 \cdot 10 = 11 \cdot 5 \cdot 13 = 715$. B is all males or 2 males and 1 female: ${}_{10}C_3 + {}_{10}C_2 \cdot {}_{11}C_1 = \left(\frac{10!}{7!3!}\right) + \left(\frac{10!}{2!8!}\right) \cdot 11 = \left(\frac{10 \cdot 9 \cdot 8 \cdot 7!}{3!7!}\right) + \left(\frac{10 \cdot 9 \cdot 8!}{2!8!}\right) \cdot 11$ ➔
$10 \cdot 3 \cdot 4 + 5 \cdot 9 \cdot 11 = 120 + 495 = 615$. $715 - 615 = 100$. **Ans. 100**

Polynomials

1. $2(A + B) - 3(A - B) = 2A + 2B - 3A + 3B = 5B - A$. Thus
$5(x^2y^2 - xy^2 - x^2y + x^3y) - (3x^2y - xy^2 - 4x^3y + 2x^2y^2) =$
$5x^2y^2 - 5xy^2 - 5x^2y + 5x^3y - 3x^2y + xy^2 + 4x^3y - 2x^2y^2$
Ans. $3x^2y^2 - 4xy^2 - 8x^2y + 9x^3y$

2. $x^3 + 19x = 8x^2 + 12$ ➔ $x^3 - 8x^2 + 19x - 12 = 0$. By synthetic dividion:

4	1	-8	19	-12
		4	-16	12
	1	-4	3	

Now factoring $(x - 3)(x - 1)$. The solutions are 1, 3, 4.
Ans. 1, 3, 4

3. Dividing synthetically:

2	1	a – 2	b	a + b	4
		2	2a	4a + 2b	10a + 6b
2	1	a	2a + b	5a + 3b	(10a + 6b + 4 = 0)
		2	2a + 4	8a + 2b + 8	
	1	a + 2	4a + b + 4	(13a + 5b + 8 = 0)	

Thus $5a + 3b = -2 \Rightarrow 25a + 15b = -10$

$13a + 5b = -8 \Rightarrow -39a - 15b = 24$

$-14a = 14$ So $a = -1$. $5(-1) + 3b = -2$, $b = 1$. **Ans. a = -1 and b = 1**

Areas and Volumes

1. The area of the square is 1. The two triangles can be put together to make a square whose diagonals are 1, the area being half the product of the diagonals. **Ans. 1.5**

2. The cone and hemisphere have same base and same height. The volume of the cone is $1/3\ \pi r^3$, the hemisphere is $2/3\ \pi r^3$. Ratio is 1 to 2. **Ans. 1 to 2**

3. Let R be the radius of the large circle and r be the radius of the smaller circle. Then the area of the annulus is $\pi R^2 - \pi r^2$ or $\pi(R^2 - r^2)$. $(1/2C)^2 = R^2 - r^2$. **Ans.** $\frac{\pi}{4}C^2$

Team

1. 196 is the last. $196 = 7 + (n - 1)\,7 \rightarrow 28 = 1 + n - 1 \rightarrow n = 28$. Sum $= \frac{28}{2}(7 + 196) = 14(203) = 2842$. **Ans. 2842**

2. $3 = a_1 + 2/3(4) \rightarrow 3 = a_1 + 2\frac{2}{3}$, so $a_1 = 1/3$. 8^{th} term: $\frac{1}{3} + 7\left(\frac{2}{3}\right) = 5$. **Ans. 5**

3. Each row of Pascal's Δ has a sum of 2^n where n is the power. $2^9 = 512$. **Ans. 512**

4. $18x^4 - 33x^3 - 117x^2 + 22x + 70 \rightarrow (-33x^3 + 22x) + (18x^4 - 117x^2 + 70) \rightarrow$
$-11x(3x^2 - 2) + (3x^2 - 2)(6x^2 - 5) \rightarrow (3x^2 - 2)(6x^2 - 11x - 35) = 0$. So (1) $3x^2 - 2 = 0$, or (2) $6x^2 - 11x - 35 = 0$. In (1) $x = \pm\sqrt{6}/3$. In (2) $(3x + 5)(2x - 7) = 0$, so $x = -5/3, 7/2$.
Ans. $\pm\sqrt{6}$**/3, -5/3, 7/2**

5. $(x - y):(x + y):(xy) = 1a:4a:15a$. (1) $x - y = 1a$, (2) $x + y = 2a$, thus $2x = 5a$ or $x = \frac{5}{2}a$ and $y = \frac{3}{2}a$. Since $xy = 15a$, then $\left(\frac{5}{2}a\right)\left(\frac{3}{2}a\right) = 15a$ or $\frac{15}{4}a^2 = 15a \rightarrow a^2 = 4a$, so $a = 4$, since a cannot be 0. In (1) $x + y = 4$, in (2) $x - y = 16$. (1) + (2): $2x = 20$, so $x = 10$ and $y = 6$.
Ans. 6 and 10

6. $\binom{x+1}{2} = \frac{(x+1)x(x-1)!}{2(x+1-2)!} = \frac{(x+1)x}{2}$. $\binom{x}{2} = \frac{x(x-1)(x-2)!}{2(x-2)!} = \frac{x}{2}\frac{-1)}{}$.

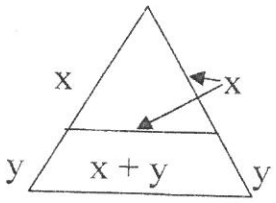
$\left(\frac{x^2+x}{2}\right)^2 - \left(\frac{x^2-x}{2}\right)^2 = \frac{x^4+2x^3+x^2-(x^4-3x^3+x^2)}{4} = \frac{4x^3}{} = x^3$ **Ans. x^3**

7. From the exponents: $\binom{6}{3} = 20$. 4320/20 = 216. Sir the coefficients of x and y had to be cubed and each integers, then factoring 216 produ , $2^3 3^3$, which could also have been $6^3 1^3$. Being that this is the 4th term and positive, ther he coefficients had to be both positive or both negative. Possibilities are: $\pm(6x + y$, $\pm(x + 6y)$, $\pm(2x + 3y)$, $\pm(3x + 2y)$

Ans. $\pm(6x +$, $\pm(x + 6y)$, $\pm(2x + 3y)$, $\pm(3x + 2y)$

8. Let r be the radius of the sphere. Then the sur e area of the sphere is $4\pi r^2$. The inner diagonal of the cube is 2r. Let the edge of e cube be a, then $a^2 + a^2 + a^2 = (2r)^2$. $3a^2 = 4r^2$ or $a = \frac{2r}{\sqrt{3}}$. Total surface area of cub , $6\left(\frac{4r^2}{3}\right) = 8r^2$. $\frac{4\pi r^2}{8r^2} = \frac{\pi}{2}$. **Ans. $\pi/2$**

9. In the figure at right: $3x = 2x + 3y$ or $x = 3y$. The area of the triangle is $(\frac{1}{2}x)(\frac{1}{2}x\sqrt{3}) = \frac{\sqrt{3}}{4}x^2$. The area of the trapezoid is $\frac{1}{2}(\frac{1}{2}y\sqrt{3})(2x + y) = \frac{1}{4}\left(\frac{x}{3}\right)\sqrt{3}\,(2x + \frac{x}{3}) = \frac{7\sqrt{3}}{36}x^2$. Ratio of areas:



$$\frac{\frac{\sqrt{3}}{4}x^2}{\frac{7\sqrt{3}}{36}x^2} = \frac{\sqrt{3}}{4}\cdot\frac{36}{7\sqrt{3}} = \frac{9}{7}.$$

Ans. 9/7

Answer Sheet Nov 2013

Arithmetic with Ratio and Proportion

1. 1:5 or $\frac{1}{5}$ or 1 to 5
2. 3/5
3. 59/72

Series and Sequences

1. 6 1/2, $\frac{13}{2}$, 6.5
2. 81/256
3. 5750

Counting Principles and Binomial Theorem

1. 286
2. $5120x^5$
3. 100

Polynomials

1. $3x^2y^2 - 4xy^2 - 8x^2y + 9x^3y$
2. 1, 3, 4
3. a = -1, b = 1
   (-1, 1)

Areas and Volumes

1. 1.5 or 1 1/2 or 3/2
2. 1 to 2 or 1:2 or 1/2
3. $\frac{\pi}{4}C^2$ or .7854 C^2

Team

1. 2842
2. 5
3. 512
4. $\pm\sqrt{6}/3$, -5/3, 7/2
5. 6 and 10
6. x^3
7. $\pm(6x + y)$, $\pm(x + 6y)$, $\pm(2x + 3y)$, $\pm(3x + 2y)$
8. $\frac{\pi}{2}$ or π:2 or π to 2
9. 9/7 or 9:7 or 9 to 7