Meet #4 Feb. 2000

SOLUTIONS – LOGS AND LOG EQUATIONS

1. Let $x = \log_8 \sqrt[3]{16}$, then $8^x = \sqrt[3]{16}$ or $2^{3x} = 4^{4/3}$. Thus $3x = 4/3$ and $x = 4/9$. Ans. 4/9

2. $\frac{\log(x^2y)\ \log y}{2} + \log\left(\frac{1}{x}\right) = \frac{2\log x + \log y - \log y}{2} - \log x = 0$. Ans. 0

3. $2\log_7(x+3) = \log_7(x^2-9) + \log_7(x-1) \Rightarrow \log_7 \frac{(x+3)^2}{(x-3)(x+3)} = \log_7(x-1)$. And thus $\frac{x+3}{x-3} = x-1$ or $x+3 = x^2 - 4x + 3 \Rightarrow x^2 - 5x = 0$. Thus $x = 0$ or 5. $x \neq 0$. Ans. 5

FUNCTIONS

1. Since $f(x) = 2x-1$ and $g(x) = 6-4x$, then $f(g(x)) = 2(6-4x)-1 = 11-8x$.
$f(g(h(x))) = 11-8h(x) = 3x \Rightarrow -8h(x) = 3x-11$ or $h(x) = \frac{11-3x}{8}$. $h(2) = 5/8$. Ans. 5/8

2. Working from right to left of $f^{-1}(f(f(f^{-1}(f^{-1}(f(f^{-1}(3)))))))$:
$f^{-1}(3) = 4$. $f(4) = 3$. $f^{-1}(3) = 4$. $f^{-1}(3) = 4$. $f^{-1}(4) = 5$. $f(5) = 4$. $f(4) = 3$.
And finally $f^{-1}(3) = 4$. Ans. 4

3. $f(x) = \frac{4}{x^2-x-6}$, $g(x) = \frac{1}{x+1}$. $g \circ f(x) = \frac{1}{\frac{4}{x^2-x-6}+1}$. Here $x \neq 3$ or -2. Continuing to simplify the function: $\frac{1}{\frac{4}{x^2-x-6}+1} = \frac{1}{\frac{4+x^2-x-6}{x^2-x-6}} = \frac{x^2-x-6}{x^2-x-2}$. Here $x \neq 2$ or -1.

Ans. All reals $\neq -1, 3, \pm 2$

ARITHMETIC WITH STATISTICS

1.
$$\begin{array}{r} 413_5 \\ \underline{32_5} \\ 1331 \\ \underline{2244} \\ 24321 \end{array}$$

Ans. $24{,}321_5$ or 24,321

2. Since the median is 1, then $\frac{1+1+3+A+1-4}{7} = 1$. Thus $A = 5$. Ans. 5

3. $\sqrt{2} = 1.4142$, $\sqrt[3]{2} = 1.2599$, $\pi/3 = 1.0472$, $\sqrt[4]{3} = 1.3161$, $\sqrt[4]{4} = 1.4142$, $\sqrt[5]{5} = 1.3797$.
Median = (1.3161+1.3797)/2 = 1.3479. Mean = 7.8313/6 = 1.3052. Mode = 1.4142.
The sum of these is 4.0673. Rounded to 100^{th} = 4.07. Ans. 4.07

LINEAR COORDINATE GEOMETRY

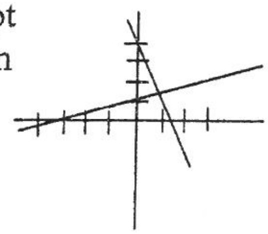
1. Slope through (-2,3), (15,-1) = $\frac{3+1}{-2-15} = -\frac{4}{17}$. The line through these points takes on the form $y = -\frac{4}{17}x$ or in standard form: 4x + 17y = . Plugging in (17,19): 4(17) + 17(19) = 391.
Thus the equation in standard form 4x + 17y = 391, or in intercept-slope form y = – 4/17x + 23.

Ans. 4x + 17y = 391 or y = – 4/17x + 23

2. The median from C to AB is 11x – 7y = -7, and the median from A to BC is 5x – y = 7. These lines intersect at the point (7/3,14/3) or (2 1/3, 4 2/3). This answer can be gotten very simply by finding the mean of the x and y coordinates; $\left(\frac{2+5+0}{3}, \frac{3+10+1}{3}\right)$ = (7/3,14/3).

Ans. (7/3,14/3) or (2 1/3, 4 2/3)

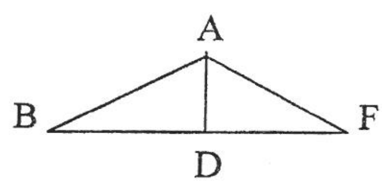
3. The line 5x + 2y = 7 (1) rotated about the line x = y is 2x + 5y = 7, which is x = - 5/2 y + 7/2. (2) Reflecting this about the y axis: x = 5/2 y – 7/2, which is 2x – 5y = -7. Rotating this 90° about the origin means the line perpendicular to this has the form 5x + 2y =
As you can see in the figure, the y-intercept becomes the x- intercept. Thus plugging in the point (7/5,0); 5x + 2y = 7.



Ans. 5x + 2y = 7

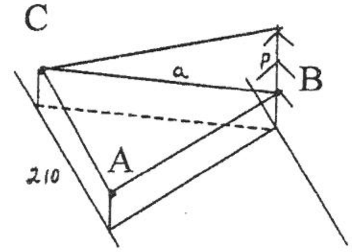
TRIG MECHANICS

1. m∠ABF = 30°, so dropping the perpendicular from A to meet BF at D makes a 30-60-90 Δ. Since AB = 6, then AD = 3 and BD = $3\sqrt{3}$. Thus BF = $6\sqrt{3}$.
Ans. $6\sqrt{3}$



2. Using cosine law: $(6\sqrt{3})^2 = 64 + 100 - 2(8)(10)\cos D$ or $\cos D = \frac{64+100-108}{160} = .35$
$\cos^{-1} .35 = 69.512$.

Ans. 69.5°



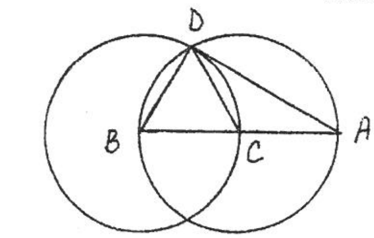
3. $\angle CBA = 41°40'$ so $\frac{a}{\sin 65°30'} = \frac{210}{\sin 41°40'}$,
thus a = 287.44. sin 13°40' = $\frac{p}{287.44}$, thus p = 69.9.

Add this to the 5 ft tripod height and the tree is 9 ft high to the nearest 10th. Ans. 74.9 ft

TEAM

1. $5(78) + 4(82) + 3(86) + 2(90) = 1156$. This is 14 quizzes. $15(82) = 1230$. The least that this could be in order to round up to 82 is $1230 - 7 = 1223$. $1223 - 1156 = 67$. Ans. 67

2. Connecting the center B to D and A and then connecting D to point C as shown, produces the 30-60-90Δ ABD, since Δ BCD is equilateral and $\angle$ABD is 90° because it is inscribed in a semicircle. $BD = 12/\sqrt{3} = 4\sqrt{3}$. Ans. $4\sqrt{3}$



3. $f(x) = \frac{1}{x} + 1$, $g(x) = \frac{1}{x}$, then $g(f(x)) = \frac{1}{\frac{1}{x}+1}$. Here $x \neq 0$. Simplifying further:

$g(f(x)) = \frac{x}{x+1}$. Here $x \neq -1$. Ans. all real $\neq$ 0 or -1

4. $\log x + \log x^2 + \log x^3 + \ldots + \log x^8 = \log x^{36}$. If $x = 2$, then $\log 2^{36}$ or $18 \log 4$. Ans. 18q

5. $\log(\tan 1°) + \log(\tan 89°) = \log(\tan 1°) + \log(\cot 1°) = \log(\tan 1°)(\cot 1°) = \log 1 = 0$. Thus $\log(\tan 1°) + \log(\tan 2°) + \log(\tan 3°) + \ldots + \log(\tan 89°) = \log(\tan 45°)$ Ans. 0

6. By the Emperical Rule of the Bell-shaped curve, 95% of the samples lie with 2 standard deviations of the mean. Where being symetrical, there are 4 standard deviations, between 52 [illegible]d 70 inches, 2 on each side of the mean. Therefore $(70 - 52)/4 = 4.5$. Ans. 4.5

7. The function has the form $f(x) = ax^2 + bx + c$. The point (0,-3) produces $-3 = c$. The point (1,-1) produces $-1 = a + b + c$, or since $c = -3$ ➔ (1) $a + b = 2$. The point (2,-7) produces the equation $-7 = 4a + 2b + c$. Since $c = -3$, then (2) $4a + 2b = -4$. Multiplying (1) by -2 and adding it to (2) yields $2a = -8$, or $a = -4$. Plugging this into (1): $-4 + b = 2$, thus $b = 6$. The polynomial function is $f(x) = -4x^2 + 6x - 3$. The sum of these is -1. Ans. -1
Some students may not consider -3 a coefficient, thus 2 may be an acceptable answer.

8. A(-3,2), B(7,-4), C(15,0), D(5,6). The equation of the diagonal AC: slope $= \frac{2-0}{-3-15} = -\frac{1}{9}$.

Thus the equation takes on the form $y = -\frac{1}{9}x$ or $x + 9y =$ and plugging in (15,0): $x + 9y = 15$

The altitude from D to AB: Slope of AB $= \frac{2+4}{-3-7} = -\frac{3}{5}$, AB has form $3x + 5y =$. The line perpendicular to this has the form $5x - 3y =$. Plugging in (5,6): $5x - 3y = 7$.

$x + 9y = 15$ ➔ $x + 9y = 15$ Thus $16x = 36$ and $x = 2\frac{1}{4}$. Subbing this in: $2\frac{1}{4} + 9y = 15$.

$5x - 3y = 7$ ➔ $15x - 9y = 21$ $9y = 12\frac{3}{4}$, $y = \frac{51}{4}\cdot\frac{1}{9} = \frac{17}{12} = 1\ 5/12$.

Ans. (9/4,17/12) or $(2\frac{1}{4}, 1\frac{5}{12})$

9. The centers of the circles are on the angle bisectors of the angles made by the lines. The angle bisector equations are: $\frac{|3x-4y-7|}{5}=\frac{|5x-12y-1|}{13}$ → 39x – 52y – 91 = ±(25x –60y –5) or in simplest form: 7x + 4y = 43 and 4x – 7y = 6. 1 unit from 3x – 4y = 7 makes the two parallel equations: 3x – 4y = 2 and 3x – 4y = 12. Each of these will intersect the line 4x – 7y = 6 one unit from the intersection of the given two lines. Finding where these intersect: 3x – 4y = 2 and 4x – 7y = 6 will intersect at (-2,2). 3x – 4y = 12 and 4x – 7y = 6 at (12,6).

1 unit from 5x – 12y = 1 makes the equations 5x – 12y = 14 and 5x – 12y = -12. Each of these will intersect the line 7x + 4y = 43 one unit from the intersection of the given two lines. Finding where they intersect: 5x –12y = 14 and 7x + 4y = 43 intersect at (5 ½,1 ⅛) and 5x – 12y = -12 will intersect 7x + 4y = 43 at (4 ½, 2 ⅞)

Ans. (-2,2), (12,6), (5 ½,1 ⅛), (4 ½, 2 ⅞)

Meet #4 Feb 2000

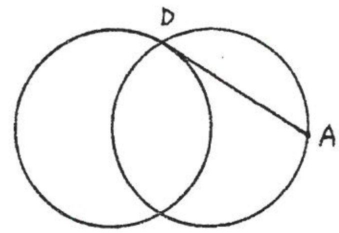
VI TEAM

3 pts 1. Sergio has an algebra quiz every week. On the first 5 quizzes, he averaged exactly 78; on the nest 4 quizzes, he averaged exactly 82; on the next 3 quizzes he averaged exactly 86; and on the next 2 quizzes, he averaged exactly 90. There is one quiz left. What is the lowest Sergio can score on his last quiz in order to average at least 82, when rounded, for all the quizzes?

Ans. ____________

3 pts 2. Each of the circles contains the center of the other. If A is on the line of centers, and AD = 12, find the exact length of the radius of the circles.

Ans. ____________



3 pts 3. If $f(x) = \frac{1}{x} + 1$ and $g(x) = \frac{1}{x}$, find the domain of $g(f(x))$. Ans. ____________

4 pts 4. Let $q = \log 4$. In terms of q, find $\log x + \log x^2 + \log x^3 + \ldots + \log x^8$ for $x = 2$. Ans. ____________

4 pts 5. Evaluate $\log(\tan 1°) + \log(\tan 2°) + \log(\tan 3°) + \ldots + \log(\tan 89°)$

Ans. ____________

4 pts 6. 2500 14-year olds were measured for their height. The results were graphed and a Bell-shaped curve was formed, symmetric about the mean. 95% of the students had a height between 52 inches and 70 inches. Determine the value of one standard deviation for this data.

Ans. ____________

5 pts 7. Only one quadratic polynomial obeys the ordered pairs below. Find the sum of the coefficients of the polynomial.

x	0	1	2	3
f(x)	-3	-1	-7	-21

Ans. ____________

5 pts 8. The vertices of parallelogram ABCD are A(-3,2), B(7,-4), C(15,0) and D(5,6). The altitude from D to side AB meets diagonal AC at P. Find the exact coordinates of P.

Ans. ____________

5 pts 9. Four circles of radius 1 are each tangent to both lines $3x - 4y = 7$ and $5x - 12y = 1$. Find the centers of each of the four circles.

Ans. ____________