1 Arithmetic with Literal Equations Feb 2013 (No Calculators)

3 pts 1. Solve $L = a + (n - 1)d$ for n. Express your answer as a single fraction in simplest form.

Ans. _____

4 pts 2. P is increased by 20%. The new value of P is then decreased by 30%. What percent of P is the final result?

Ans. _____

5 pts 3. If $\frac{a+2}{m-1} + \frac{b-2}{m+1} = \frac{a-2b+8}{m^2-1}$, where $m \neq 1$ or -1, solve for m. Express your answer as a single fraction in simplest form.

Ans. _____

2 Logs and Log Equations Feb 2013 (No calculators)

3 pts 1. Find x, if $\log_x 128 = 7/3$

Ans. _____

4 pts 2. If $\log_4 P = \frac{2}{3}\log_4 27 - \frac{3}{2}\log_4 25 + 4\log_4 \sqrt{15} - 6\log_4 \sqrt{6}$, find P in simplest form.

Ans. _____

5 pts 3. Find all values of x such that

$$\log_5(2x^2 - 5) + \log_5(x - 1) = \log_5(13x^2 - 42x + 35)$$

Ans. _____

3 Linear Coordinate Geometry Feb 2013 (No Calculators)

3 pts 1. Two lines pass through the point (-4, -7) in the xy-coordinate plane. One line has slope m_1 and a y-intercept at (0, 9). The other line has slope m_2 and a y-intercept at (0, 10). Find the value of $m_2 - m_1$.

Ans. _____

4 pts 2. Line segment L has endpoints at $(a, 2a)$ and $(5a, 4a)$ for some $a \neq 0$. In terms of a, find the point at which the perpendicular bisector of L has its y-intercept.

Ans. _____

5 pts 3. The coordinates of ΔABC are A(6, 11), B(-2, 1) and C(10. -1). $\overline{AD}$ is the median from A to side BC. Point E on line AD has x-coordinate of 8. Find the x-intercept of the line through E perpendicular to line AD.

Ans. _____

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4 Functions Feb 2013 (No Calculators)

3 pts 1. $f(x) = x^2 - 3x + 7$. If $x > 0$ and $f(x) = 11$, find x.

Ans. _____

4 pts 2. When Alice gets a number, she doubles it and tells the result to Bill. Bill adds 7 and tells the result to Carol. Carol subtracts what she gets from 100 and tells the result to Don. Don adds 65 and says the result. Call these functions A(x), B(x), C(x), D(x), respectively and let $E(x) = D(C(B(A(x))))$. Find $E^{-1}(52)$.

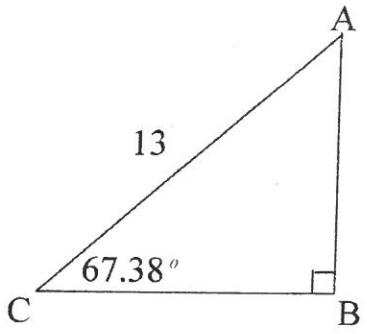
Ans. _____

5 pts 3. Let f be a real-valued function such that $f(x) + 2f(2002/x) = 3x$. Find f(2) in simplest form.

Ans. _____

5 Trigonometric Mechanics Feb 2013 (You may use calculators)

3 pts 1. Find the perimeter of triangle ABC. Round to the nearest unit.



Ans. _____

4 pts 2. At time t = 0, ship A is 40 nm west of ship B and is traveling due east at 10 knots. Ship B is heading due north at 8 knots. Find the value of t in hours when ship B will be 50 degrees north of east as measured from ship A. Round answer to nearest 100th of an hour.

Ans. _____

5 pts 3. The ground in a field slopes upward 3° over the horizontal in the direction from a kite string holder to her kite. The kite is flying on 400 feet of straight string and its shadow is 327 ft from the holder (measured along the ground) when the sun is directly vertical. Because the wind is so strong, she is standing on the spool of string so the kite won't blow away. Measured perpendicular to the ground, how high is the kite in feet? Round answer to nearest tenth of a foot.

Ans. _____

6 Team Feb 2012 (You may use calculators)

3 pts. 1. If $f(1-x) = 3x - 2$, find $f^{-1}(4)$. (1)**Ans.** _____ **3 pts**

3 pts 2. Simplify : $\sqrt{2}^{(\log_2 9)}$ **(2)Ans.** _____ **3 pts**

3 pts 3. In a popular model for aerodynamic drag, $F = kv^{\frac{3}{2}}$. Find v rounded to two decimal places, if $\log F = 0.8$ and $\log k = -1.30$.

(3)**Ans.** _____ **3 pts**

4 pts 4. The number of people waiting in a line at an ice cream stand in Aroostook on the warmest night of the summer equals $28[\sin\left(\frac{t-5}{8}\pi\right) + \cos\left(\frac{t-5}{8}\pi\right)]$, where t = the number of hours past noon. If the expression is rounded to the nearest natural number, find the greatest number of people in line at any one time from 5PM to 10 PM.

(4)**Ans.** _____ **4 pts**

4 pts 5. ΔPQR has vertices P(-2, 15), Q(0, 0) and R(0, 12). If $\overline{QT}$ is the altitude to $\overline{PR}$, find the measure of angle RQT to the nearest minute.

(5)**Ans.** _____ **4 pts**

4 pts 6. If $\log_b AC - \log_b(C - A) = D$, find C as a single fraction in simplest form.

(6)**Ans.** _____ **4 pts**

5 pts 7. Consider the following Chart:

x	1	2	3	4	5	6	7	8
f(x)	3	7	5	1	8	6	4	2
g(x)	2	3	7	8	4	6	5	1

Let $h(x) = g^{-1}(f^{-1}(f^{-1}(f^{-1}(x))))$. Find the product:
$[h(8) + h(7) + h(6) + h(5)]\,[h(4) + h(3) + h(2) + h(1)]$. (7) **Ans.** _____ **5 pts**

5 pts 8. The seating at a soccer match consists of parallel sections of bleachers on opposite sides of the field. Pele spots Mia 53 meters to the right of the seat directly across from him. If David is to Mia's immediate left one meter away, and if it is 140 meters from Pele's seat to a seat directly across from him, find the measure of the angle Mia-Pele-David in degrees rounded to 4 decimal places.

(8)**Ans.** _____ **5 pts**

5 pts 9. Quadrilateral ABCD is such that A(-2, -4), B(9, -1), C(10, 4) and D(-3, 4). $\overline{AE} \perp \overline{BD}$ at E, and $\overline{CF} \perp \overline{BD}$ at F. Find the length of $\overline{EF}$.

(9)**Ans.** _____ **5 pts**

Solutions – Arithmetic with Literal Equations

1. $L = a + (n-1)d \Rightarrow (L-a)/d = n-1 \Rightarrow n = \frac{L-a}{d}+1 \Rightarrow n = \frac{L-a+d}{d}$. **Ans.** $\frac{L-a+d}{d}$

2. $P + .2P = 1.2P$. $1.2P - .3(1.2P) = .7(1.2P) = .84P$. $.84 = 84\%$. **Ans. 84%**

3. $\frac{a+2}{m-1}+\frac{b-2}{m+1}=\frac{a-2b+8}{m^2-1} = (a+2)(m+1)+(b-2)(m-1) = a-2b+8 =$
$am + 2m + a + 2 + bm - 2m - b + 2 = a - 2b + 8 \Rightarrow m(a+b) + a - b + 4 = a - 2b + 8$
$m(a+b) = -b + 4$. Thus $m = \frac{4-b}{a+b}$. **Ans.** $\frac{4-b}{a+b}$

Logs and Logarithmic Equations

1. $\log_x 128 = 7/3 \Rightarrow x^{\frac{7}{3}} = 128 \Rightarrow \left(x^{7/3}\right)^{3/7} = \left(2^7\right)^{3/7} = 2^3 = 8$. **Ans. 8**

2. $\frac{2}{3}\log_4 27 - \frac{3}{2}\log_4 25 + 4\log_4\sqrt{15} - 6\log_4\sqrt{6} \Rightarrow$
$\log_4\left(3^3\right)^{2/3} - \log_4\left(5^2\right)^{3/2} + \log_4\left(15^{1/2}\right)^4 - \log_4\left(6^{1/2}\right)^6 = \log_4\frac{3^2 15^2}{5^3 6^3} = \log_4\frac{3^2 3^2 5^2}{5^3 3^3 2^3} =$
$\log_4\frac{3}{40} = \log_4 P$. So $P = \frac{3}{40}$. **Ans.** $\frac{3}{40}$

3. $\log_5(2x^2 - 5) + \log_5(x-1) = \log_5(13x^2 - 42x + 35) \Rightarrow$
$(2x^2 - 5)(x-1) = 13x^2 - 42x + 35 \Rightarrow 2x^3 - 2x^2 - 5x + 5 = 13x^2 - 42x + 35 \Rightarrow$
$2x^3 - 15x^2 + 37x - 30 = 0$. Solving synthetically:

3	2	-15	37	-30
		6	-27	30
2	2	-9	10	0
		4	-10	
	2	-5	0	

$\Rightarrow 2x - 5 = 0$

Ans. 3, 2 or 5/2

Linear Coordinate Geometry

1. $m_2 = \frac{10-(-7)}{0-(-4)} = \frac{17}{4}$. $m_1 = \frac{9-(-7)}{0-(-4)} = \frac{16}{4}$. $m_2 - m_1 = ¼$. **Ans. ¼**

2. Slope of L $= \frac{4a-2a}{5a-a} = \frac{1}{2}$. Slope of Perpendicular line -2. Midpoint: $\left(\frac{5a+a}{2}, \frac{4a+2a}{2}\right)$
$= (3a, 3a)$. Perpendicular bisecting equation: $y = -2x + b \Rightarrow 2x + y = b$. $2(3a) + 3a = b$,
thus $b = 9a$.

Ans. 9a or **(0, 9a)**

3. A(6, 11), B(-2, 1) and C(10. -1). Mdpt. D = (4, 0). Line AD: $y = \frac{11-0}{6-4}x = \frac{11}{2}x$ form, so $11x - 2y = 11(6) - 2(11) = 44 \Rightarrow 11x - 2y = 44$. For x coordinate 8: $11(8) - 2y = 44 \Rightarrow$ the y-coordinate is 22. The line perpendicular to line AD has the form $2x + 11y = C$. Passing through E, the equation is: $2(8) + 11(22) = C \Rightarrow 16 + 242 = C$, $2x + 11y = 258$. The x intercept of this line is: $2x + 11(0) = 258 \Rightarrow x = 129$. **Ans. 129**

Functions

1. $11 = x^2 - 3x + 7 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0$. $x = 4$ for $x > 0$. **Ans. 4**

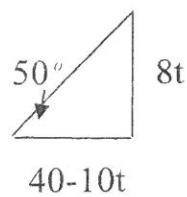
2. Working backward: 52 => 13 => 113 => 106 => 53. **Ans. 53**

3. $f(x) + 2f(2002/x) = 3x$. (1) $f(2) + 2f(1001) = 6$. (2) $f(1001) + 2f(2) = 3003$. In (2): $f(1001) = 3003 - 2f(2)$. Subbing this into (1): $f(2) + 2[3003 - 2f(2)] = 6$. Thus $-3f(2) = 6 - 6006 \Rightarrow f(2) = 2000$. **Ans. 2000**

Trigonometric Mechanics

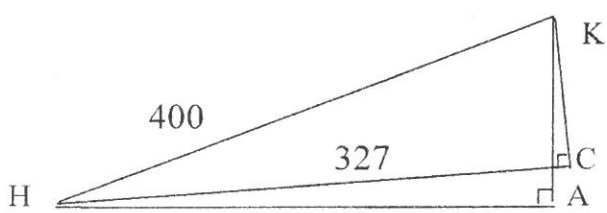
1. $AB = \sin 67.38^\circ$. $BC = \cos 67.38^\circ$. Perimeter = 30.0000165. **Ans. 30**

2. $\tan 50^\circ = \frac{8t}{40 - 10t} \Rightarrow t = \frac{40 \tan 50^\circ}{8 + 10 \tan 50^\circ} = 2.39337$



Ans. 2.39

3.



$AH = 327 \cos 3^\circ$. $m\angle KHA = \cos^{-1}\left(\frac{327 \cos 3^\circ}{400}\right)$. $m\angle KHC = m\angle KHA - 3^\circ$.

$KC = 400 \sin\left[\cos^{-1}\left(\frac{327 \cos 3^\circ}{400}\right) - 3^\circ\right] = 213.59925$. **Ans. 213.6**

Team

1. $4 = 3x - 2$, thus $x = 2$. $1 - x = 1 - (2) = -1$. **Ans. -1**

2. $\sqrt{2}^{\log_2 9} = (2)^{\frac{1}{2}\log_2 9} = 2^{\log_2 3} = 3$. **Ans. 3**

3. $\log F = \log K + \frac{3}{2}\log v \Rightarrow \log v = \frac{2}{3}(\log F - \log k) = \frac{2}{3}(2.1) = 1.4$.
$v = 10^{1.4} = 25.11886$. **Ans. 25.12**

4. Analysis of the graph of $\sin\theta + \cos\theta$ on this domain shows the greatest value at $\pi/4$, the equivalent of 7 PM. $28[\sin \pi/4 + \cos \pi/4)] = 39.59$. **Ans. 40**

5. line PR ➔ $y = \frac{15-12}{-2-0}x = -\frac{3}{2}x$, so $3x + 2y = 3(0) + 2(12) = 24$. The line parallel to PR through Q is $3x + 2y = 0$. $QT = \frac{|24-(0)|}{\sqrt{9+4}} = \frac{24}{\sqrt{13}}$. $\cos \angle RQT = \frac{\frac{24}{\sqrt{13}}}{12} = \frac{2}{\sqrt{13}}$. So $m\angle RQT$ equals $\cos^{-1}\frac{2}{\sqrt{13}} = 56^\circ 19'$

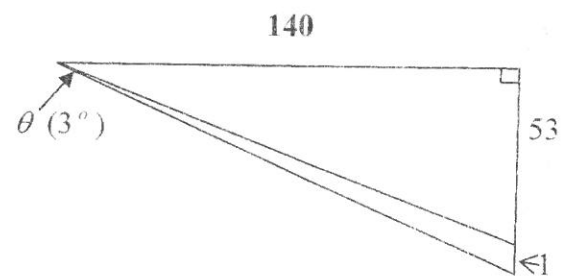
Ans. $56^\circ 19'$

6. $\log_b AC - \log_b(C - A) = D$ ➔ $\log_b \frac{AC}{C-A} = D$ ➔ $b^D = \frac{AC}{C-A}$. Thus $b^D(C - A) = AC$.
$b^D C - b^D A = AC$ ➔ $C(b^D - A) = b^D A$, so $C = \frac{b^D A}{b^D - A}$.

Ans. $\frac{b^D A}{b^D - A}$

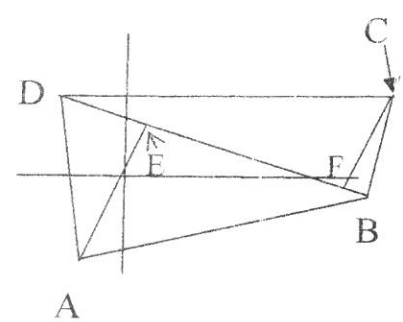
7. Observe that $f(f(f(g(x)))) = x$ for all x. Therefore $h(x) = x$ for all x and thus $[8 + 7 + 6 + 5]\,[4 + 3 + 2 + 1] = 26[10] = 260$. **Ans. 260**

8. In the figure at right
$\theta = \tan^{-1}\left(\frac{54}{140}\right) - \tan^{-1}\left(\frac{53}{140}\right) = .3571^\circ$.



Ans. .3571 degrees

9. At right: $BD = \sqrt{(-3-9)^2 + (4+1)^2} = 13$.
Line BD: $y = \frac{4-(-1)}{-3-9}x$ ➔ $y = -\frac{5}{12}x$ ➔ $5x + 12y = 33$.
Line through C(10, 4) parallel to $\overline{BD}$: $5x + 12y = 98$.
$CF = \frac{|F|}{\sqrt{a^2+b^2}} = \frac{|98-33|}{13} = 5$. $BC = \sqrt{(10-9)^2 + (4+1)} = \sqrt{26}$.
Thus $BF = \sqrt{26-25} = 1$.
Line through A(-2, -4) parallel to BC: $5x + 12y = -58$.
$AE = \frac{|33-(-58)|}{13} = 7$. $AD = \sqrt{(-3+2)^2 + (4+4)^2} = \sqrt{65}$. Thus $DE = \sqrt{65-49} = 4$.
$FE = BD - BF - DE = 13 - 1 - 4 = 8$.



Ans. 8

Answer Sheet Feb 2013

Arithmetic with Literal Equations

1. $\frac{L-a+d}{d}$
2. 84 or 84%
3. $\frac{4-b}{a+b}$

Logs and Logarithmic Equations

1. 8
2. 3/40
3. 3, 2, 2½ or 3, 2, 5/2

Linear Coordinate Geometry

1. 1/4
2. 9a or (0, 9a)
3. 129 or (129, 0)

Functions

1. 4
2. 53
3. 2000

Trigonometric Mechanics

1. 30
2. 2.39
3. 213.6

Team

1. -1
2. 3
3. 25.12
4. 40 or 40 people
5. 56° 19'
6. $\frac{b^D A}{b^D - A}$
7. 260
8. 0.3571 degrees
9. 8