1 Arithmetic with Ratio and Proportion Nov 2016-17 (No Calculators)

3 pts 1. Kelly is negotiating to buy a used car. The listed price is $8,000. Kathy offers to pay 9/10 of the listed price. The owner counters with a price that is 11/10 of Kathy's offer. What is the owner's counter offer amount?

Ans. _____

4 pts 2. The average age of a group consisting of doctors and lawyers is forty years exactly. If the average of the doctor's ages is 35 years exactly and that of the lawyers is 50 years exactly, find the ratio of the number of doctors to the number of lawyers.

Ans. _____

5 pts 3. Solve for N, if $6!7! = N!$

Ans. _____

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2 Series and Sequences Nov 2016-17 (No Calculators)

3 pts 1. What is the sum of the first 100 positive odd integers?

Ans. _____

4 pts 2. Find k given that $17 + 19 + 21 + 23 + \ldots + k = 1700 + k$.

Ans. _____

5 pts 3. If you continued the triangular array of numbers shown in the figure, what number would be directly below 122?

1

2 3 4

5 6 7 8 9

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2016-17 (No Calculators)

3 pts 1. From a group of 12 boys a basketball team of 5 boys is to be made. How many different teams are possible?

Ans. _____

4 pts 2. On a musical instrument there are four valves, which are either open or shut to vary the sounds. How many different valve arrangements are possible?

Ans. _____

5 pts 3. What is the quotient, in simplest form, of the fifth term divided by the eighth term of $(x+y)^{13}$.

Ans. _____

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4 Polynomials Nov 2016-17 (No Calculators)

3pts 1. Find all values of a such that $2(a^2-7a)-5(a-8)+2=0$.

Ans. _____

4 pts 2. Find k so that a^2+5a-8 is a factor of $a^4+7a^3-5a^2+ka+56$.

Ans. _____

5 pts 3. Find p so that the roots of $x^3+3px^2+2px+24=0$ are integers and are in arithmetic progression.

Ans. _____

5 Areas and Volumes Nov 2016-17 (Calculators Allowed)

3 pts 1. Find the positive difference between the area of a circle with radius 1 and the area of a square with side length 2. Give exact answer or answer rounded to four decimal places.

Ans. _____

4 pts 2. The volume of a right hexagonal pyramid is $48\sqrt{3}$. The side length of the base is 4. Find the height of the pyramid.

Ans. _____

5 pts 3. Find the volume of the largest cube that can fit inside a sphere of radius 6. State your answer as exact or rounded to four decimal places.

Ans. _____

6 Team Nov 2016-17 (Calculators Allowed)

3 pts 1. Find the greatest monomial factor of the expression:

$117a^3b^3c^3 - 52a^2bc^2 + 143ab^3c^2$ (1)**Ans.**_____**3 pts**

3 pts 2. A cylindrical oil tank is 2/5 full. If 6 more kiloliters are added, the tank will be 5/8 full. What is the capacity of the tank?

(2) Ans. _____**3 pts**

3 pts 3. In a collection of nickels, dimes and quarters, the ratio of nickels to dimes is 3 to 5 and the ratio of dimes to quarters is 2 to 5. If there is \$43.75 in quarters, what is the value of the collection?

(3) Ans. _____**3 pts**

4 pts 4. Near the beginning of a party, everyone shakes hands with everyone else. Later a stranger arrives and shakes hands with only those people he knows. Altogether 68 handshakes occur. How many people, other than himself, did the stranger know?

(4) Ans. _____**4 pts**

4 pts 5. Find all values of k for which the equation $12x^2 - kx + 25 = 0$ has two distinct real solutions. **(5) Ans.** _____**4 pts**

4 pts 6. The three sides of a right triangle are in an arithmetic progression. The area of the triangle is 150 square feet. Find the perimeter of the triangle.

(6) Ans. _____**4 pts**

4 pts 7. A ball dropped from 100 feet to an asphalt surface rebounded 75 feet, and continued at the same rate until it came to rest. A second ball dropped from the same height to a wooden surface rebounded 62½ feet and continued at the same rate until it came to rest. Find the ratio of the total distance the ball rebounding from the wooden surface is to the total distance both covered before coming to rest?

(7) **Ans.** _____**5 pts**

5 pts 8. An integer is called "decreasing" if each digit is less than the digit to its left. For example, 9762 is "decreasing". How many "decreasing" integers occur between 2000 and 7000?

(8) Ans. _____**5 pts**

5 pts 9. A circle of radius 1 is circumscribed by a square. The square is then circumscribed by a circle, and the then circle is circumscribed by a square, and so on. What is the area of the four regions between the n^{th} square and the circle inscribed in it?

(9) **Ans.** _____**5 pts**

Solutions – Arithmetic with Ratio and Pro portion

1. $8000 \cdot \frac{9}{10} \cdot \frac{11}{10} = 7920.$ **Ans. $7920**

2. $35D + 50L = 40(D + L) = 40D + 40L \rightarrow 10L = 5D \rightarrow \frac{D}{L} = \frac{10}{5} = \frac{2}{1}.$ **Ans. 2 to 1**

3. $6!7! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = (4 \cdot 2)(7!)(6 \cdot 5 \cdot 3) = 8 \cdot (5 \cdot 2)(3 \cdot 3)7! = 10!$ **Ans. 10**

Series and Sequences

1. 1st term is 1. Sum of first 2 terms is 1 + 3 = 4. Sum of first 3 terms is 4 + 5 = 9. So the sum of the first 100 terms is $100^2 = 10000$. **Ans. 10,000**

2. $1700 = n\left(\frac{17+17+(n-1)2}{2}\right) \rightarrow 3400 = n(32 + 2n) \rightarrow n^2 + 16n - 1700 = 0 \rightarrow$

$(n - 34)(n + 50) = 0$, so $n = 34$. The last number for the sum to make 1700 is $L = 17 + 33(2) = 83$. So the next number, or k, is 85. **Ans. 85**

3. The numbers on the end of each row are perfect squares. So the end of the 11th row would be 121. And the beginning of the 12th row would be 122. The twelth row finishes with 144, so the nest row starts with 145, and 146 falls right underneath 122. **Ans. 146**

Counting Principles and Binomial Theorem

1. ${}_{12}C_5 = \frac{12!}{7!5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 7!} = 11 \cdot 9 \cdot 8 = 792.$ **Ans. 792**

2. No valves – 1 way: 1 valve - 4 ways: 2 valves - ${}_4C_2 = 6$; 3 valves – 4 ways; 4 valves – 1 way.

In all there are: 1 + 4 + 6 + 4 + 1 = 16 ways. **Ans. 16**

3. $\frac{\binom{13}{4}x^9y^4}{\binom{13}{7}x^6y^7} = \frac{7!6!}{4!9!}\left(\frac{x^3}{y^3}\right) = \frac{6 \cdot 5}{9 \cdot 8}\left(\frac{x^3}{y^3}\right) = \frac{5x^3}{12y^3}$ **Ans.** $\frac{5x^3}{12y^3}$

Polynomials

1. $2(a^2 - 7a) - 5(a - 8) + 2 = 0 \rightarrow 2a^2 - 14a - 5a + 40 + 2 = 0 \rightarrow 2a^2 - 19a + 42 = 0.$

$(2a - 7)(a - 6) = 0$. Therefore $a = 6$ or $3\frac{1}{2}$. **Ans. 6 or 3½**

2.

$$a^2 + 2a - 7$$
$$a^2 + 5a - 8 \;\overline{)\; a^4 + 7a^3 - 5a^2 + ka + 56}$$
$$\underline{a^4 + 5a^3 - 8a^2}$$
$$2a^3 + 3a^2 + ka$$
$$\underline{2a^3 + 10a^2 - 16a}$$
$$-7a^2 + (k+16)a + 56$$
$$\underline{-7a^2 \quad -35a + 56}$$
$$(k+16+35)a$$

If $a^2 + 5a - 8$ is a factor, then $k + 16 + 35 = 0$ and k = -51.

Ans. -51

3. The possible roots of $x^3 + 3px^2 + 2px + 24 = 0$, since their product is -24 are 1, 1, 24; 1, 2, 12; 1, 3, 8; 1, 4, 6; 2, 2, 6; and 2, 3, 4. One of the roots or all of the roots have to be negative. To form an arithmetic progression either -2, -3, -4 or -2, 2, 6 are the only possible choices.

$(x + 2)(x + 3)(x + 4) = x^3 + 9x^2 + 26x + 24$. The 3p, 2p ratio does not work with 9 and 26.

$(x - 2)(x + 2)(x - 6) = x^3 - 6x^2 - 4x + 24$. This produces the proper ratio. So p = -2. **Ans. -2**

Areas and Volumes

1. Square area = 4. Circle area = π. 4 is greater than π. **Ans. 4 – π** or **0.8584**

2. The area of the base, $B = 6\left(\frac{e^2\sqrt{3}}{4}\right) = 6\left(\frac{4^2\sqrt{3}}{4}\right) = 6(4\sqrt{3}) = 24\sqrt{3}$. The volume is $V = \frac{1}{3}\pi Bh$.

$48\sqrt{3} = \frac{1}{3}\left(24\sqrt{3}\right)h$ ➔ 2 = 1/3 h, so h = 6. **Ans. 6**

3. The longest diagonal of the cube will equal the radius of the sphere. $12 = \sqrt{3s^2}$ ➔

$144 = 3s^2$ ➔ $48 = s^2$. So $s = 4\sqrt{3}$. $V = \left(4\sqrt{3}\right)^3 = 192\sqrt{3}$. **Ans. $192\sqrt{3}$** or **332.5538**

Team

1. $117a^3b^3c^3 - 52a^2bc^2 + 143ab^3c^2 = 13abc^2(9a^2b^2c - 4a + 11b^2)$. **Ans. $13abc^2$**

2. 5/8 – 2/5 = 25/40 – 16/40 = 9/40. 9/40 x = 6 ➔ $x = 6 \cdot \frac{40}{9} = \frac{80}{3} = 26\frac{2}{3}$. **Ans. 26 2/3**

3. 4375/25 = 175. The # of dimes: $\frac{2}{5} = \frac{d}{175}$ ➔ 5d = 2(175) ➔ d = 70. The # of nickels: $\frac{3}{5} = \frac{n}{70}$

$5n = 210$, so there are 42 nickels. $42n + 70d + 175q = 2.10 + 7.00 + 43.75$. **Ans. $52.85**

4. For n people there are ${}_nP_2$ handshakes. If 11 people there would be 55 handshakes with 13 left over, not. 12 would be 66 with 2 left over. Thus he knew only 2 people. **Ans. 2**

5. In order to have two distinct roots, the discriminant of the quadratic, $b^2 - 4ac$, must be greater than zero. Thus $k^2 - 1200 > 0 \rightarrow k^2 > 1200$ and $|k| > 20\sqrt{3}$. So $k < -20\sqrt{3}$ or $k > 20\sqrt{3}$.

Ans. $k < -20\sqrt{3}$ or $k > 20\sqrt{3}$

6. 3, 4, 5 are in an arithmetic progression. All multiples are also. So $(3x)(4x)/2 = 150 \rightarrow$ $6x^2 = 150$, thus $x = 5$. The three sides are 15, 20, 25. Thus the perimeter is 60. **Ans. 60**

7. First ball distance: $100 + 2\left(\frac{75}{1-3/4}\right) = 100 + 2(300) = 700$. Second ball:

$100 + 2\left(\frac{62\frac{1}{2}}{1-5/8}\right) = 100 + 2(\frac{125}{2} \cdot \frac{8}{3}) = 100 + \frac{1000}{3} = \frac{1300}{3}$. Ratio: $\frac{1300/3}{700 + 1300/3} = \frac{1300}{3400} = \frac{13}{34}$. **Ans. 13/34**

8. None in the 2000's. Only one in the 3000's, 3210. In the 4000's: 4321, 4320, 4310, 4210. There are 4. In the 5000's: 5432, 5431, 5430, 5421, 5420, 5410, 5321, 5320, 5310, 5210. There are 10. In the 6000's: 6543, 6542, 6541, 6540, 6532, 6531, 6530, 6521, 6520, 6510, 6432, 6431, 6430, 6421, 6420, 6410, 6321, 6320, 6310, 6210. There are 20. **Ans. 35**

9. The first circle has an area of π. Its circumscribed square has area 4. The next circle has a radius of $\sqrt{2}$, so the area is 2π. The area of its circumscribed square is 8. The area for the next larger circle or square doubles each time. The nth square would have an area of 2^{n+1}. The circle inscribed in it would have an area of $2^{n-1}\pi$. The area of the four segment formed by the square and the circle would there be $2^{n+1} - 2^{n-1}\pi$.

Ans. $2^{n+1} - 2^{n-1}\pi$

Answer Sheet – Nov 2016 – 2017

Arithmetic with Ratio and Proportion

1. **$7920** or **7920**
2. **2 to 1** or **2:1** or **2/1**
3. **10** or **N = 10**

Series and Sequences

1. **10, 000**
2. **85**
3. **146**

Counting Principles and Binomial Theorem

1. **792**
2. **16**
3. $\frac{.5x^3}{12y^3}$

Polynomials

1. **6 or 3½** or **6 or 7/2** or **6 or 3.5**
2. **-51** or **k = -51**
3. **-2** or **p = -2**

Areas and Volumes

1. **4 – π** or **0.8584**
2. **6**
3. **$192\sqrt{3}$** or **332.5538**

Team

1. **$13abc^2$**
2. **26 2/3 kiloliters** or **$26.\overline{6}$** or **26.6667**
3. **$52.85**
4. **2 people**
5. **$k < -20\sqrt{3}$ or $k > 20\sqrt{3}$**
6. **60** or **60 ft**
7. **13/34**
8. **35**
9. $2^{n+1} - 2^{n-1}\pi$ or $2^{n-1}(4-\pi)$