

1 Probability Dec 2016-17 (No Calculators)

3 pts 1. If you draw a card at random from a regular deck of 52 playing cards, what is the probability that it is a jack, queen or king?

Ans. _____

4 pts 2. A student council of a certain school has 4 students from each class. The senior class has 2 boys and 2 girls. The junior class also has 2 boys and 2 girls. If two seniors and two juniors are chosen at random to represent the school at a student council event, what is the probability that 2 boys and 2 girls are chosen?

Ans. _____

5 pts 3. It has been determined that at a certain intersection, cars arriving from the west go straight ahead 10% of the time, turn left 70% of the time and turn right 20% of the time. It is also known that 80% of the drivers use their signals properly. You, who are heading into the intersection from the west, are in a car behind a driver who does not have his or her signal on. What is the probability that the driver will turn left?

Ans. _____

2 Exponents and Radicals Dec 2016-2017 (No Calculators)

3 pts 1. Find $a + b$, if m is not zero, given that $(m^3)^{2a} = m^{54}$ and $m^b \cdot m^{2b} \cdot m^{4b} = m^{28}$.

Ans. _____

4 pts 2. Solve for x : $\left(\frac{81}{625}\right)^{x-2} = \left(\frac{125}{27}\right)^{5-2x}$.

Ans. _____

5 pts 3. Find the greatest real solution to the equation $3x^8 + 5x^4 = 2$.

3 Lines, Angles and Polygons Dec 2016-17 (No Calculators)

3 pts 1. Consider the parallelogram ABCD, where $m\angle A = 54 + x$ and the $m\angle B = 84 + 2x$, both measured in degrees. Find the measure of angle D in degrees.

Ans. _____

4 pts 2. The complement of the supplement of angle A is equal to twice the complement of half of angle A. Give the degree measure of angle A.

Ans. _____

5 pts 3. Suppose that a sheet of paper with dimensions 8 inches by 8 inches has the four corners folded toward the center in such a way that it forms a regular octagon. What is the length of each side of the octagon?

Ans. _____

4 Complex Numbers Dec 2016-17 (No Calculators)

3 pts 1. Find the value of the expression $T - B$, given that $T + Bi = (8 + 2i)(4 - 3i)$.

Ans. _____

4 pts 2. Evaluate $\left| \frac{i + 3i^2 - 2i^3 + 4i^4 - 6i^5}{2i + 1} \right|$.

Ans. _____

5 pts 3. Find the value of $f(2)$, if a zero of $f(x) = x^3 + ax^2 + bx + 4$ is $2i$, where a and b are real numbers.

Ans. _____

5 Arithmetic with Percent Dec 2016-17 (You may use calculators)

3 pts 1. Evaluate: 10% of 10 plus 20% of 20 plus 30% of 30 plus ... plus 120% of 120.

Ans. _____

4 pts 2. Pete, a financial planner invested \$8,000, some in stocks earning 15% annually and the rest in bonds earning 6% annually. The end-of-year investment returns were \$930. Determine the amount he invested at 6%.

Ans. _____

5 pts 3. The product of two natural numbers is 28 more than their sum. Find the value of the larger of the two numbers.

Ans. _____

6 Team Dec 2016-17 (You may use calculators)

3 pts 1. Express $(64x^2)^{\frac{-1}{6}}(32x^{-5})^{\frac{-2}{5}}$ in simplest form. (1) Ans. _____ **3 pts**

3 pts 2. A used car dealer sold two cars receiving \$5,600 for each. He netted a 40% profit on one car and a 20% loss on the second, both based on the cost. What was the dealer's profit or loss, as a result of both transactions? Make sure to state your value and the word "profit" or "loss."

(2) Ans. _____ **3 pts**

3 pts 3. A number is randomly picked from the first 100 positive integers. What is the probability that it is a perfect square or a perfect cube, but not both?

(3) Ans. _____ **3 pts**

4 pts 4. The measure of an exterior angle of a regular polygon with $2x + 6$ sides is x degrees. How many diagonals does this polygon have?

(4) Ans. _____ **4 pts**

4 pts 5. A triangle has side lengths of 3, 25 and 26. Find the length of the altitude to the shortest side.

(5) Ans. _____ **4 pts**

4 pts 6. In $\triangle ABC$, $AB = 7a$, $BC = 5a$ and $AC = 8a$, for some number $a > 0$. If point D is on \overline{AC} and \overline{BD} bisects $\angle ABC$, find the least value of a so that \overline{AD} , \overline{DC} and the sides of $\triangle ABC$ have whole number lengths.

(6) Ans. _____ **4 pts**

5 pts 7. If $Z = a + bi$, where a and b are integers, and $(4 + 3i)Z = 14 + 23i$, what will $(4 + 3i)/Z$ equal?

(7) Ans. _____ **5 pts**

5 pts 8. A bingo game board has 25 squares arranged in a 5 by 5 grid of columns and rows. If two of the 25 squares are selected at random, what is the probability both squares are in the same row, the same column or the same corner to corner diagonal?

(8) Ans. _____ **5 pts**

5 pts 9. A , B , C and D are positive integers. If B is 35% greater than A , C is 35% less than D , and C is 15% greater than B , find the least possible value of C .

(9) Ans. _____ **5 pts**

Solutions – Probability

- 4 jacks, 4 queens, 4 kings out of 52 cards $\rightarrow 12/52 = 3/13$. **Ans. 3/13**
- The possible results are 2 senior boys and 2 junior girls, 2 senior girls and 2 junior boys, or a boy and a girl from each class. Thus: $\frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{1}{3} + \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{1}{3} + \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{2}{4} \cdot \frac{2}{3} = \frac{4(2+4)}{4 \cdot 3 \cdot 4 \cdot 3} = \frac{6}{36}$. **Ans. 1/6**
- If the person is turning left he must not have his signal light on $(0.7)(0.2)$. All the possibilities that the driver could do are go left, go straight ahead, or turn right. Those are $(0.7)(0.2) + 0.1 + (0.2)(0.2)$. Thus the probability he goes left is $\frac{(0.7)(0.2)}{(0.7)(0.2) + 0.1 + (0.2)(0.2)} = \frac{1}{2}$

Ans. 1/2

Exponents and Radicals

- $6a = 54$, so $a = 9$. $7b = 28$, so $b = 4$. $9 + 4 = 13$. **Ans. 13**
- $\left(\frac{81}{625}\right)^{x-2} = \left(\frac{125}{27}\right)^{5-2x} \rightarrow \left(\frac{3}{5}\right)^{4(x-2)} = \left(\frac{3}{5}\right)^{-3(5-2x)}$. $4x - 8 = -15 + 6x \rightarrow 2x = 7$. **Ans. 7/2**
- $3x^8 + 5x^4 - 2 = 0 \rightarrow (3x^4 - 1)(x^4 + 2) = 0$. Only $3x^4 - 1 = 0$, so $x^4 = \frac{1}{3}$ or $x = \frac{\sqrt[4]{27}}{3}$. **Ans. $\frac{\sqrt[4]{27}}{3}$**

Lines, Angles and Polygons

- $54 + x + 84 + 2x = 180 \rightarrow 3x + 138 = 180 \rightarrow 3x = 42$, so $x = 14$ and $D = 84 + 28$. **Ans. 112**
- $90 - (180 - A) = 2(90 - \frac{1}{2}A) \rightarrow A - 90 = 180 - A \rightarrow 2A = 270$, so $A = 135$. **Ans. 135**
- Let the side of the octagon be s . Then $8 = s + 2\left(\frac{s}{\sqrt{2}}\right) = s + s\sqrt{2} = s(1 + \sqrt{2})$. Therefore

$$s = \frac{8(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} = \frac{8 - 8\sqrt{2}}{1 - 2} = 8\sqrt{2} - 8$$

Ans. $8\sqrt{2} - 8$

Complex Numbers

- $T + Bi = (8 + 2i)(4 - 3i) = 32 - 24i + 8i - 6i^2 = 38 - 16i$. $T - B = 38 - (-16) = 54$. **Ans. 54**
- $\left|\frac{i + 3i^2 - 2i^3 + 4i^4 - 6i^5}{2i + 1}\right| = \left|\frac{i - 3 + 2i + 4 - 6i}{2i + 1}\right| = \left|\frac{1 - 3i \cdot \frac{2i - 1}{2i - 1}}{2i + 1}\right| = \left|\frac{2i - 1 - 6i^2 + 3i}{4i^2 - 1}\right| = \left|\frac{5i + 5}{-5}\right| = |-i - 1| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$. **Ans. $\sqrt{2}$**

3. $f(2i) = (2i)^3 + a(2i)^2 + b(2i) + 4 = 0 \rightarrow -8i - 4a + 2bi + 4 = 0$. Therefore $-8 + 2b = 0$ or $b = 4$ and $-4a + 4 = 0$ or $a = 1$. Thus $f(x) = x^3 + x^2 + 4x + 4$, and $f(2) = (2)^3 + (2)^2 + 4(2) + 4 = 8 + 4 + 8 + 4 = 24$.

Ans. 24

Arithmetic with Percent

1. The numbers form a pattern of squares: $1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 + 121 + 144 = 650$.

Ans. 650

2. $.15x + .06(8000 - x) = 930 \rightarrow 15x + 48000 - 6x = 93000 \rightarrow 9x = 45000$, so

$x = 5000$. The amount invested at 6% is $\$8000 - \$5000 = \$3000$.

Ans. \$3,000

3. Let the two natural numbers be x and y , where $x \leq y$. Then $xy = x + y + 28 \rightarrow x(y - 1) = y + 28 \rightarrow x = \frac{y+28}{y-1} = 1 + \frac{29}{y-1}$. Thus $x - 1 = \frac{29}{y-1}$ or $(x - 1)(y - 1) = 29$. Since 29 is prime, then $y - 1 = 29$, and $x - 1 = 1$. Thus $y = 30$ and $x = 2$.

Ans. 30

Team

1. $(64x^2)^{\frac{-1}{6}} (32x^{-5})^{\frac{-2}{5}} \rightarrow (2^6 x^2)^{\frac{-1}{6}} (2^5 x^{-5})^{\frac{-2}{5}} \rightarrow 2^{-1} x^{\frac{-1}{3}} 2^{-2} x^2 \rightarrow 2^{-1} x^{\frac{-1}{3}} 2^{-2} x^2 = 2^{-3} x^{\frac{5}{3}} = \text{Ans. } \frac{1}{8} x^3 \sqrt{x^2}$

2. $1.4P = 5600$, so $P = 4000$ and the profit $\$1600$. $.8L = 5600$, so $L = 7000$ and the loss is $\$1400$. Thus the overall profit is $\$200$.

Ans. \$200

3. $1, 4, 9, 16, 25, 36, 49, 64, 81, 100$ and $1, 8, 27, 64$. $10/100 = 1/10$.

Ans. 1/10

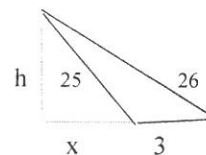
4. $\frac{360}{2x+6} = x \rightarrow 2x^2 + 6x - 360 = 0 \rightarrow x^2 + 3x - 180 = 0 \rightarrow (x + 15)(x - 12) = 0$. Thus it has

12 sides and $\frac{12(12-3)}{2} = 54$ diagonals. $x=12$
 $\# \text{ of sides} = 2x+6 = 30 \rightarrow \frac{30(30-3)}{2} = 405$ **Ans. 54**

5. In the figure: (1) $h^2 + x^2 = 625$ and (2) $h^2 + (x + 3)^2 = 676$

(2) - (1): $(x + 3)^2 - x^2 = 51 \rightarrow x^2 + 6x + 9 - x^2 = 51 \rightarrow$

$6x = 42$, so $x = 7$ and thus $h = 24$.



Ans. 24

6. By the angle bisector theorem: $\frac{AD}{DC} = \frac{AB}{BC} = \frac{7}{5}$. Therefore $AD = \frac{7}{12} \cdot 8a = \frac{14}{3}a$ and

$DC = \frac{5}{12} \cdot 8a = \frac{10}{3}a$. The least possible value of a is 3.

Ans. 3

7. $(4 + 3i)(a + bi) = 14 + 23i \rightarrow 4a + 4bi + 3ai - 3b = 14 + 23i$. Thus (1) $4a - 3b = 14$ and (2) $3a + 4b = 23$. 4(1): $16a - 12b = 56$; 3(2): $9a + 12b = 69$. Adding these: $25a = 125$, so $a = 5$. In (1): $4(5) - 3b = 14 \rightarrow 6 = 3b$, so $b = 2$. $Z = 5 + 2i$.

$$\frac{4+3i}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{20-8i+15i-6i^2}{25-4i^2} = \frac{26+7i}{29}$$

Ans. $\frac{26+7i}{29}$

8. In the first row there are ${}_5C_2 = 10$ possibilities, therefore 50 possibilities in all 5 rows. 50 in the columns. And 20 in both diagonals. There are ${}_{25}C_2 = 300$ total possibilities. $120/300 = 2/5$.

Ans. 2/5

9. "35% greater" is the ratio of 27:20. "35% less" is the ratio of 13:20. "15% greater" is the ratio of 23:20. Then A:B:C:D = $1 : \frac{27}{20} : \frac{27}{20} \cdot \frac{23}{20} : \frac{27}{20} \cdot \frac{23}{20} \cdot \frac{20}{13} = 1 : \frac{27}{20} : \frac{621}{400} : \frac{621}{260}$. Since the LCM of

20, 400 and 260 is 5200, then the ratio in integers is 5200:7020: 8073:12,420. $C = 8073$.

Ans. 8073

Answer Sheet Dec 2016-17

Probability

1. $3/13$ or $\overline{.230769}$
2. ~~$1/6$ or $\overline{.16}$~~ $1/2$ or $.5$ or 50%
3. $1/2$ or $.5$ or 50%

Exponents and Radicals

1. 13 or *Alternate acceptable answer*
If $m=1$, $A+B = \text{anything}$,
if $m \neq 1$, $A+B=13$
2. $3\frac{1}{2}$ or 3.5 or $7/2$ or
3. $\frac{\sqrt[4]{27}}{3}$ or $\frac{3^{3/4}}{3}$

Lines, Angles and Polygons

1. 112 or 112°
2. 135 or 135°
3. $8\sqrt{2} - 8$

Complex Numbers

1. 54
2. $\sqrt{2}$
3. 24

Arithmetic with Percent

1. 650
2. $\$3000$ or 3000
3. 30

Team

1. $\frac{1}{8}x\sqrt[3]{x^2}$
2. $\$200$, profit or 200 , profit
3. $1/10$ or $.1$
4. ~~54~~ 405
5. 24
6. 3
7. $\frac{26+7i}{29}$ or $\frac{26}{29} + \frac{7}{29}i$ or $\frac{7}{29}i + \frac{26}{29}$
8. $2/5$ or $.4$ or 40%
9. 8073