1. Arithmetic with Ratio and Proportion ***November 1991***

1. Simplify $5+\dfrac{2}{3-\dfrac{5}{2-\dfrac{3}{5-\dfrac{2}{3}}}}$

Ans. _____

2. Find the units digit of the number N given that $57^{100}+56^{100}+55^{100}=N$.

Ans. _____

3. Find the smallest number such that if each of the following fractions were multiplied by it, the products would each be a whole number: $\dfrac{9}{20}, \dfrac{15}{28}, \dfrac{21}{32}$

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1992***

1. In a stairway there are 36 steps of equal height. If each step were 1 inch higher, there would have been 32steps. How high is each step?

Ans. _____

2. Simplify and reduce to the lowest form: $\dfrac{\left(\frac{3}{2}\div\frac{9}{16}\right)+\left(\frac{1}{2}-\frac{1}{3}+2\frac{5}{6}\right)}{\left(\frac{1}{3}+\frac{1}{4}\right)\div 7}$

Ans. _____

3. Lucy, Linus, and Charlie Brown's other friends decided to form a club. Dues were the same for everyone, and everyone paid with five coins. If \$4.37 was collected, how many pennies were there if each person used the same 5 coins, and all paid their dues?

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1994***

1. Henry paid for a meal at a restaurant. The bill included a tax that was 4½% of the meal, and a tip that was 15% of the meal. The total bill was $33.94. To the nearest cent, what was the price of the meal?

Ans. _____

2. Find the ratio of a to b if: $\frac{2a+3b-3c}{a-2b-4c}=\frac{3}{4}$

Ans. _____

3. 3 tabs cost the same as 5 taps, while 10 tabs cost the same as 7 tops. If 25 taps cost $16, find the cost of 105 tops.

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1995***

1. An 8-foot piece of tin is to be cut into three pieces in the ratio of 1:2:3. How long will the middle sized piece be if the cut is made by a pair of tin snips. Give answer in feet and inches.

Ans. _____

2. How many pairs of factors of 84 are in a ratio of 1:3?

Ans. _____

3. To use a concentrated liquid household cleaner, you mix 1 part cleaner with 8 parts water. One quart of this solution will clean an area of 250 sq. ft. How many gallons of water are needed to clean a 30 ft. by 25ft.? Five answer as exact or to nearest 100th.

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1996***

1. Find x, if the ratio of x to 36 is equal to the ratio of 4 to x.

Ans. _____

2. Jack has \$10.24 and Jill has \$8.96. When they go shopping they each agree to purchase item(s) by spending exactly half of the money that each has at the time of purchase, until each can no longer make a purchase. After all the possible purchases each can make, how much more money does Jill have then Jack?

Ans. _____

3. In a factory, 3 goobers are manufactured for every 4 gibbles. For every 3 gibbles, 5 gimlings are manufactured. And for every 5 gimlings, 6 gixcogs are manufactured. If the factory manufactures 130 objects in one day, how many of those objects are goobers?

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1998***

1. At the Sweet Shop, one can buy 8 Pips for a dive, 4 Quips for a quarter, and 13 Rips for a nickel. How many Rips can be purchased for the same cost as what one would pay for a total of 40 Pips and 40 Quips?

Ans. _____

2. Given: $\frac{a}{b} = 2, \frac{a}{c} = 12$, and $\frac{a}{d} = 2$, find the value of $\frac{ab}{cd}$.

Ans. _____

3. Each student in the third grade made an identical donation to the room fund. Each contributed by using exactly the same 10 coins. \$3.91 was collected. How many Student are in the third grade class?

Ans. _____

1. Arithmetic with Ratio and Proportion ***November 1999***

1. Simplify: $\frac{8 \cdot 9 \div 19 \cdot 8}{8 + 19 - 19 + 8}$ Give exact answer.

Ans. _____

2. If 3 pips = 5 pops and 7 pops= 9 paps, how many pips are in 105 paps?

Ans. _____

3. It takes 15 workers to build 500 ft. of fencing in 6 hours. At this same rate, how many workers are needed to build 1000 ft. of fencing in 9 hours?

Ans. _____

2. Sequences and Series ***November 1988***

1. How many terms are there in the following sequence: 4, 11, 18, 25, 32, …, 1124?

Ans. _____

2. The sum to infinity of the terms of an infinite geometric progression is 6. Find the first term(s) of the progression if the sum of the first two terms is 4 ½.

Ans. _____

3. The difference between two positive numbers is 10. The arithmetic mean of these two numbers is one more that its geometric mean. Find the product of the arithmetic mean and the geometric mean.

Ans. _____

2. Sequences and Series ***November 1989***

1. Find the sum of all the integers between 50 and 350 that end in 1.

Ans. _____

2. The first and fourth terms of a geometric progression are x^{-7} and x^{t} respectively. If the ninth term is x^{29}, find the value of t.

Ans. _____

3. The three infinite arithmetic progressions P, Q, and R have exactly one number in common whose value is between 700 and 800. Find this common value.

   P = 10, 14, 18, 22, 26, …
   Q = 47, 52, 57, 62, 67, …
   R = 23, 30, 37, 44, 51, …

Ans. _____

2. Sequences and Series ***November 1991***

1. If $3(1) + 3(2) + 3(3) + \ldots + 3(N) = 165$, find N.

Ans. _____

2. Find the sum of the following series: $48 + 40 + 36 + 30 + 27 + 22.5 \ldots$

Ans. _____

3. The second term of a geometric sequence is 54, and its fifth term is 16. An arithmetic sequence has the same 3rd term as the geometric sequence, but its 5th term equals the 4^{th} term of the geometric sequence. Find the sum of the first ten terms of the arithmetic sequence.

Ans. _____

2. Sequences and Series ***November 1992***

1. Mark's javelin throws increase by 4% for the first four throws. Thereafter his throws vary as much as 2% of his fourth throw. If his first throw is 200 meters, find his longest possible throw. Express your answer to the nearest meter.

Ans. _____

2. The first four terms of an arithmetic sequence are $3x + 2$, $5x + 1$, $6x + 3$ and $10x - 4$. Find the 20^{th}.

Ans. _____

3. The second term of a geometric series is 21 more than the first. The third term is 84 more than the second. Find the three terms.

Ans. _____

2. Sequences and Series ***November 1993***

1. What is the common difference, if one inserts 23 arithmetic means between –5 and 59?

Ans. _____

2. Find all numerical value(s) of k, so that the sequence $2k^2 - 5k + 2$, $k^2 - k + 3$, $3k^2 - 3k - 5$ is an arithmetic progression.

Ans. _____

3. Three non-zero numbers form an arithmetic progression. Increasing the first term by 2 results in a geometric progression. Increasing the third term of the original by 4 results in yet another different geometric progression. Find all value(s) of the second number.

Ans. _____

2. Sequences and Series ***November 1994***

1. Which term of the sequence 85, 82, 79, … is –200?

Ans. _____

2. In a certain sequence of numbers, the first term is 1, and for $2 \geq n$, the product of the first n numbers in the sequence is n^2. The sum of the third and fifth terms of this sequence is P. Find the numerical value of P.

Ans. _____

3. A rubber ball bounces back two-thirds of the distance it falls. It is dropped from a height of six feet. The ball is caught at the apex of the bounce after the ball has bounced from the floor five times. How many feet has the ball traveled in all? Express your answer in exact form.

Ans. _____

2. Sequences and Series ***November 1995***

1. Find three positive geometric means between 8 and 1152.

Ans. _____

2. A substance initially weighing 64 grams is decaying at a rate such that after 4 hours only 32 grams are left, after 2 hours only 16 grams are left, after 1 hour only 8 grams are left and so on. How long will it take before nothing is left?

Ans. _____

3. The first three terms of a sequence are in an arithmetic series with the first term being 4 more than the second term. All terms after the third are found by adding the two previous terms. Is the fifth term is 340, find the ninth term.

Ans. _____

2. Sequences and Series ***November 1996***

1. Insert 3 arithmetic means between 11 and 20.

Ans. _____

2. Find the sum of the 1st 12 terms of the sequence: 1, 3, 9, 2, 7, 9, 4, 11, 9, …

Ans. _____

3. The 22^{nd} term of an arithmetic series is 119, the 30^{th} term is 47. Find the 8^{th} term.

Ans. _____

2. Sequences and Series ***November 1998***

1. How many terms of the sequence 29, 24, 19, … are necessary so that the sequence ends with -101?

Ans. _____

2. If an arithmetic sequence has 1 for a first term, 37/64 for the 82^{nd} term, and 0 for the last term, how many terms are there in the sequence?

Ans. _____

3. During 1998, 1400 acre-feet of water will flow over Harmony Falls. If the flow of water over the falls decreases at a rate of 1% per year, how many acre-feet of water will flow over the falls from January 1, 1999 through the end of time?

Ans. _____

2. Sequences and Series ***November 1999***

1. Find the 10^{th} term of the sequence which begins 5, 3.0, 1.8, … Round answer to the nearest $10{,}000^{th}$.

Ans. _____

2. The 19^{th} term of an arithmetic sequence is -8. the 85^{th} term is 1114. Find the 65^{th} term.

Ans. _____

3. The first three terms of a geometric series of positive integers is represented in x as follows: $x - 8$, $2x - 1$, $10x - 5$. Find the numerical value of the 4^{th} term.

Ans. _____

3. Counting Principles and Binomial Theorem ***December 1988***

1. Two fair dice are thrown. If the sum is 6, what is the probability that one of the numbers is a two?

Ans. _____

2. In the expansion of $\left(4x^3 - \frac{y^2}{2}\right)^8$ find the coefficient of the term containing x^9y^{10}.

Ans. _____

3. How many different arrangements of the letters of the word THEATRE begin and end with a vowel? Consider only 7 letter arrangements.

Ans. _____

3. Counting Principles and Binomial Theorem ***December 1989***

1. How many natural number less than 450 can be formed using the digits 1, 2, 4, 5, 7 and 8 if no digit can be repeated in any number?

Ans. _____

2. Find *n* if the coefficient of the third term in the expansion of $(x + 4y)^n$ is 576.

Ans. _____

3. Find the probability of selecting at least two prime numbers when drawing three distinct numbers at random from the first twenty natural numbers. Give your answer in lowest terms.

Ans. _____

3. Counting Principles and Binomial Theorem ***November 1993***

1. 7 students from Central High, 6 students from Washington High and 7 students from North High have been selected to participate on a committee. The students from each high school shake hands once with each student from the other high school. How many handshakes take place?

Ans. _____

2. Mrs. Lumn's students are making a flag. It has 13 stripes altering red and white beginning and ending with white. 7 stripes are shorter to make room for the stars in the upper right corner. Each student is to make a stripe. In how many ways can the students place them correctly on the background?

Ans. _____

3. There are 8 seniors, 7 juniors and 4 sophomores on a committee. A leadership committee will be selected randomly, containing one member from each class. No siblings can sit on the committee at the same time. Each of 3 of the seniors has a junior sibling. How many possible combinations are there of different subcommittees which fit this format?

Ans. _____

3. Counting Principles and Binomial Theorem ***November 1994***

1. For what value of c is $-15{,}120x^4$ the 4^{th} term of $(2x + c)^7$?

Ans. _____

2. A pizza may be ordered with any, all, or none of the following toppings: pepperoni, sausage, ground beef, salami, mushroom, peppers, onions, and anchovies. How many different kinds of pizza can be made?

Ans. _____

3. After a group elects a president and a vice-president, it needs to select a committee of two from the remaining members of the group. The sum of the number of ways to select the officers and the number of ways of selecting the committee is 71. Find the number of people in the group.

Ans. _____

3. Counting Principles and Binomial Theorem ***November 1995***

1. Determine the 4^{th} term in the expansion of $(2x\text{-}1/2\ y)^6$

Ans. _____

2. A searching party has 12 people. They have decided to split up into 3 groups of 4 and will go to 3 separate locations to search. In how many ways can the groups be composed and sent out?

Ans. _____

3. Mr. Bumsted found that doing grocery shopping was not as easy as he thought. Among other things, he needed to get a toothbrush and toothpaste. At the store he had a choice of 3 brands of toothbrushes, each with 3 standards of softness and each in 5 different colors. That was the easy part. He then had a choice of 3 different brands of toothpaste, two of which had 3 different flavors and 3 different sizes of each. The other had 2 flavors and 2 different sizes of each. How many different choices does he have, because his wife told him that he can not come back with a pack of Dentine?

Ans. _____

3. Counting Principles and Binomial Theorem ***November 1996***

1. On Mike's 18th birthday he received 3 new shirts, 2 pairs of pants and a pair of shoes. He already had 14 shirts, 10 pairs of pants, and 5 pairs of shoes. If Mike is not concerned about matching his outfits and wears a different outfit each day, how old will he be when he runs through all possible outfits?

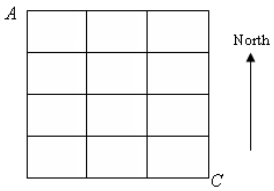
Ans. _____

2. Find the sum of the coefficients in the expansion of $(x + 1)^{11}$

Ans. _____

3. If Freddy the fly starts at point *A*, how many paths can he take to get to *C*, by moving only south and east? Freddy does not fly.

Ans. _____



3. Counting Principles and Binomial Theorem ***November 1998***

1. Seven people enter a cribbage tournament. Each entry is to play the other entries exactly once. How many games will be played in all?

Ans. _____

2. From the letters *A*, *B*, *C*, *D*, *E*, *F*, how many three letter "words" can be formed using the following criteria:
   A) The word must begin and end with a consonant.
   B) The middle letter must be a vowel.
   C) No letter may be used twice.

Ans. _____

3. Express the 4^{th} term in the expansion of the following polynomial in simplest form:
   $\left(x^2-(y-2)\right)^6$

Ans. _____

3. Counting Principles and Binomial Theorem ***November 1999***

1. Sara's family has a porch swing that holds three people. There are 10 ways of selecting the three people to sit on the swing. How many people are there in Sara's family?

Ans. _____

2. Find the numerical coefficient of *abc* in the expansion of: $\left((a+b)=c\right)^3$

Ans. _____

3. Al, Ben and Carl play youth football. Eight parents, each with a vehicle that can carry at least 4 growing boys, have volunteered to transport the youths to and from the game. Any one, two or all three of the boys can return home in any one or some of the 8 vehicles. In how many different ways can they board the vehicles and return home?

Ans. _____

4. Polynomials ***November 1988***

1. Find the value(s) of k so that $3x^2 - 7x - k = 0$.

Ans. _____

2. For what value(s) T will the polynomial $P : P(x) = 3x^3 + 6x^2 + Tx + k$,where T and k are rational numbers, have a zero equal to $\sqrt{2}$.

Ans. _____

3. Find $(p + q)$r if $x^4 + 4x^3 + 6px^2 + 4qx + r$ is exactly divisible by $x^3 + 3x^2 + 9x + 3$.

Ans. _____

4. Polynomials ***November 1989***

1. If $f : f(x) = 3x^2 + Px + Q,\ f(2) = 17$ and $f(-1) = -10,$ find $f(x)$ with integer coefficients.

Ans. _____

2. Two roots of the equation $x^3 + 3x^2 + Kx + Q = 0$are 1 and 3. Find the third root.

Ans. _____

3. If 2 is a root of $8x^3 + 14x^2 - 17kx + k^2 = 0$, find all the other positive rational roots.

Ans. _____

4. Polynomials ***November 1991***

1. If $f(x) = 12x^3 - x^2 - 24x - 12$ and $f(-2/3) = 0$, find all other zeroes of f.

Ans. _____

2. Determine all values of k so that the first polynomial is a factor of the second.
   $x^2 - x + 2,\ x^3 + kx^2 + 5x - 6$

Ans. _____

3. One of the roots of the equation below is –2i, what are the others?
   $2x^4 - x^3 + 2x^2 - 4x - 24 = 0$

Ans. _____

4. Polynomials ***November 1992***

1. Find k so that $2x^2 - 3x - 3k$ has only one root.

Ans. _____

2. Solve for x: $(3x - 8/x)^2 - 7(3x - 8/x) + 10 = 0$.

Ans. _____

3. Find all value(s) of k so that $(x + k)^4 - 5(x + k)^2 + 6$ has a remainder of 2 when divided by $x - 2$.

Ans. _____

4. Polynomials ***November 1993***

1. The polynomial function $f(x) = ax^2 + bx + c$ has zeroes of -1 and $-1\frac{1}{2}$. Find $f(x)$, where a, b and c are integers and are relatively prime.

Ans. _____

2. Solve for x: $x^3 - 2x - 1 = 0$ Express answers in simplest exact form, or to nearest thousandth, if not integers.

Ans. _____

3. Find a and b so that $2x^4 + x^2 + ax + b$ is divisible by both $x + 1$ and $x - 2$.

Ans. _____

4. Polynomials ***November 1994***

1. Find the remainder when $x^4 - 6x^3 + 7x^2 + 6x - 4$ is divided by $x^2 - 3x - 1$.

Ans. _____

2. For what values of c are the solutions to $2x^2 + cx + 2 = 2x$ imaginary?

Ans. _____

3. $P(x)$ is a cubic function. Two of its zeroes are -3 and -1. The average of the zeroes of $P(x)$ is the same as the average of the zeroes of $Q(x)$ which equals $x^4 - 8x^3 + 4x^2 + 28x + 15$. Find the equation of $P(x)$ in an unfactored form, if the y-intercept of $P(x)$ is -2.

Ans. _____

4. Polynomials ***November 1995***

1. Determine the sum of $A + B + C + D + E$, if
   $(x+2)(x+3)(x-2)(x+5) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

Ans. _____

2. Determine the value of k so that $x^2 - 5x + 6$ is a factor of $x^3 + kx^2 - 14x + 24$.

Ans. _____

3. If $P(x + 2) = x^3 + 9x^2 + 26x + 23$, find $P(x-2)$.

Ans. _____

4. Polynomials ***November 1996***

1. Find the zeroes of the function f, if $f(x) = 6x^2 + x - 12$

Ans. _____

2. Find the value of the constant c, if the difference of the roots of $x^2 - 11x + c = 0$ is 3.

Ans. _____

3. Which roots of $x^3 - 4x^2 + x + 6 = 0$ are also roots of $x^3 - 3x^2 - 4x + 12 = 0$.

Ans. _____

4. Polynomials ***November 1998***

1. Solve: $x^2 - 2x - 323 = 0$

Ans. _____

2. Given: $f(x) = x^3 - 2x^2 + 10x$ *and* $g(x) = \frac{-1}{4}x^2 + 3x + 3$ find all real values of x for which $f(x) = 4 \cdot g(x)$.

Ans. _____

3. Factor completely: $x^4 - 3x^3 - 7x^2 + 27x - 18$

Ans. _____

4. Polynomials ***November 1999***

1. Find the sum of the coefficients of the polynomial resulting from the product $(x+2)(x+3)(x+5)(x-1)$.

Ans. _____

2. Find the zeroes of the polynomial $12x^3 + 16x^2 - 27x - 36$.

Ans. _____

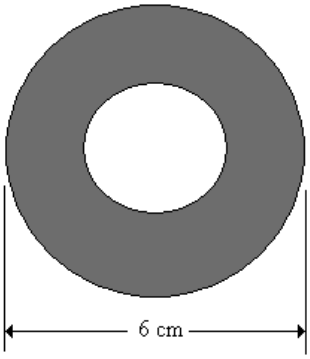
3. Find all the solutions of the equation $x^4 + 4x^3 - x^2 - 8x + 4 = 0$.

Ans. _____

5. Areas and Volumes ***November 1988***

1. Find r such that the area of the circular washer and the hole are equal. Give your answer in centimeters with a rational denominator.

Ans. _____



2. If the altitude, h, of a triangle is increased by a length, m, how much must be taken from the corresponding base b so that the area of the new triangle is one-half that of the original triangle? Give your answer in terms of b, h, and m as a single rational expression.

Ans. _____

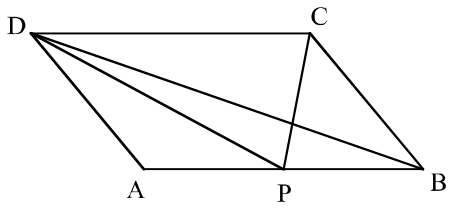
3. Find the volume in cubic centimeters of a cube with its diagonal 4 centimeters more that its edge.

Ans. _____

5. Areas and Volumes ***November 1991***

1. *ABCD* is a parallelogram. Segment *DP* is a median of triangle *ABD*. The area of quadrilateral *APCD* is 18. Find the area of quadrilateral *ABCD*.

Ans. _____



2. The area of the sector of a circle is 24π. The length of the arc of the sector is 6π. How long is the radius of the circle?

Ans. _____

3. The volumes of two similar cones are 375πand 1536π. What is the ratio of the surface area of the smaller to that of the larger?

Ans. _____

5. Areas and Volumes ***November 1992***

1. The bases of a trapezoid are 12 ft. and 26 ft. and its area is 285 sq. ft. Find the distance between the bases.

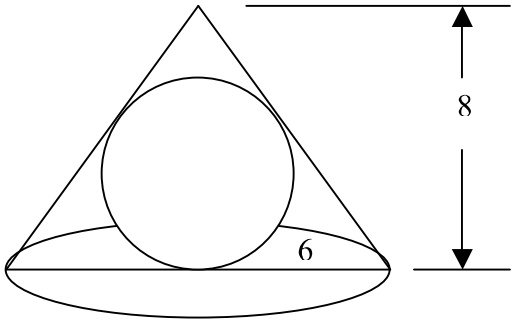
Ans. _____

2. A square and an equilateral triangle have the same perimeters. If the area of the triangle is $32\sqrt{3}$ cm, find the length of the diagonal of the square.

Ans. _____

3. Find the ratio of the volume of the right circular cone to the volume of the inscribed sphere below. The height of the cone is 8 and the base radius is 6.

Ans. _____



5. Areas and Volumes ***November 1993***

1. A pathway is cemented completely around a garden, which measures 4m by 6m. The pathway is 1.5m wide. Find the area of the pathway in square meters.

Ans. _____

2. How many different triangles with different base and height can be made with the following criteria:

   i. The area of each is 24 sq. in.
   ii. The measure of each base and height is an integer
   iii. The measure of the base is greater than the height

Ans. _____

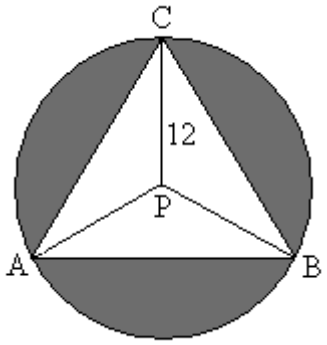
3. A ten-inch high cone with a twenty-inch base diameter is truncated. The truncated cone is half the volume of the original. Find the radius of the upper base of the truncated cone in inches. Express answer is simplest and exact form.

Ans. _____

5. Areas and Volumes ***November 1994***

1. Equilateral triangle *ABC* is inscribed in circle *P*. If the radius of the circle is *12*, find the area of the circle that is outside of triangle *ABC*.

Ans. _____

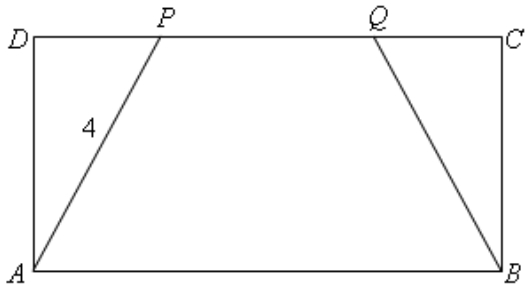


2. Find the volume of a regular square pyramid whose base diagonals and lateral edges have a length of 8 meters. Express your answer to nearest hundredth of a cubic meter or in exact form.

Ans. _____

3. Rectangle *ABCD* has an area of 21 sq. ft. Isosceles trapezoid *ABQP* has bases $\overline{AB}$ and $\overline{PQ}$ in a ratio of 7 : 3 respectively. If $\overline{AP}$ = 4 ft, find the longest possible length of $\overline{AD}$. Express answer to nearest 1000^{th}.

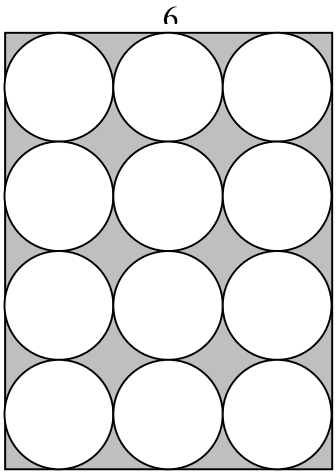
Ans. _____



5. Areas and Volumes ***November 1995***

1. Find the area of the shaded region. Give exact answer or to nearest 100th.

Ans. _____

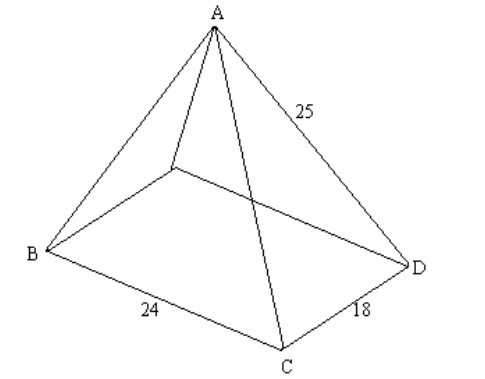


2. A rectangle pyramid has base dimensions of 8 by 12 and is 16 units high. Find the area of the cross-section 4 units from the base.

Ans. _____

3. Find the volume of this right rectangular Pyramid if $BC = 24$ cm. $CD = 18$cm and $AD = 25$ cm.

Ans. _____



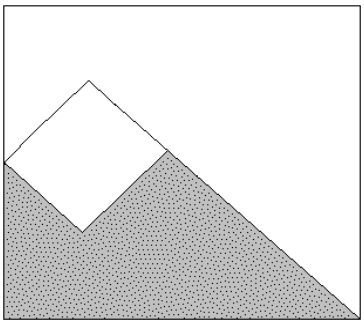
5. Areas and Volumes ***November 1996***

1. What is the volume of a spherical shell with inner radius 4 inches and outer radius 5 inches? Express answer in terms of π, in simplest form.

Ans. _____

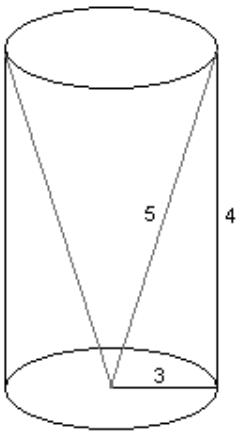
2. A child's puzzle is created by cutting a 10 by 10 inch piece of cardboard as shown below, with each cut being made at a 45° angle to a side of the original square. What is the area of the shaded region? Give exact answer.

Ans. _____



3. If a triangle with sides 3, 4, and 5 is rotated about one vertex as shown (with the axis of revolution parallel to the side of length 4), what is the volume of the resulting solid? Express your answer in terms of π.

Ans. _____



5. Areas and Volumes ***November 1998***

1. Find the diameter of a sphere that has a volume equal to its surface area.

Ans. _____

2. Sandy has a water glass in the shape of a right circular cylinder and a rack holding many right circular cones, point down. Each cone has the same radius and height as the water glass. How many of the cones can be filled half way to the top with all the water from the glass?

Ans. _____

3. The point of a compass is placed at the midpoint of side BC in equilateral triangle ABC. A circle passing through vertices B and C is drawn with the compass. If the length of a side of triangle ABC is 2, find the exact area of the region inside triangle ABC and outside the circle. Express your answer in simplest radical form in terms of π.

Ans. _____

5. Areas and Volumes ***November 1999***

1. Find the volume of a square pyramid whose height is twice as long as its base edge, if its base area is 144 cm^2.

Ans. _____

2. Find the area of a triangle whose sides are 8, 12, and 16.

Ans. _____

3. 7 ball bearings have diameters of 1 cm are placed in a shallow can of water which is filled to the top with water. The inside height of the can and of the water is the same height of each ball bearing. Find the volume of water left in the can, if all the ball bearings are tangent and these are tangent to the can, as pictured. Give exact answer.

Ans. _____