1 Probability Dec 2014 (No Calculators)

3 pts 1. On the toss of a pair of dice, what is the probability of getting a pair of numbers whose product is 12?

Ans. _____

4 pts 2. Grandma Dote has 12 presents for her 4 grandsons, all in boxes. 3 of the presents are games and the rest are clothes and a variety of other things. She doesn't put names on the boxes and each grandson gets three presents. What is the probability that one of the grandsons gets all three of the games?

Ans. _____

5 pts 3. Each of the letters A, B, C, D, and E are placed on separate cards. Each of the letters are then drawn randomly one at a time and placed side by side from left to right. What is the probability that no vowels are placed consecutively?

Ans. _____

2 Exponents and Radicals Dec 2014 (No Calculators)

3 pts 1. Express $\sqrt[4]{16^6}$ in simplest form.

Ans. _____

4 pts 2. $a^{2/3} \cdot b^{1/4} \cdot c^{7/5} = \sqrt[N]{a^X b^Y c^Z}$. Find the number which is the sum of N + X + Y + Z.

Ans. _____

5 pts 3. Find all real values of x such that $\sqrt{5x+4} - 2 = \sqrt{3x-2}$.

Ans. _____

3 Lines, Angles and Polygons Dec 2014 (No Calculators)

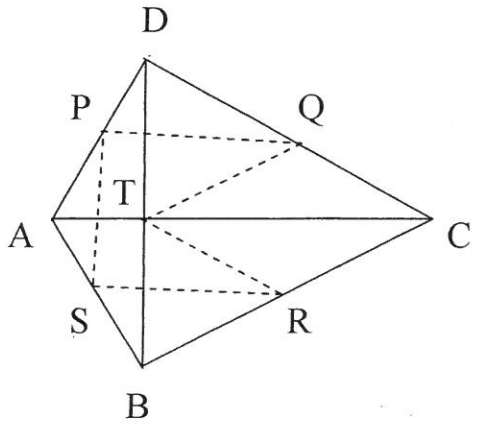
3 pts 1. Find the measure of an angle if the supplement of the angle is 40° larger than twice its complement.

Ans. _____

4 pts 2. The measure of each angle of a regular polygon is $162\frac{6}{7}$ degrees. How many diagonals does the polygon have?

Ans. _____

5 pts 3. In the figure at right $\overline{AC} \perp \overline{BD}$ at T. P, Q, R, and S are midpoints. AT = 5, BT = 10, CT = 16, and DT = 12. Find the perimeter of pentagon TRSPQ.



Ans. _____

4 Complex Numbers Dec 2014 (No Calculators)

3 pts 1. If $M = 1 + 3i$ and $N = 2 - i$, find $M(M + N)$.

Ans. _____

4 pts 2. Let $P = 1 + 3i$. Let $Q = 2 - i$. Find $\frac{P - Q}{Q^4}$.

Ans. _____

5 pts 3. Find all values of a such that $([a - i][a + i])^2 = 36$.

Ans. _____

5 Arithmetic with Percent Dec 2014 (You may use Calculators)

3 pts 1. A certain article has a price tag of \$24, and with that a state sales tax of 7% of the price tag is attached before the selling price is reached. In another state the article sold at the same selling price, but this state's sales tax rate was $5\frac{1}{2}\%$. What was the price tag on the article in the second state?

Ans. _____

4 pts 2. Find the base 5 value of the expression $243_5 + 314_5 - 323_5$.

Ans. _____

5 pts 3. The budget for a trip includes the following components: Plane Fare – 31.25%, Hotel – 25%, Food – 12.50%, Car Rental – 9.375%, Other – 21.875% for a total of 100.00%. If a way is found to save 25% of the Hotel component and 2/7 of the budgeted Other component, by what fraction (or percent) is the total budget reduced?

Ans. _____

6 Team Dec 2014 (You may use calculators)

3 pts 1. Find the number of degrees in each angle of a regular 14-gon.

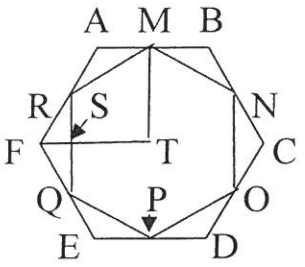
(1) Ans. _____ **3 pts**

3 pts 2. A six-sided die is tossed two times. What is the probability that the second throw number is larger than the first throw number?

(2) Ans. _____ **3 pts**

3 pts 3. If $N = 2 + i$, then what does N^{-2} equal? **(3) Ans.** _____ **3 pts**

4 pts 4. Regular hexagon ABCDEF has consecutive side midpoints connected to form hexagon MNOPQR. Connecting M and F to T, the center of the hexagon, produces quadrilateral MTSR where S is the point where $\overline{TF}$ intersects $\overline{RQ}$. If AB = 4. Find the perimeter of quadrilateral MTSR.



(4) Ans. _____ **4 pts**

4 pts 5. 3 red socks, four blue socks and 2 white socks are in a container. Five socks are drawn from the container one at a time and placed on a shelf. What is the probability that only red and blue socks are drawn?

(5) Ans. _____ **4 pts**

4 pts 6. Find n so that $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}=100.$

(6) Ans. _____ **4 pts**

5 pts 7. Baseball teams A, B, C, and D have power ratings of 40, 30, 20, and 10 respectively. Power ratings are designed so that if teams X and Y have power ratings of x and y respectively, and they play each other, team X has $\frac{x}{x+y}$ probability of winning and Y has $\frac{y}{x+y}$ probability of winning. The teams have a tournament in which A plays D, B plays C, and the two winners play each other for the championship. Find the probability that D wins the championship.

(7) Ans. _____ **5 pts**

5 pts 8. Find the least positive integer that is both 15% less than one whole number and 35% greater than another whole number.

(8) Ans. _____ **5 pts**

5 pts 9. If $\frac{X-2Yi}{X+2Yi}=\frac{21-20i}{29}$, find all ordered pairs (X, Y) that satisfy the equation.

(9)Ans. _____ **5 pts**

Solutions – Probability

1. 2(6), 6(2), 3(4), 4(3) are ways to get a product of 12. 4/36 = 1/9. **Ans. 1/9**

2. For a grandson to get all games: $\frac{3}{12}\cdot\frac{2}{11}\cdot\frac{1}{10}=\frac{1}{220}$. For all 4, 1/55. **Ans. 1/55**

3. All possible ways: 5(4)(3)(2)(1) = 120. For all ways that the vowels are together: Pretend the two are glued, so they only move as one in the group, thus 4! = 24, but those two can be switched. 2(24) = 48. (120 – 48)/120 = 72/120 = 3/5. **Ans. 3/5**

Exponents and Radicals

1. $\sqrt[4]{16^6}=(2^4)^{6/4}=2^6=64$. **Ans. 64**

2. $a^{2/3}\cdot b^{1/4}\cdot c^{7/5}=a^{40/60}\cdot b^{15/60}c^{84/60}=\sqrt[60]{a^{40}b^{15}c^{84}}$. 60+40+15+84 = 199. **Ans. 199**

3. $\sqrt{5x+4}-\sqrt{3x-2}=2$. Squaring: $5x+4-2\sqrt{15x^2+2x-8}+3x-2=4$.
$\sqrt{15x^2+2x-8}=4x-1$. Squaring: $15x^2+2x-8=16x^2-8x+1$ ➔ $0=x^2-10x+9$.
Factoring: $(x-9)(x-1)=0$. $x=9$ or 1. **Ans. 9 or 1**

Lines, Angles and Polygons

1. $180-n=40+2(90-n)$ ➔ $-n=40-2n$. $n=40$. **Ans. 40**

2. $180-162\frac{6}{7}=17\frac{1}{7}$. $\frac{360}{17\frac{1}{7}}=\frac{360}{\frac{120}{7}}=360\cdot\frac{7}{120}=21$(# of sides). $\frac{21(18)}{2}=189$. **Ans. 189**

3. Using the Pythagorean Theorem: DC = 20, BC = $\sqrt{356}=2\sqrt{89}$, and. TQ = ½ DC = 10, TR = ½ BC = $\sqrt{89}$, because the median to the hypotenuse is half as long as the hypotenuse. SR = ½ (21) = 10.5, PS = ½ BD = 11, and PQ = ½ (21) = 10.5, because the segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long. $10+\sqrt{89}+10.5+11+10.5=\sqrt{89}+42$. **Ans. 42 + $\sqrt{89}$**

Complex Numbers

1. $M+N=3+2i$. $(3+2i)(1+3i)=3+11i+6i^2=-3+11i$ **Ans.** ~~$-3i+11$~~ $-3+11i$

2. $\frac{1+3i-(2-i)}{(2-i)^4}=\frac{-1+4i}{(3-4i)^2}=\frac{(-1+4i)(-7+24i)}{(-7-24i)(-7+24i)}=\frac{7-24i-28i+96i^2}{49-576i^2}=$ **Ans.** $\frac{-89-52i}{625}$

3. $([a-i][a+i])^2=36$ ➔ $(a^2+1)^2=36$ ➔ $a^2+1=\pm6$ ➔ $a^2=-1\pm6$, therefore $a^2=5$ or $a^2=-7$. $a=\pm\sqrt{5}$ or $a=\pm\sqrt{7}\,i$. **Ans.** $\pm\sqrt{5}$, $\pm\sqrt{7}\,i$

Arithmetic with Percent

1. With 7% sales tax, the cost for the article = 1.07(24) = 25.68. 1.055x = 25.68 ➔ 25.68/1.055 = 24.34 rounded. **Ans. $24.34**

2. $243_5 + 314_5 = 1112_5$. $1112_5 - 323_5 = 234_5$. **Ans. 234_5**

3. 25% = ¼. $\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{16}$ (amt. reduced) = .0625%. 21.875% = 7/32. $\frac{7}{32}\cdot\frac{2}{7}=\frac{1}{16}$.

$2\left(\frac{1}{16}\right)$ = 1/8. 2(.0625) =.125 = 12.5% **Ans. 1/8 or 12.5%**

Team

1. 360/14 = $25\frac{5}{7}$ = each exterior angle. 180 - $25\frac{5}{7}$ = $154\frac{2}{7}$. **Ans. $154\frac{2}{7}^{o}$**

2. There are 6 even, so 30 uneven, 15 greater, 15 less. 15/36 = 5/12. **Ans. 5/12**

3. $\frac{1}{(2+i)^2}=\frac{1}{4+4i+i^2}=\frac{1}{3+4i}\cdot\frac{3-4i}{3-4i}=\frac{3-4i}{9+16}$ **Ans.** $\frac{3-4i}{25}$ **or** $\frac{3}{25}-\frac{4}{25}i$

4. Since AB = 4, then TM = $2\sqrt{3}$ and so does RM. Since RQ = $2\sqrt{3}$, then RS = $\sqrt{3}$ and ST = 3. $2\sqrt{3}+2\sqrt{3}+\sqrt{3}+3=3+5\sqrt{3}$. **Ans. $3+5\sqrt{3}$**

5. Since there has to be both blue and red socks, the possible results are 1red/4blue or 2red/3blue or 3 red/2blue: $\frac{{}_3C_1\cdot{}_4C_4+{}_3C_2\cdot{}_4C_3+{}_3C_3\cdot{}_4C_2}{{}_9C_5}=\frac{3+12+6}{126}=\frac{21}{126}=\frac{1}{6}$. **Ans. 1/6**

6. Rationalizing each denominator yields:

$\frac{1-\sqrt{3}}{-2}+\frac{\sqrt{3}-\sqrt{5}}{-2}+\frac{\sqrt{5}-\sqrt{7}}{-2}+\cdots+\frac{\sqrt{2n-1}-\sqrt{2n+1}}{-2}=100$ or $\frac{1-\sqrt{2n+1}}{-2}=100$.

1 - $\sqrt{2n+1}$ = -200 or $\sqrt{2n+1}$ = 201. 2n + 1 = 40401 ➔ n = 20,200 **Ans. 20,200**

7. A plays D and wins: $\frac{10}{40+10}=\frac{1}{5}$. B plays C and wins: $\frac{30}{30+20}=\frac{3}{5}$.

C plays B and wins: $\frac{20}{30+20}=\frac{2}{5}$. D plays B and wins: $\frac{10}{30+10}=\frac{1}{4}$. D plays C and wins: $\frac{10}{20+10}=\frac{1}{3}$. After D beats A, d must play the winner of either B or C: $\frac{1}{5}\cdot\frac{3}{5}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{2}{5}\cdot\frac{1}{3}=\frac{3}{100}+\frac{2}{75}=\frac{9+8}{300}=\frac{17}{300}$.

Ans. 17/300

8. Let a be the greater whole number and b be the lesser. If N = the least positive integer then $N = \frac{17}{20}a = \frac{27}{20}b$ ➔ 17a = 27b. Then N = the least common multiple of 17 and 27. Since they have no common factors, then N = 17(27) = 459. Note that a = 540 and b = 340. **Ans. 459**

9. Using the complex conjugate approach to rationalize the denominator, produces the two answers (5, 1) and (-5, -1): $\frac{(X-2Yi)}{(X+2Yi)} \cdot \frac{(X-2Yi)}{(X-2Yi)} = \frac{X^2-4XYi-4Y^2}{X^2+4Y^2} = \frac{21-20i}{29}$. Therefore (1) $X^2 - 4Y^2 = 21$ and (2) $X^2 + 4Y^2 = 29$. Adding (1), (2): $2X^2 = 50$ ➔ $X^2 = 25$, so X = ± 5. Using 4XY = 20, XY = 5; so when X = 5, Y =1, and when X = -5, Y = -1.

However, using cross-multiplication produces ($5a$, a): using lower case x and y,
$29x - 58yi = 21x - 20xi + 42yi - 40yi^2$ ➔ $8x + 20xi = 100yi + 40y$ ➔
$2x + 5xi = 25yi + 10y$ ➔ $x(2 + 5i) = 5y(2 + 5i)$ ➔ x = 5y. Thus the ordered pairs ($5a$, a), as long as a does not equal 0. Rationalizing the denominator only produces two of these.
Ans. All ordered pairs (5a, a), a ≠ 0

Answer Sheet Dec 2014

Probability

1. 1/9
2. 1/55
3. 3/5

Exponents and Radicals

1. 64
2. 199
3. 1 or 9

Lines, angles and Polygons

1. 40 or 40^{o}
2. 189
3. $42 + \sqrt{89}$

Complex Numbers

1. $-3 + 11i$
2. $\frac{-89-52i}{625}$
3. $\pm\sqrt{5}, \pm\sqrt{7}\,\mathrm{i}$

Arithmetic with Percent

1. $24.34
2. 234_5
3. 1/8 or 12.5%

Team

1. $154\frac{2}{7}^{o}$ 154.2857°
2. 5/12 or .4167
3. $\frac{3-4i}{25}$ or $\frac{3}{25}-\frac{4}{25}i$
4. $3 + 5\sqrt{3}$ or 11.6603
5. 1/6 or .16667
6. 20,200
7. 17/300 or .0567
8. 459
9. (5a, a), where $a \neq 0$