

Solutions – Arithmetic with Literal Equations

1. $\frac{2A}{h} = B + b \rightarrow b = \frac{2A}{h} - B = \frac{2A - Bh}{h}$ **Ans.** $\frac{2A - Bh}{h}$

2. $(3a - 2b)(x + 2y) = 3ax - 2by + a \rightarrow 3ax + 6ay - 2bx - 4by = 3ax - 2by + a \rightarrow$
 $6ay - 2bx - 2by = a \rightarrow 6ay - a = 2bx + 2by \rightarrow a(6y - 1) = 2bx + 2by$ **Ans.** $\frac{2bx + 2by}{6y - 1}$

3. $7 \cdot 13 \cdot 9 = 1001$. If the sum of the digits is 26 then the digits have to be 8, 9 and 9. Whatever way you multiply these by 1001, you will get four 9's. **Ans.** 4

Logs and Log Equations

1. $\log_5 625 = 4$, $\log_3 \left(\frac{1}{81} \right) = -4$. The sum is 0. **Ans.** 0

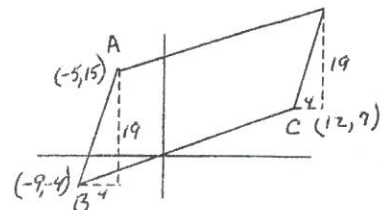
2. $(x + 1)(x - 5) = 7 \rightarrow x^2 - 4x - 5 = 7 \rightarrow x^2 - 4x - 12 = 0 \rightarrow (x - 6)(x + 2) = 0$.
 Therefore $x = 6$ or -2 . But x cannot be -2 . **Ans.** 6

3. Since $\log_b a = k$, then (1) $\frac{\log a}{\log b} = k$. If $c = a^2$, then $\log c = 2 \log a$, or (2) $\frac{\log c}{\log a} = 2$.
 Multiplying (1) and (2): $\frac{\log c}{\log b} = 2k$ or $\log_b c = 2k$. **Ans.** 2k

Linear Coordinate Geometry

1. The line $5y - 3x = 7$ is $y = \frac{3}{5}x + \frac{7}{5}$. Slope of line perpendicular is $-\frac{5}{3}$. **Ans.** $-\frac{5}{3}$

2. Plotting the three points and making a parallelogram that would have its fourth point in the first quadrant is as shown. Going from point B to A in directed run and rise, the run is 4 and the rise is 19. Similarly to go from point C to point D: $(12 + 4, 7 + 19) = (16, 26)$. **Ans.** (16, 26)



3. The midpoint of side BC is $\left(\frac{3+8}{2}, \frac{7-4}{2} \right) = (5 \frac{1}{2}, 1 \frac{1}{2})$. Finding the equation of the line through this point and point A(-6, 2) would give the equation of the line containing the

median to side BC. Slope of line AD: $\frac{2-1\frac{1}{2}}{-6-5\frac{1}{2}} = \frac{\frac{1}{2}}{-11\frac{1}{2}} = -\frac{1}{23}$. The form of the line AD is $y = -\frac{1}{23}x + b$, or $x + 23y =$ Plugging in $(-6, 2) : -6 + 23(2) = 40$. **Ans. $x + 23y = 40$**

Functions

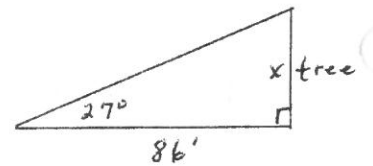
1. The smallest value that $f(x)$ can take on is when $x = 0$, which is 9. **Ans. All reals ≥ 9**

2. $\text{gof}(x) = \frac{\frac{x-3}{x+2} + 2}{\frac{x-3}{x+2} - 3} = \frac{\frac{x-3+2x+4}{x+2}}{\frac{x-3-3x-6}{x+2}} = \frac{3x+1}{-2x-9}$. If $\frac{3x+1}{-2x-9} = 0$, $x = -\frac{1}{3}$. **Ans. $-\frac{1}{3}$**

3. $P \circ M \circ P(6) \rightarrow (6,5) \Rightarrow (5,4) \Rightarrow (4,-3) = -3$
 $P \circ M \circ N(7) \rightarrow (7,3) \Rightarrow (3,5) \Rightarrow (5,9) = 9$
 $M \circ N \circ N(6) \rightarrow (6,7) \Rightarrow (7,3) \Rightarrow (3,5) = 5$
 $M \circ N \circ P(5) \rightarrow (5,9) \Rightarrow (9,5) \Rightarrow (5,4) = 4$ Thus $-3 + 9 - 5 + 4 = 5$. **Ans. 5**

Trigonometric Mechanics

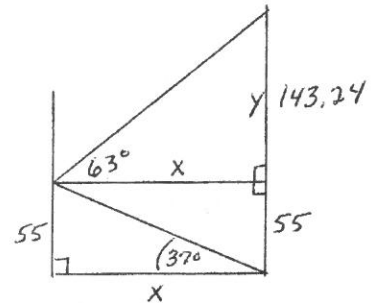
1. In the figure as shown: $\tan 27^\circ = \frac{x}{86}$. Thus $x = 86 \tan 27^\circ$, which equals 43.819. Rounded 43.8. **Ans. 43.8 ft**



2. Using the figure at right: to find the distance x between the buildings: $\tan 37^\circ = \frac{55}{x}$ or $x = \frac{55}{\tan 37^\circ}$. To find y , $\tan 63^\circ = \frac{y}{x}$.

Thus $y = x \tan 63^\circ = \frac{55 \tan 63^\circ}{\tan 37^\circ} = 143.24$. The height of the Hirsh Building is $55 + 143.24 = 198.24$, rounded 198.2.

Ans. 198.2 ft



3. $AP^2 = AC - CP^2 = 56^2 - 38^2 = 1692$.

$$PB^2 = BC^2 - CP^2 = 82^2 - 38^2 = 5280$$

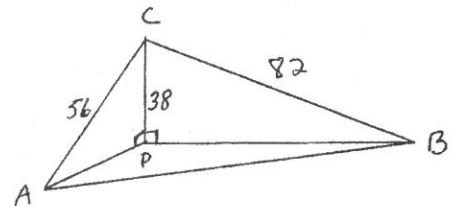
Using Cosine Law in $\triangle ABP$:

$$AB^2 = AP^2 + PB^2 - 2(AP)(PB) \cos P$$

$$\text{or } AB^2 = 6972 + 2044.555 = 9016.555. \quad AB = 94.9555$$

Rounding to 100ths:

Ans. 94.96



Team

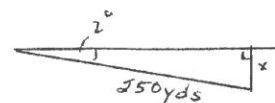
1. The last reflection (e,f) are at points $(-a,-b)$. Therefore $ab - ef = 0$.

Ans. 0

2. Plugging in 7 for each will make b) $6/8$ which is the smallest fraction.

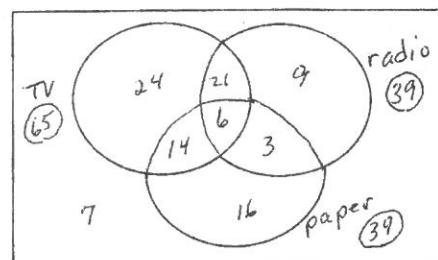
Ans. b

3. In the figure: $\sin 2^\circ = \frac{x}{250}$ or $x = 250 \sin 2^\circ = 8.724$. Rounding:



Ans. 8.7 yds

4. Making a Venn diagram as at right and fitting all the information starting with the 6 people that get their news from all three sources, you will find that 7 got no news from these 3 sources and that $24 + 9 + 16$ got their news from only one source. Total is 49.



Ans. 49

5. $g(x) = 1/x$, therefore $x \neq 0$. $f(g(x)) = f(1/x) = 1/x - 1$. $h(f(g(x))) = h(1/x - 1) = \sqrt{1/x - 1}$.
 $\sqrt{1/x - 1} = \sqrt{\frac{1-x}{x}}$. If $x < 0$, then $\frac{1-x}{x} < 0$, which it cannot be. If $x > 1$, then $\frac{1-x}{x} < 0$ also.

If $0 < x \leq 1$, then $\frac{1-x}{x}$ is positive or 0.

Ans. $0 < x \leq 1$

6. The slope of $\overline{PQ} = \frac{3-2}{-2-12} = -\frac{1}{14}$. The altitude to \overline{PQ} has slope of 14, thus form $y = 14x$ or $14x - y =$. Plugging in $R(7,-3)$ produces the equation (1) $14x - y = 101$.

The slope of $\overline{RQ} = \frac{-3-2}{7-12} = 1$. The altitude to \overline{RQ} has slope of -1 , thus the form

$y = -x$ or $x + y =$. Plugging in $P(-2,3)$ produces (2) $x + y = 1$. Solving simultaneous equations (1) and (2), by adding them: $15x = 102$, thus $x = 6\frac{2}{5}$. In (2): $6\frac{2}{5} + y = 1$, so $y = -5\frac{3}{5}$. Thus the orthocenter is $(6\frac{2}{5}, -5\frac{3}{5})$

Ans. $(6\frac{2}{5}, -5\frac{3}{5})$

$$7. \log_c a \cdot \log_b d = \frac{\log a}{\log c} \cdot \frac{\log d}{\log b} = \frac{\log a}{\log b} \cdot \frac{\log d}{\log c} = \log_b a \cdot \log_c d = \log_b a \cdot \frac{1}{\log_d c} = \frac{\log_b a}{\log_d c} = \frac{6.324}{.8524} =$$

7.419052. Rounding to the nearest 10,000th:

Ans. 7.4191

$$8. f_1(x) = f_o(f_o(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}} = \frac{1}{\frac{-x}{1-x}} = \frac{x-1}{x}.$$

$$f_2(x) = f_o(f_1(x)) = \frac{1}{1 - \frac{x-1}{x}} = \frac{1}{\frac{x-x+1}{x}} = \frac{1}{\frac{1}{x}} = x. \text{ And for every even numbered function of}$$

f , the $f(x) = x$. So $f_{2000}(2000) = 2000$.

Ans. 2000

9. Since $AC = 5$, then $\cos \angle BAD = 5/9$.

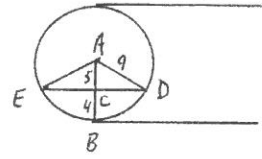
And $m \angle DAE = 2\cos^{-1} 5/9$. The area of the sector is

$\frac{2\cos^{-1} 5/9}{360} \cdot \pi(81) = 79.523$. The area of the triangle is

$(5)\sqrt{81 - 25} = 37.4166$. Therefore the area of the segment of the circle is 42.1064. This multiplied by the length of the drum which is 40, yields 1684.257 cu ft. gives the volume of the oil in the drum. The number of gallons of oil = $7.48(1684.257) = 12,598.242$ gal.

Rounded to the nearest 10 gallons = 12,600 gal.

40



Ans. 12,600 gal