1 Probability Dec 2013 (No Calculators)

3 pts 1. There are 8 red, 7 white, 5 blue chips. A chip is chosen at random and then replaced. A second chip is then chosen. What is the probability that both are red?

Ans. _____

4 pts 2. A positive integer, n, is picked at random from the integers 1 to 30 inclusive. What is the probability that the greatest common factor of n and 30 is greater than 1?

Ans. _____

5 pts 3. Four people are selected at random. What is the probability that at least two of them were born on the same day of the week?

Ans. _____

2 Exponents and Radicals Dec 2013 (No Calculators)

3 pts 1. If $9^6 \cdot 27^5 \cdot 81^4 = 3^x$, find x.

Ans. _____

4 pts 2. Solve the following equation for x:

$$\left(\frac{1}{4}\right)^{-1} \cdot 8^{x-1} = \left(\frac{1}{16}\right)^{\frac{1}{2}x-1}$$

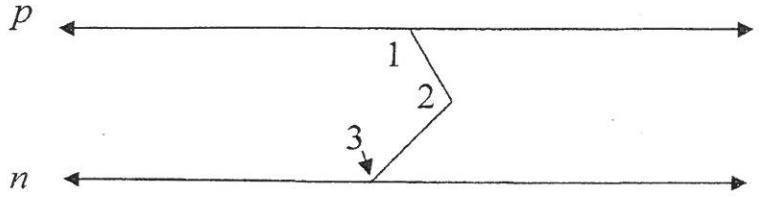
Ans. _____

5 pts 3. What does $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$ equal?

Ans. _____

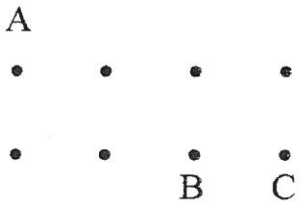
3 Lines, Angles, and Polygons Dec 2013 (No Calculators)

3 pts 1. In the figure, line p is parallel to line n, $m\angle 1 = 100°$, and $m\angle 2 = 120°$. Find the measure of angle 3.



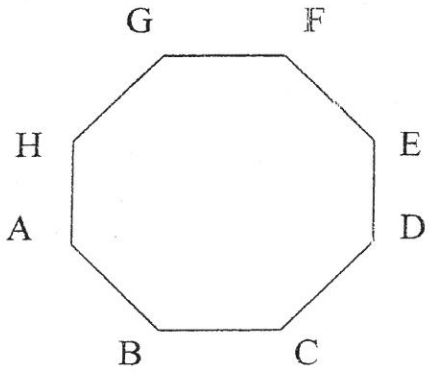
Ans. _____

4 pts 2. On the square grid shown each of the points, vertical and horizontal, is one unit apart and $AB = 3\sqrt{5}$. Find the length AC.



Ans. _____

5 pts 3. Regular octagon ABCDEFGH has a perimeter of 48. Find the length AE.



Ans. _____

4 Complex Numbers Dec 2013 (No Calculators)

3 pts 1. Simplify: $(3-4i)(4-3i)-(3-4i)(3-4i)$

Ans. _____

4 pts 2. Solve for z : $(3+i)z+4-i=2i.$

Ans. _____

5 pts 3. Express the following in simplest form:

$$\left(1-\sqrt{3}i\right)^8$$

Ans. _____

---

5 Arithmetic with Percent Dec 2013 (Calculators allowed)

3 pts 1. If x is 250 percent of y, then what percent of x is $3y$?

Ans. _____

4 pts 2. A square is changed into a rectangle by increasing its length by 20% and decreasing its width by 20%. By what percent is the area changed? Be sure to state percent increase or percent decrease.

Ans. _____

5 pts 3. 60 quarts of an orange juice solution is 70% pure orange juice. How many quarts of pure orange juice should be added to make a solution which is 75% pure orange juice?

Ans. _____

6 Team Dec 2013 (Calculators allowed)

3 pts 1. Find the probability that if a regular polygon of n sides is selected at random, where $n \leq 24$, then the number of degrees in each of the interior angles is an integer.

(1) **Ans.** _____ **3 pts**

3 pts 2. Simplify: $i^7(3-4i)(-3+4i)$. (2) **Ans.** _____ **3 pts**

3 pts 3. Express $2^{3/5} \cdot 4^{2/3} \cdot 8^{1/2}$ in simplest form with no exponents.

(3) **Ans.** _____ **3 pts**

4 pts 4. If $\left(\frac{2\sqrt[3]{4}}{\sqrt[6]{243}}\right)^5 \left(\frac{3\sqrt[4]{27}}{\sqrt[5]{8}}\right)^3 = 2^a 3^b$, find the value of $a + b$ as a mixed number.

(4) **Ans.** _____ **4 pts**

4 pts 5. \$8000 was put into a certain bank at 2½ % for a child's college education. The interest compounded annually. Three generations passed (82 years) before a descendent was college ready. How much money was ready for her education at the end of 82 years?

(5) **Ans.** _____ **4 pts**

4 pts 6. A rectangle measuring 11 units by 13 units is tiled with unit-square tiles. How many tiles are crossed by one diagonal of the rectangle?

(6) **Ans.** _____ **4 pts**

5 pts 7. Solve for z over the set of complex numbers: $z^2 = 7 + 24i$.

(7) **Ans.** _____ **5 pts**

5 pts 8. Find all ordered pairs (a, b), such that $(ai - b)(ai + b) = -29$ and $(ai)^4 = 625$.

(8) **Ans.** _____ **5 pts**

5 pts 9. What is the probability that a positive integer less than 124 is divisible by 2, 3, or 5?

(9)**Ans.** _____ **5 pts**

Solutions – Probability

1. $rr = \frac{8}{20}\cdot\frac{8}{20} = \frac{2}{5}\cdot\frac{2}{5} = \frac{4}{25}$ **Ans. 4/25**

2. The numbers 1 through 30 that do not have a GCF greater than one are 1, 7, 11, 13, 17, 19, 23, and 29. So 22 do. 22/30 = 11/15. **Ans. 11/15**

3. Let p = the probability that they were born on the same day of the week. Then there are 4 scenarios' of at least two of them. (1) only two, (2) three, (3) four, (4) two on one day and two others on a different day. You may want to consider the individuals as A, B, C, and D, especially on (4).

(1): $p\cdot p\cdot \not{p}\cdot \not{p}\binom{4}{2}$ + (2): $p\cdot p\cdot p\cdot \not{p}\binom{4}{1}$ + (3): $p\cdot p\cdot p\cdot p$ + (4): $p\cdot p\cdot d\cdot d\,(3)$

$$1\cdot\frac{1}{7}\cdot\frac{6}{7}\cdot\frac{5}{7}(6) \quad + \quad 1\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{6}{7}(4) \quad + \quad 1\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{7} \quad + \quad 1\cdot\frac{1}{7}\cdot\frac{6}{7}\cdot\frac{1}{7}(3) =$$

$$\frac{180+24+1+18}{343} = \frac{223}{343}$$ **Ans.** $\frac{223}{343}$

Exponents and Radicals

1. $9^6\cdot 27^5\cdot 81^4 = (3^2)^6\cdot(3^3)^5\cdot(3^4)^4 = 3^{12}\cdot 3^{15}\cdot 3^{16} = 3^{43} = 3^x$. So x = 43. **Ans. 43**

2. $\left(\frac{1}{4}\right)^{-1}\cdot 8^{x-1} = \left(\frac{1}{16}\right)^{\frac{1}{2}x-1} \rightarrow 2^2(2)^{3x-3} = 2^{-2x+4} \rightarrow 3x-1 = -2x+4 \rightarrow 5x = 5.$ **Ans. 1**

3. Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$, then $x = \sqrt{6+x} \rightarrow x^2 = 6+x \rightarrow x^2 - x - 6 = 0$.
Factoring: (x – 3)(x + 2) = 0. x cannot = -2. So x = 3. **Ans. 3**

Lines, Angles and Polygons

1. If you run line *m* parallel to the other two lines but passing through the vertex at angle 2, then angle two is split into an 80°, the supplement of $\angle 1$, and a 40° angle. The supplement of 40° is 140°, the measure of $\angle 3$. **Ans. 140°**

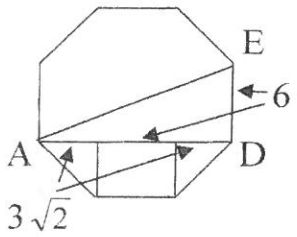
2. Let the unit between dots be x. Then $x^2 + (2x)^2 = (3\sqrt{5})^2 \rightarrow 5x^2 = 45$, so x = 3.
So $AC^2 = 3^2 + 9^2 = 90$, thus $AC = 3\sqrt{10}$. **Ans. $3\sqrt{10}$**

3. At right $AD = 6 + 6\sqrt{2}$. $AE^2 = (6 + 6\sqrt{2})^2 + 6^2 =$

$36 + 72\sqrt{2} + 72 + 36 = 144 + 72\sqrt{2}$. $AE = \sqrt{144 + 72\sqrt{2}} =$

$\sqrt{36(4 + 2\sqrt{2})} = \mathbf{6}\sqrt{4 + 2\sqrt{2}}$.

Ans. $\mathbf{6}\sqrt{4 + 2\sqrt{2}}$



Complex Numbers

1. $(3 - 4i)(4 - 3i) - (3 - 4i)(3 - 4i) = (3 - 4i)(1 + i) = 3 - 4i + 3i - 4i^2 = 7 - i.$ **Ans.** $\mathbf{7 - i}$

2. $(3 + i)z + 4 - i = 2i \rightarrow (3 + i)z = -4 + 3i \rightarrow z = \frac{-4 + 3i}{3 + i} \cdot \frac{3 - i}{3 - i} = \frac{-12 + 4i + 9i - 3i^2}{9 + 1} =$

$\frac{-9 + 13i}{10}$.

Ans. $\frac{-9 + 13i}{10}$

3. $(1 - \sqrt{3}\,i)^8 = (((1 - \sqrt{3}\,i)^2)^2)^2 = ((-2 + 2\sqrt{3}\,i)^2)^2 = (-8 - 8\sqrt{3}\,i)^2 = -128 - 128\sqrt{3}$.

Ans. $\mathbf{-128 - 128\sqrt{3}}$

Arithmetic with Percent

1. $x = 2.5\,y$, $\frac{3y}{2.5y} = \frac{6}{5} = 1.2 = 120\%$ **Ans. 120%**

2. Area of square x^2. Area of rectangle $(1.2x)(.8x) = .96x^2$. **Ans. decreased 4%**

3. $.70(60) + 1.00x = .75(60 + x) \rightarrow 70(60) + 100x = 75(60) + 75x \rightarrow 25x = 5(60) \rightarrow$
$25x = 300$ so $x = 12$. **Ans. 12 qts**

Team

1. If the number of degrees in each exterior angle is a fraction, then the interior angle measure must also be. Since the exterior angle sum is 360, then n must divide 360 evenly. Those that do not are 7, 11, 13, 14, 16, 17, 19, 21, 22 and 23. So 10 do not. Since $n \geq 3$, there are 22 polygons. $12/22 = 6/11$ **Ans. 6/11**

2. $i^7(3 - 4i)(-3 + 4i) = -i(-9 + 24i - 16i^2) = -i(7 + 24i) = -7i - 24i^2$ **Ans.** $\mathbf{24 - 7i}$

3. $2^{3/5} \cdot 4^{2/3} \cdot 8^{1/2} = 2^{18/30} \cdot 2^{20/30} \cdot 2^{15/30} = 2^{103/30} = 2^3 \cdot 2^{13/30} = 8\sqrt[30]{8192}$ **Ans.** $\mathbf{8\sqrt[30]{8192}}$

4. $\left(\frac{2\sqrt[3]{4}}{\sqrt[6]{243}}\right)^5 \left(\frac{3\sqrt[4]{27}}{\sqrt[5]{8}}\right)^3 = \left(\frac{2^{5/3}}{3^{5/6}}\right)^5 \cdot \left(\frac{3^{7/4}}{2^{3/5}}\right)^3 = \frac{2^{25/3}}{3^{25/6}} \cdot \frac{3^{21/4}}{2^{9/5}} = 2^{\frac{125-27}{15}} 3^{\frac{63-50}{12}}$. $a + b = \frac{98}{15} + \frac{13}{12} =$

$6\frac{8}{15} + 1\frac{1}{12} = 7\frac{32 + 5}{60} = 7\frac{37}{60}$

Ans. $7\frac{37}{60}$

5. Using calculator: $8000(1.025)^{82} = 60{,}596.4175$ **Ans. $60,596.42**

6. Each time the diagonal crosses a horizontal or a vertical line, it exits one tile. The exception occurs when it crosses an intersection. Therefore the number of tiles it crosses is the sum of the horizontal and vertical tiles minus the greatest common factor of these two numbers. Here: $11 + 13 - 1 = 23$. **Ans. 23 tiles**

7. $z^2 = 7 + 24i$. Let $z = a + bi$. Then $(a + bi)^2 = a^2 + 2abi + bi^2$ or $a^2 - b^2 + 2abi$. Then (1) $a^2 - b^2 = 7$ and $2ab = 24$ or (2) $ab = 12$. In (1), since the difference of squares is 7, then a is 4 and b is 3. These can be (+) or (-), but since in (2) $ab = 12$, they must have the same sign.

Ans: $\pm$ (4 + 3*i*)

8. $(ai - b)(ai + b) = -29 \rightarrow -a^2 - b^2 = -29$ or (1) $a^2 + b^2 = 29$.
$(ai)^4 = 625$, therefore $(ai)^2 = \pm 25$ or $-a^2 = \pm 25$. Thus $a = \pm 5$ or $\pm 5i$. Subbing into (2): (± 5): $25 + b^2 = 29 \rightarrow b^2 = 4$, or $b = \pm 2$. $(\pm 5i)$: $-25 + b^2 = 29 \rightarrow b^2 = 54$ or $b = \pm 3\sqrt{6}$.

Ans. $(\pm 5, \pm 2), (\pm 5i, \pm 3\sqrt{6})$

9. The least common multiple of 2, 3, 5 is 30. The numbers in (2) above are 22 out of 30. In 120 there are 88. 122 and 123 are the other ones. 90/123 = 30/41. **Ans. 30/41**

Answer Sheet Dec 2013

Probability

1. 4/25
2. 11/15
3. 223/343

Exponents and Radicals

1. 43
2. 1
3. 3

Lines, Angles and Polygons

1. 140 or 140°
2. $3\sqrt{10}$
3. $6\sqrt{4+2\sqrt{2}}$

Complex Numbers

1. $7-i$
2. $\frac{-9+13i}{10}$
3. $-128 - 128\sqrt{3}\,i$

Arithmetic with Percent

1. 120 or 120%
2. decreased by 4%
3. 12 or 12 qts

Team

1. 6/11
2. $24-7i$
3. $8\sqrt[30]{8192} = 10.8028$
4. $7\frac{37}{60}$
5. $60,596.42
6. 23 or 23 tiles
7. $4+3i, -4+3i$
8. $(\pm 5, \pm 2), (\pm 5i, \pm 3\sqrt{6})$
9. 30/41