1 Arithmetic with Ratio and Proportion Nov 2017 (No Calculators)

3 pts 1. A rope 115 ft. long is to be cut into three lengths with a ratio of 5:7:11. What is the length of the middle-sized piece?

Ans. _____

4 pts 2. A merchant purchased a model of bikes from a dealer at a cost of $540 each. On June 1st he put a selling price on the bikes so as to make a 30% profit over the cost. The bikes did not sell well, so on July 1st he offered a 10% discount sale of the selling price. The bikes started selling well, so on August 1st he boosted the July 1st selling price by 5%. The bikes continued to sell well. What was this final selling price?

Ans. _____

5 pts 3. a and b are in the ratio of 3 to 8. b and c are in the ratio of 2 to 5. c and d are in a ratio of 7 to 11. What is the ratio of a to d in simplest form?

Ans. _____

2 Series and Sequences Nov 2017 (No Calculators)

3 pts 1. In the sequence 7, 15, 23, …, the nth term is 143. Find n.

Ans. _____

4 pts 2. The 10^{th} term of an arithmetic sequence is 142. The 15^{th} term is 187. Find the 25^{th} term.

Ans. _____

5 pts 3. The 5^{th} term of a geometric sequence is 648. The 8^{th} term is 2187. Find the sum of the first three terms.

Ans. _____

3 Counting Principles and Binomial Theorem Nov 2017 (No Calculators)

3 pts 1. A set consists of 9 distinct elements. How many different subsets of 3 distinct elements can be formed from this set?

Ans. _____

4 pts 2. Find the 4th term in the expansion of $(3x-5y)^7$.

Ans. _____

5 pts 3. How many even 4-digit numbers can be formed using 0, 2, 3, 5, 6, 7, 8 or 9 for each digit in the numbers, and in each number the digits are distinct?

Ans. _____

4 Polynomials Nov 2017 (No Calculators)

3 pts 1. Find the product of $4x-9$ and $3x^2-5x+2$.

Ans. _____

4 pts 2. Find all value(s) of x such that $15x^3+71x^2+86x+24=0$.

Ans. _____

5 pts 3. If $p(x+3) = 5x^2+27x+38$, find $p(x-3)$.

Ans. _____

5 Areas and Volumes Nov 2017 (You may use Calculators)

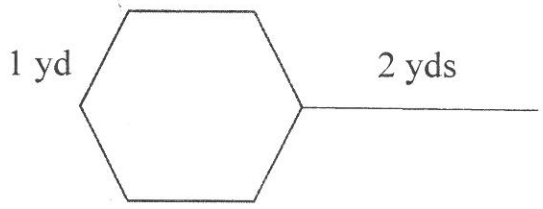
3 pts 1. A right circular cone is 12 inches high and has a base diameter of 10 inches. Find its volume.

Ans. _____

4 pts 2. A rhombus is formed by two radii and two chords of a circle whose diameter is 32. What is the area of the rhombus?

Ans. _____

5 pts 3. Ruby's doghouse is in the shape of a regular hexagon. Ruby's rope, attached to the front of the doghouse as shown, is 2 yards long. Each side of the doghouse is 1 yard long. What is the area that the rope can reach outside of the doghouse?



Ans. _____

6 Team Nov 2017 (You may use calculators)

3 pts 1. What is the sum of the coefficients of the expansion of $(x+y)^5$? **(1) Ans. _____3 pts**

3 pts 2. Sarah is training for a long-distance cross country skiing event. On Jan 1 she skied a distance of 2 miles. On each subsequent day she skied 1.5 miles farther than the day before. On what day will she ski more than 30 miles in a single day?

(2) Ans. _____3 pts

3 pts 3. Give the constant term in the expansion of $\left(x^2-\frac{2}{x^5}\right)^7$. **(3) Ans. _____3pts**

4 pts 4. The first three terms of an arithmetic sequence are 2x – 3, 5x – 11 and 3x+1. What is the numerical value of the 10^{th} term?

(4) Ans. _____4 pts

4 pts 5. P(x) is a third degree polynomial with the coefficient of the x^3 term being 1. When P(x) is divided by x – 1, the remainder is 3 and when divided by x + 1 the remainder is 5. The graph of P(x) intersects the y-axis at y = 2. If P(4) = r, and s is the remainder when P(x) is divided by x – 6, find $r + s$.

(5) Ans. _____ 4 pts

4 pts 6. Pat, Terry and four friends are sitting at a round table. How many ways can they be arranged, if Pat and Terry refuse to sit next to each other? Assume that any arrangement in one direction is the same as the going in the other direction.

(6) Ans. _____ 4 pts

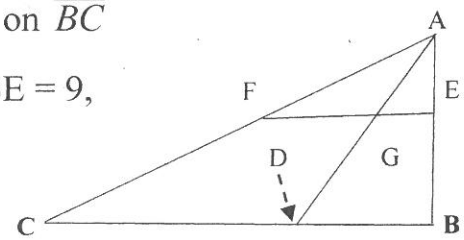
5 pts 7. A regular square pyramid, which is not oblique, has a base edge of 8. A plane parallel to the base intersects the pyramid to form a square 6 inches on a side. If the distance between the planes is 3, what is the volume of the frustum formed?

(7) Ans. _____ 5 pts

5 pts 8. We say that a number is "arithmetically sequenced" if the digits, in order, form an arithmetic sequence. Compute the number of 4-digit positive integers (none beginning with 0) which are "arithmetically sequenced".

(8) Ans. _____ 5 pts

5 pts 9. In the figure at right $m\angle ABC = 90^\circ$. Point D lies on $\overline{BC}$ and $\overline{AD}$ bisects $\angle CAB$. $\overline{FE}$ is parallel to $\overline{BC}$. AE = 3, BE = 9, and CF = 30. Find the area of quadrilateral DCFG.



(9) Ans. _____ 5 pts

Solutions – Arithmetic with Ratio and Proportion

1. $\frac{7}{23}(115) = 7(5) = 35.$ **Ans. 35 ft**

2. $540(1.3) = 702.$ $.9(702) = 631.8.$ $631.8(1.05) = 663.39$ **Ans. $663.39**

3. $\frac{a}{b} = \frac{3}{8} \rightarrow 8a = 3b \rightarrow b = \frac{8}{3}a.$ $\frac{b}{c} = \frac{2}{5} \rightarrow 5b = 2c,\ c = \frac{5}{2}b = \frac{5}{2}\cdot\frac{8}{3}a = \frac{20}{3}a.$ $\frac{c}{d} = \frac{7}{11} \rightarrow 11c = 7d$

$d = \frac{11}{7}c = \frac{11}{7}\cdot\frac{20}{3}a = \frac{220}{21}a.$ So $21d = 220a \rightarrow \frac{a}{d} = \frac{21}{220}.$ **Ans. 21:220**

Series and Sequences

1. $143 = 7 + (n-1)8 \rightarrow 136 = (n-1)8 \rightarrow 17 = n - 1 \rightarrow n = 18.$ **Ans. 18**

2. (1) $142 = a + 14d$, (2) $187 = a + 24d.$ (2) – (1): $45 = 5d,\ d = 9.$ $187 = a + 14(9) = a + 126,$ $a = 61.$ 25^{th} term: $61 + 24(9) = 61 + 216 = 277.$ Alternate solution: Since the 10^{th} term is 143 and the 15^{th} term is 187, then the difference is 45 for 5 terms. For 10 terms the difference is 90. $187 + 90 = 277.$ **Ans. 277**

3. $648n^3 = 2187,\ n^3 = \frac{2187}{648} = \frac{243}{72} = \frac{27}{8}.$ So $n = 3/2.$ $648 = a\left(\frac{3}{2}\right)^4 = a\left(\frac{81}{16}\right).$ $a = 648\left(\frac{16}{81}\right) =$ $8(16) = 128.$ $128(3/2) = 64(3) = 192.$ $192(3/2) = 96(3) = 288.$ $288 + 192 + 128.$ **Ans. 608**

Counting Principles and Binomial Theorem

1. ${}_9C_3 = 84.$ **Ans. 84**

2. $\binom{7}{3}(3x)^4(-5y)^3 = 35(81)x^4(-125)y^3 = -354{,}375x^4y^3$ **Ans. $-354{,}375x^4y^3$**

3. Since the numbers have to be even, then the unit's digit has to be 0, 2, 6, or 8. If it is 0, then there are 7 possible numbers for the first digit, then 6, and then 5. Multiplying = 210. If 2,6, 8 are the unit's digit, then the first digit cannot be 0, so that leaves 6 for the first digit. Then 0 can go in the next digit's place making 6 and the last digit 5 possible numbers left. Multiplying $6(6)(5)(3) = 30(18) = 540.$ $540 + 210 = 750.$ **Ans. 750**

Polynomials

1. $(4x - 9)(3x^2 - 5x + 2) = 12x^3 - 20x^2 + 8x - 27x^2 + 45x - 18.$ **Ans. $12x^3 - 47x^2 + 53x - 18$**

2.

	15	71	86	24
-3		-45	-78	-24
	15	26	8	0

So $15x^2 + 26x + 8 = 0$

$(3x + 4)(5x + 2) = 0.$ $x = -3, -4/3, -2/5$

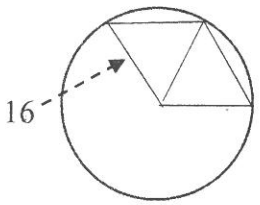
Ans. -3, -4/3, -2/5

3. $P(x-3)$ is 6 units less than $P(x+3)$ Thus $P(x-6) = 5(x-6)^2 + 27(x-6) + 38 \rightarrow$ $5(x^2 - 12x + 36) + 27x - 162 + 38 \rightarrow 5x^2 - 60x + 180 + 27x - 124$. **Ans. $5x^2 - 33x + 56$**

Areas and Volumes

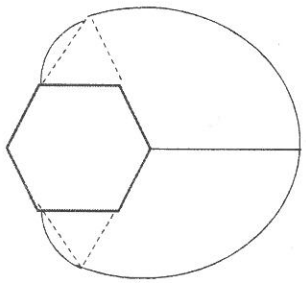
1. $V = \frac{1}{3}(5)^2 \pi(12) = 100\pi$. **Ans. 100π**

2. The figure at right shows two equilateral triangles with 16 ft.



Area $= 2\left(\frac{16^2\sqrt{3}}{4}\right) = 128\sqrt{3}$. **Ans. $128\sqrt{3}$**

3. The area will be 2/3 of a 2 yard radius circle, then 1/6 of a 1 yard radius circle two times gives $\frac{2}{3}\pi(4) + 2\left(\frac{1}{6}\pi(1)\right) =$



$\frac{8}{3}\pi + \frac{1}{3}\pi = 3\pi$. **Ans. 3π**

Team

1. The sum of the coefficients in each power has a power of 2. $2^5 = 32$. **Ans. 32**

2. $2 + 1.5(d-1) = 30 \rightarrow 1.5d - 1.5 = 28 \rightarrow 1.5d = 29.5 \rightarrow d = 19\frac{2}{3}$. So on the 20th day Sarah would be skiing over 30 miles on that day. **Ans. Jan 20**

3. $\binom{7}{2}(x^2)^5\left(\frac{-2}{x^5}\right)^2 = 21(4) = 84$. **Ans. 84**

4. The common difference is the difference between successive terms. Thus

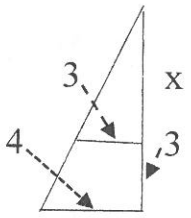
$5x - 11 - (2x - 3) = 3x + 1 - (5x - 11) \rightarrow 3x - 8 = -2x + 12 \rightarrow 5x = 20$, so $x = 4$. $2x - 3 = 5$, $5x - 11 = 9$, $3x + 1 = 13$. The 10th term: $5 + 9(4) = 41$. **Ans. 41**

5. $P(x) = x^3 + ax^2 + bx + c$, $P(1) = 3 \rightarrow 3 = 1 + a + b + c \rightarrow$ (1) $a + b + c = 2$; $P(-1) = 5 \rightarrow$ $5 = -1 + a - b + c = 0 \rightarrow$ (2) $a - b + c = 6$; $P(0) = 2$, $2 = c$. In (1): $a + b = 0$; in (2) $a - b = 4$. Adding these two: $2a = 4$, so $a = 2$ and $b = -2$. Thus $P(x) = x^3 + 2x^2 - 2x + 2$.

$P(4) = 4^3 + 2(16) - 2(4) + 2 = 64 + 32 - 6 = 90$. $P(6) = 6^3 + 2(36) - 2(6) + 2 = 216 + 72 - 10 = 278$. $P(4) + P(6) = 90 + 278 = 368$. **Ans. 368**

6. Arranging people around a round table is (n – 1)! If Pat sits down, 5 people have a chance to sit to his right. Once that person is seated, then there are 4 people that can sit to his right, etc. So putting them all around the table is 5!. Putting Pat and Terry side by side would be 4!. Subtracting 5! – 4! = 120 – 24 = 96. **Ans. 96**

7. Dropping the altitude from the vertex of the pyramid, the frustum could have been made from, and using the triangle formed by half the base and the altitude to one of the triangular lateral faces gives us a way to find the length of the altitude of that pyramid. $\frac{x}{x+3}=\frac{3}{4}$ ➔ 4x = 3x + 9, so x = 9.



The volume of the large pyramid is $\frac{1}{3}(64)(12) = 256$. The volume of the small pyramid is $\frac{1}{3}(36)(9) = 108$. Thus 256 – 108= 148, the volume of the frustum. **Ans. 148**

8. There are 9 numbers with sequence difference of 0, from 1111 through 9999. There are 6 with a difference of 1, 1234 through 6789. There are 3 with a difference of 2, 1357 through 3579. There are 7 with a difference of -1, 9876 through 3210. There are four with a difference of -2, 9753 through 6420. There is 1 with a difference of -3, 9630. 9+6+3+7+4+1. **Ans. 30**

9. AF = 10, so by the Pythagorean Theorem BC = $4\sqrt{91}$. Let CD = x, then by the angle bisector theorem: $\frac{AB}{AC}=\frac{BD}{DC}$ ➔ $\frac{3}{10}=\frac{4\sqrt{91}-x}{x}$ ➔ 3x = $40\sqrt{91}$ - 10x, thus CD = $\frac{40\sqrt{91}}{13}$. Since ΔAFG is similar to ΔACD and the corresponding sides are in a 1 to 4 ratio, then FG = $\frac{10\sqrt{91}}{13}$.

The area of trapezoid DCFG = $\frac{1}{2}\left(\frac{50\sqrt{91}}{13}\right)9 = \frac{225}{13}\sqrt{91}$. **Ans.** $\frac{225}{13}\sqrt{91}$

Answer Sheet Nov 2017

Arithmetic with Ratio and Proportion

1. **35 ft** or **35**
2. **$663.39** or **663.39**
3. **21:220** or **21 to 220** or $\frac{21}{220}$

Series and Sequences

1. **18** or 18th
2. **277**
3. **608**

Team

1. **32**
2. **Jan 20** or 20
3. **84**
4. **41**
5. **368**
6. ~~**96**~~ 36
7. **148**
8. **30** or 21 → Not counting the 9 sequences from 1,1,1,1 through 9,9,9,9 as they are considered trivial (d=0)
9. $\frac{225}{13}\sqrt{91}$ or **165.1049**

Counting Principles and Binomial Theorem

1. **84**
2. **$-354{,}375x^4y^3$**
3. **750**

Polynomials

1. **$12x^3 - 47x^2 + 53x - 18$**
2. **-3, -4/3, -2/5 (check variations)**
3. **$5x^2 - 33x + 56$**

Areas and Volumes

1. **100π** or **314.1593**
2. **$128\sqrt{3}$** or **221.7025**
3. **3π** or **9.4248**