1 Arithmetic with Ratio and Proportion Nov 2014 (No Calculators)

3 pts 1. If you add up all the whole numbers from 1 to 7, you get 28. The ratio of 28 to 7 is 4:1. What is the ratio of the sum of all the whole numbers from 1 to 13 to the number 13?

Ans. _____

4 pts 2. If a varies directly with b, b varies inversely with c, and c varies directly with the square of d, by what is d multiplied, if a doubles?

Ans. _____

5 pts 3. Let k be the smallest real number, greater than 0, such that the product of k and each member of the set $\left\{\frac{25}{18}, \frac{80}{27}, \frac{5}{12}\right\}$ is a whole number. Find $\frac{25}{18}k + \frac{80}{27}k + \frac{5}{12}k$.

Ans. _____

2 Series and Sequences Nov 2014 (No Calculators)

3 pts 1. Find the 47^{th} term of the sequence -47, -36, -25, …

Ans. _____

4 pts 2. An infinite geometric series has a sum of 100 and a first term of 40. What is the fifth term in the series?

Ans. _____

5 pts 3. The 4^{th} term of a geometric sequence is 567. The 8^{th} term is 45,927. What is the sum of the first three terms?

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2014 (No Calculators)

3 pts 1. President, vice-president, secretary and treasurer are to be chosen from a club of 10 members. How many different choices are possible?

Ans. _____

4 pts 2. A classroom contains 7 girls, 8 boys, and 1 teacher. If the order of departure from the classroom during a fire drill is recorded in terms of G's, B's, and the T, how many orders are possible?

Ans. _____

5 pts 3. In the expansion of $(2x + 3y)^9$, what is the sum of the coefficients of the 5th and the $6t^h$ terms?

Ans. _____

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4 Polynomials Nov 2014 (No Calculators)

3 pts 1. Find all values of x such that $x^2 - 12x + 20 = 0$.

Ans. _____

4 pts 2. Find the remainder when dividing $x^9 + 2x^8 + 3x^7 + 4x^6 + \ldots + 8x^2 + 9x + 10$ by $x + 2$.

Ans. _____

5 pts 3. For any integer values of a and b, let S be the set of all rational roots for all cubic polynomials of the form $6x^3 + ax^2 + bx - 12$. How many elements are in the set?

Ans. _____

5 Areas and Volumes Nov 2014 (You may use calculators)

3 pts 1. Cube A has base edges of 6. Cube B has edges half as long as cube A. What is their combined volumes?

Ans. _____

4 pts 2. A rectangular pyramid has base edges of 6 and 8 and a height of 4. All the lateral edges are congruent. Find the total surface area of the pyramid.

Ans. _____

5 pts 3. . A spherical balloon has a volume of 288π cubic cm. If it begins losing volume at the rate of 18π cubic cm. per hour, how many hours will it take until the number for its surface area equals the number for its volume?

Ans. _____

6 Team Nov 2014 (You may use calculators)

3 pts 1. A clear glass box 7 by 12 by 18 cm sealed on all six sides, is partially filled with colored water prior to being sealed tight. When the box is placed on one of its 7 by 12 faces, the water is 15 cm high. If the box is placed on one of its 7 by 18 faces, how high will the water level be in cm?

(1)**Ans.** _____ **3 pts**

3 pts 2. If a scale is set up such that ½ inch represents 1 mm, how many meters would be represented by one mile?

(2)**Ans.** _____ **3 pts**

3 pts 3. Grandma Moses has made 2 dozen cookies to give to her grandchildren, 9 to Peter who is 9 years old, 8 to Paul who is 8, and 7 to Mary who is 7. In how many distinguishable ways can the 24 cookies be sorted for the three containers?

(3)**Ans.** _____ **3 pts**

4 pts 4. Find the total surface area of a regular hexagonal pyramid whose volume is $48\sqrt{3}$ and whose height is 150% of a base edge. Its lateral edges are all the same length.

(4)**Ans.** _____ **4 pts**

4 pts 5. Find all 6 solutions of the equation
$x^6 - 2x^5 - 13x^4 + 26x^3 + 36x^2 - 72x = 0$. (5)**Ans.** _____ **4 pts**

4 pts 6. Two of the roots of the equation $2x^3 - 3x^2 + px + q = 0$ are 3 and -2. Find the values of p and q.

(6)**Ans.** _____ **4 pts**

5 pts 7. Jimmy's one-foot ruler is marked every 32^{nd} of an inch beginning at 0 and ending with the 12-inch mark. To kill time one afternoon, Jimmy writes down the exact measure at each mark, beginning, 0, $\frac{1}{32}, \frac{1}{16}, \frac{3}{32}, \ldots$ and ending $11\frac{31}{32}$, 12. Then he finds the sum of all the numbers he wrote. What total does he get? (7)**Ans.** _____ **5 pts**

5 pts 8. There are 100 members in the US Senate. In how many ways can a committee of 5 senators be formed, if no state may be represented more than once on the committee? Each of the 50 states has two senators.

(8)**Ans.** _____ **5 pts**

5 pts 9. On a 10 column by 10 row blank grid of squares, a 1 is entered in the upper left, in row 1 column 1. Then row 1 is completed by entering the terms of an arithmetic sequence with a constant difference of +2. Each column is then completed by entering the terms of a geometric sequence with a common multiple of 2. Thus, column 4 reads from top to bottom: 7, 14, 28, 56, 112, 224, 448, 896, 1792, 3584. Find the sum of all 100 entries in the grid.

(9)**Ans.** _____ **5 pts**

Solutions – Arithmetic with Ratio and Proportion

1. Sum is 91. 91:13 = 7:1. **Ans. 7:1**

2. The ratios are: $\frac{a}{b}$, bc, $\frac{c}{d^2}$. Letting $a = 2$, $b = 3$, $c = 8$, and $d = 5$: if a were to change to 4, then b would change to 6 to have same ratio. Since bc was 3(8), now bc is 6(4). $\frac{c}{d^2}$ was $\frac{8}{25}$, now it is $\frac{4}{d^2}$. Thus $8d^2 = 100$ and $d^2 = \frac{100}{8} = \frac{25}{2}$ or $5\left(\frac{\sqrt{2}}{2}\right)$. **Ans.** $\frac{\sqrt{2}}{2}$

3. $k = \frac{LCM\{18,27,12\}}{GCF\{25,80,5\}} = \frac{108}{5}$. $\frac{25}{18}\left(\frac{108}{5}\right) + \frac{80}{27}\left(\frac{108}{5}\right) + \frac{5}{12}\left(\frac{108}{5}\right) = 30 + 64 + 9$. **Ans. 103**

Series and Sequences

1. $-47 + 46(11) = -47 + 506 = 459$. **Ans. 459**

2. $\frac{40}{1-r} = 100$. $40 = 100 - 100r$. $r = 3/5$. $40(3/5)^4 = \frac{40 \cdot 81}{5 \cdot 125} = \frac{8 \cdot 81}{125} = \frac{648}{125}$. **Ans.** $\frac{648}{125}$

3. Multiplying the 4th term by r^4 will give the 8th term: $567r^4 = 45{,}927$.
$r^4 = 45{,}927/567 = 81$. So $r = \pm 3$. Finding the 1st term a: $567 = a(3)^3$, $a = 567/27 = 21$.
So the 1st three terms are 21, ± 63, 189. The sums are 273 or 147. **Ans. 273 or 147**

Counting Principles and Binomial Theorem

1. ${}_{10}P_4 = \frac{10!}{6!} = 10(9)(8)(7) = 720(7) = 5040$. **Ans. 5040**

2. $\frac{16!}{7!\cdot 8!\cdot 1!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 52(22)90 = 1144(90) = 102{,}960$. **Ans. 102,960**

3. The 5th term is $\binom{9}{4}(2x)^5(3y)^4$. The 6th term is $\binom{9}{5}(2x)^4(3y)^5$. $\binom{9}{5} = 126$.
$126(2^5 3^4 + 2^4 3^5) = 126(2^4 3^4)(2+3) = 126(80)(81) = 126(6480) = 816{,}480$. **Ans. 816,480**

Polynomials

1. $x^2 - 12x + 20 = (x - 10)(x - 2) = 0$. $x = 10$ or 2. **Ans. 10 or 2**

2. Dividing synthetically using -2:

	1	2	3	4	5	6	7	8	9	10
-2		-2	0	-6	4	-18	24	-62	108	-234
	1	0	3	-2	9	-12	31	-54	117	-224.

Ans. -224

3. By the rational root test, all rational roots of the polynomial with integral coefficients must be of the form m/n, where m is the an integer factor of the constant (-12) and n is an integer factor of the leading coefficient (6).

-12 has 12 factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$; 6 has 8 factors: $\pm 1, \pm 2, \pm 3, \pm 6$

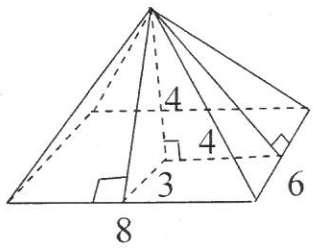
There are 24 rational roots in *S*: $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Ans. 24

Areas and Volumes Nov 2014

1. $6^3 + 3^3 = 216 + 27 = 243$. **Ans. 243**

2. In the figure at right, the right triangle whose hypotenuse is the altitude of the triangle whose base is 6, is $4\sqrt{2}$. The right triangle whose hypotenuse is the triangle whose base is 8 is 5. Both results of Pythagorean Theorems. The triangle Whose base is 8 has area 20. The triangle whose base is 6 is $12\sqrt{2}$. TSA = $48 + 40 + 24\sqrt{2}$.



Ans. $88 + 24\sqrt{2}$

3. V = SA ➔ $4/3\ \pi r^3 = 4\pi r^2$ ➔ $1/3r = 1$, so $r = 3$. $V = 4/3\ \pi 3^3 = 36\pi$. Volume change: $288\pi - 36\pi = 252\pi$. $252\pi/18\pi = 14$ hrs. **Ans. 14 hrs**

Team

1. $7x12x15 = 7x18xh$. $h = 10$. **Ans. 10**

2. $\frac{.5in}{1mm}\cdot\frac{1000mm}{1m}\cdot\frac{1ft}{12in}\cdot\frac{1mi}{5280ft} = \frac{500mi}{12\cdot 5280m} = \frac{1mi}{126.72m}$. **Ans. 126.72m**

3. ${}_{24}C_9 \cdot {}_{15}C_8 \cdot {}_7C_7 = 1{,}307{,}504(6435)(1) = 8{,}413{,}788{,}240$. **Ans. 8,413,788,240**

4. $V = \frac{1}{3}$ (hexagon area) height ➔ $48\sqrt{3} = \frac{1}{3}(\frac{3}{2}s^2\sqrt{3})(\frac{3}{2}s) = \frac{3}{4}s^3\sqrt{3}$. So $s^3 = 64$.

The sides are 4 and the height of the pyramid is 6. The area of the base is $24\sqrt{3}$. Each of the 6 triangles of the lateral surface has height of $4\sqrt{3}$ and base 4, so each triangle is $8\sqrt{3}$. For all 6 the area is $48\sqrt{3}$. TSA = $72\sqrt{3}$. **Ans. $72\sqrt{3}$**

5. Using calculators will produce $\pm 2, \pm 3, 0$. 2 is a double root. **Ans. 2, $\pm 2, \pm 3, 0$**

6. Using synthetic division:

	2	-3	p	q
3		6	9	27 + 3p
	2	3	p+9	27 + 3p + q

	2	-3	p	q
-2		-4	14	-28 – 2p
	2	-7	p+14	q – 28 – 2p

$q - 28 - 2p = q + 3p + 27$ ➔ $-55 = 5p$, $p = -11$. $q - 33 + 27 = 0$, $q = 6$. **Ans. p = -11, q = 6**

7. The average term $\left(\frac{a+l}{2}\right)$ is $\frac{0+12}{2} = 6$. The # of terms (n): $12 = 0 + (n-1)\frac{1}{32}$. $n = 385$.

The sum (S): $S = 385(6) = 2310$. **Ans. 2310**

8. $\frac{100 \cdot 98 \cdot 96 \cdot 94 \cdot 92}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 67{,}800{,}320$ **Ans. 67,800,320**

9. The sum of the numbers in column 1 is $1 + 2 + 4 + \ldots 512 = 2^{10} - 1 = 1023$.

The sum of the column N entries for N = 1, 2,3, … , 10 is 2N – 1 times the column 1 sum.
The total is: $1023(1 + 3 + 5 + \ldots + 17 + 19) = 1023(10^2) = 102{,}300$. **Ans. 102,300**

Answer Sheet – Nov 2014

Arithmetic with Ratio and Proportion

1. 7:1
2. $\sqrt{2}/2$
3. 103

Series and Sequences

1. 459
2. 648/125 → 5.184
3. -147 or 273

Counting Principles and Binomial Theorem

1. 5040
2. 102,960
3. 816,480

Polynomials

1. 10 or 2
2. -224
3. 24

Areas and Volumes

1. 243
2. $88 + 24\sqrt{2}$ or 129.5692 (121.9411)
3. 14 or 14 hrs

Team

1. 10 or 10 cm
2. 126.72 or 126.72 m (126.7200)
3. 8,413,788,240
4. $72\sqrt{3}$ or 124.7077
5. $2, \pm 2, \pm 3, 0$
6. $p = -11, q = 6$
7. 2310
8. 67,800,320
9. 102,300