1 Arithmetic with Ratio and Proportion Nov 2015 (No Calculators)

3 pts 1. The ratio of green, blue and orange m & m's is 5 to 3 to 4, respectively. If there are 156 m & m's altogether, how many are orange?

(13)

$5x+3x+4x$

$\frac{12x}{12} = \frac{156}{12}$ 13

Ans. 52

4 pts 2. The ratio of x to y is 3 to 4. Let S be the result when y is increased by 2 and let R be the result when x is decreased by 6. The ratio of R to S is 3 to 5. Find the numerical value of $x+y$.

$\frac{x}{y} = \frac{3}{4}$ $\frac{R}{S} = \frac{3}{5}$ $y \cdot \frac{x}{y} = \frac{3}{4}y$ $S = \frac{x}{y+2}$ $R = \frac{x-6}{y}$ $\frac{x-6}{y} \cdot \frac{y+2}{x} = \frac{3}{5}$

$5(x-6)(y+2) = 3xy$

$5(xy+2x-6y-12) = 3xy$

$-3xy$

$2xy+2x-6y-12=0$

Ans.

5 pts 3. In the enchanted land of Mathtinicus, the intelligence of any man is inversely proportional to his shoe size and jointly proportional to his age and the square of his height. Albert is 2 meters tall, 15 years old and his shoe size is 8. Steve is 3 meters tall and has shoe size 12. How old is Steve, if he and Albert have equal intelligence?

Ans. _____

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2 Series and Sequences Nov 2015 (No Calculators)

3 pts 1. The 24th term of this sequence marks the years that one of the MAML Board members was born. What year was it?

2015, 2012, 2009, …

Ans. _____

4 pts 2. An infinite geometric series has its fourth term equal to 3 and its common ratio equal to two-thirds. Find the sum of the series.

Ans. _____

5 pts 3. The first three terms of an arithmetic sequence are 76, 72, and 68. There are two values of n, such that the sum of the first n terms is 448. Find the sum of the two values of n.

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2015 (No Calculators)

3 pts 1. A team consisting of 3 girls and 2 boys is selected from 10 girls and 5 boys. How many different teams can be made?

Ans. _____

4 pts 2. What is the ratio of the coefficients of the x^2 term to the x^3 term in the expansion of $(2x - 3y)^7$?

Ans. _____

5 pts 3. Find the number of positive integers less than 10,000 with all distinct digits.

Ans. _____

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4 Polynomials Nov 2015 (No Calculators)

3 pts 1. From what polynomial must $5x^3 - 2x^2 + 3x$ be subtracted from so as to obtain the polynomial $x^3 - 2$?

Ans. _____

4 pts 2. The rational Root Theorem (also called the Rational Zero Theorem) can be used to generate a list of all possible rational roots of a function. What is the product of all the possible rational roots, according to the "theorems", of the function f(x), if

$$f(x) = 2x^4 - 7x^3 + 5x^2 + 14x + 8$$

Ans. _____

5 pts 3. If one root of the equation $x^4 - 5x^3 - 22x^2 + 230x - 204 = 0$ is 5 + 3i, find the sum of the remaining roots.

Ans. _____

5 Areas and Volumes Nov 2015 (You may use calculators)

3 pts 1. A 6 inch by 8 inch by 12 inch box is setting on its smallest face. What is the total surface area of the box, if it is open on the top?

Ans. _____

4 pts 2. Spherical planet A has a surface area of 270,000,000 square miles. Spherical planet B has a volume 8/27 of planet A's volume. Find the surface area of planet B in square miles.

Ans. _____

5 pts 3. A fancy salt shaker in the shape of a pyramid has an equilateral triangle measuring 2 cm on a side for a base and a height of 8 cm. Find the capacity of the shaker in grains of salt if there are 27 grains of salt per mm^3. Assume all grains of salt are whole grains. Round answer to nearest hundred grains.

Ans. _____

6 Team Nov 2015 (You may use calculators)

3 pts 1. What is the sum of the next three numbers in this sequence?

1, 1, 2, 4, 7, 11, … **(1) Ans.** _____ **3 pts**

3 pts 2. Find the sum of the coefficients in the expansion of $(2a - 3b + 5c)^2$.

(2) Ans. _____ **3 pts**

3 pts 3. Find the sum of the positive zeroes for f(x), if $f(x) = x^4 - 6x^3 + 11x^2 - 6x$.

(3) Ans. _____ **3 pts**

4 pts 4. Find the sum of the positive integers less than 1000 which are multiples of 13. **(4) Ans.** _____ **4 pts**

4 pts 5. What is the value of the constant term in the expansion of $\left(x - \frac{1}{x^3}\right)^8$?

(5) Ans. _____ **4 pts**

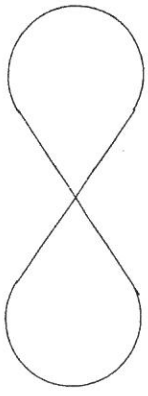
4 pts 6. A large van sits two in the front seats (one being the driver), 3 in the next row behind the front two, and four in the way back. How many seating arrangements can be made for an autumn scenic drive by 8 students, if only three can drive and both front seats are occupied?

(6) Ans. _____ **4 pts**

5 pts 7. Find the least value of N so that $17 + 34 + 51 + \ldots + N > 1{,}000{,}000$.

(7) Ans. _____ **5 pts**

5 pts 8. The figure at right is a diagram of a 1 mile race track in a *figure-eight* layout. The track was constructed using 240° arcs of circles of identical radii. The circular arcs were placed in opposition (opening toward each other) and connected with line segments meeting the edge of the arcs as tangents. Find the area inside the two lobes of the track in square feet rounded to the nearest 100 ft².



(8) Ans. _____ **5 pts**

5 pts 9. The polynomial $p(x) = x^3 - 4x^2 + 5x - 1.9$ has roots a, b and c. Find the area of a triangle with side lengths of a, b and c.

(9) Ans. _____ **5 pts**

Solutions – Arithmetic with Ratio and Proportion

1. $x + 3x + 4x = 156 \rightarrow 12x = 156$ so $x = 13$. $4x = 4(13) = 52$ **Ans. 52**

2. $\frac{x}{y} = \frac{3}{4}$, $4x = 3y$, $y = 4/3x$. Thus $\frac{x-6}{(4/3)x+2} = \frac{3}{5} \rightarrow 5x - 30 = 4x + 6$, $x = 36$, thus $y = 48$.

$36 + 48 = 84$. **Ans. 84**

3. Intelligence $= \frac{(height)^2(age)}{(shoesize)} \rightarrow \frac{(2^2)(15)}{(8)} = \frac{(3^2)(x)}{(12)} \rightarrow x = \frac{60(12)}{8(9)} = 10.$ **Ans. 10**

Series and Sequences

1. Age $= 2015 - 3(23) = 1946$. **Ans. 1946**

2. $3 = a(\frac{2}{3})^3 \rightarrow 3 = \frac{8}{27}a$, so $a = \frac{81}{8}$. Sum $= \frac{81/8}{1-2/3} = \frac{81}{8} \cdot \frac{3}{1} = \frac{243}{8}$. **Ans.** $\frac{243}{8}$

3. $448 = \frac{n}{2}(2(76) + (n-1)(-4)) \rightarrow \frac{n}{2}(152 - 4n + 4) \rightarrow 896 = 156n - 4n^2 \rightarrow$

$n^2 - 39n + 224 = 0 \rightarrow (n - 32)(n - 7) = 0$. $n = 32$ or 7, the sum being 39. **Ans. 39**

Counting Principles and Binomial Theorem

1. $\binom{10}{3}\binom{5}{2} = 120(10) = 1200.$ **Ans. 1200**

2. $\frac{\binom{7}{5}(2)^2(-3)^5}{\binom{7}{4}(2)^3(-3)^4} = \frac{21(4)(-243)}{35(8)(81)} = \frac{\cancel{7}(3)(\cancel{4})(-3)(\cancel{81})}{\cancel{7}(5)(2)(\cancel{4})(\cancel{81})} = \frac{-9}{10}.$ **Ans. -9/10**

3. Distinct one-digit numbers: $1 - 9 =$. 2–digit: $(9)(9) = 81$; 3-digit: $(9)(9)(8) = 648$;

4-digit: $(9)(9)(8)(7) = 4536$. The sum is 5274. **Ans. 5274**

Polynomials

1. $5x^3 - 2x^2 + 3x + (x^3 - 2) = 6x^3 - 2x + 3x - 2$ **Ans. $6x^3 - 2x^2 + 3x - 2$**

The factors of 8 are 1, 2, 4, 8 and the factors of 2 are 1, 2. Roots can be positive or negative.

So the possible roots are $\pm\left(1, \frac{1}{2}, 2, 4, 8\right)$ The product: $-1\left(\frac{-1}{4}\right)(-4)(-16)(-64) = -1024$. **Ans. -1024**

3. The sum of the roots is 5. The sum of the rest: $5 - (5 + 3i) = -3i$. **Ans. -3i**

Areas and Volumes

1. $6(8) + 2(8)(12) + 2(6)(12) = 48 + 192 + 144 = 384$. **Ans. 384**

2. Since the volumes are in a ratio of 8 to 27, then the radii are in a ratio of 2 to 3. This makes the surface areas ratio 4 to 9. Thus $\frac{4}{9}(270,000,000) = 120,000,000$. **Ans. 120,000,000**

3. If the density of salt is 27 grains per mm^3, then this would be 27,000 grains per cm^3.

The volume of the salt shaker is $\frac{1}{3}\left(\frac{\sqrt{3}}{4}\cdot 2^2\right)(8) = \frac{8}{3}\sqrt{3}$ cm^3. Number of grains:

$\frac{8}{3}\sqrt{3}(27,000) = 124,707.658$ ➔ Rounded: 124,700. **Ans. 124,700**

Team

1. The differences between successive terms is 0, 1, 2, 3, 4, thus the nest three terms are 16, 22, and 29. $16 + 22 + 29 = 67$. **Ans. 67**

2. Set a, b and c each equal to 1. Then $(2 - 3 + 5)^2 = 4^2 = 16$. Or, multiply out to get $4a^2 + 9b^2 + 25c^2 - 12ab + 20ac - 30bc$. $4 + 9 + 25 - 12 + 20 - 30 = 16$. **Ans. 16**

3. The sum the negative of the coeficient of the x^3 term: $-(-6) = 6$. **Ans. 6**

4. First term is 13, last term is ($100/13 = 76^+$, so there are 76 terms) ➔ $13 + 75(13)$.

Thus the sum is $\frac{76}{2}(13+13+75\cdot 13) = 8(77)(13) = 38,038$. **Ans. 38,038**

5. $\frac{8!}{6!2!}(x)^6\left(-\frac{1}{x^3}\right)^2 = 28$. **Ans. 28**

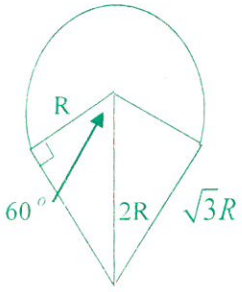
6. 3 possibilities for the driver. 7 are left to for possibilities for "shot-gun" seat. The next person has 7 seats to choose from, he next 6, etc until the last person (probably a freshman) has two to choose from. Thus $3\cdot 7\cdot 7\cdot 65\cdot 4\cdot 3\cdot 2 = 105,840$. **Ans. 105,840**

7. The sequence can be rewritten as $17(1 + 2 + 3 + \ldots + n)$. The sum of $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$. So $n(n + 1) > 2,000,000/17$. $\sqrt{\frac{2,000,000}{17}} \doteq 342.997$. So try 342 and 343:

$17(342)(343) = 1,994,202$; $17(343)(344) = 2,005,864$. Thus $n = 343$ and $17(343) = 5831$.

Ans. 5831

8. Let R = radius of circular arcs. Then one lobe of the track consists of



Of a sector and two 30-60-90 triangles. So ½ mile $= \frac{2}{3}(2\pi R)+2\sqrt{3}R)$ and

$R = \frac{1}{2\left(\frac{4}{3}\pi+2\sqrt{3}\right)} \doteq 0.06533478$ miles. Total enclosed area =

$2\left(\frac{2}{3}\pi R^2+\sqrt{3}R^2\right)=\left(\frac{4}{3}\pi+2\sqrt{3}\right)R^2 \doteq 0.03266739 \text{ mi}^2 = 910{,}714.5591 \text{ ft}^2$

Rounded to nearest hundredth: 910,700 ft^2 **Ans. 910,700 ft^2**

9. Let $s = \frac{a+b+c}{2}$ denote the semi-perimeter of this triangle. By Heron's formula, the area is $\sqrt{s(s-a)(s-b)(s-c)}$. However, Vieta's formula tells us that the sum of the roots a + b + c = -(-4), so s = 2. Thus we wish to evaluate $\sqrt{2(2-a)(2-b)(2-c)}$. Note that we can factor the polynomial in terms of its roots as p(x) = (x – a)(x – b)(x – c), so

p(2) = (2 - a)(2 -b)(2 - c) = 0.1. This gives us $\sqrt{2(2-a)(2-b)(2-c)} = \sqrt{2(0.1)} = \sqrt{0.2} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$.

Ans. $\frac{\sqrt{5}}{5}$ or 0.4472

Answer Sheet Nov 2015

Arithmetic with Ratio and Proportion

1. 52
2. 84
3. 10

Series and Sequences

1. 1946
2. 243/8 or $30\frac{3}{8}$ or 30.375
3. 39

Team

1. 67
2. 16
3. 6
4. 38,038
5. 28
6. 105,840
7. 5831
8. 910,700 or 910,700 ft^2
9. $\frac{\sqrt{5}}{5}$ or 0.4472

Counting Principles and Binomial Theoren

1. 1200
2. -9/10 or - 0.9
3. 5274

Polynomials

1. $6x^3 - 2x^2 + 3x - 2$
2. -1024
3. -3i

Areas and Volumes

1. 384 or 384 in^2
2. 120,000,000 or 120,000,000 mi^2
3. 124,700 or 124,700 grains