1 Arithmetic with Literal Equations Feb 2014 (No Calculators)

3 pts 1. Solve the following for H: $S = 2LW + 2\,LH + 2WH$.

Ans. ____________________

4 pts 2. Solve the following for c: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ans. ____________________

5 pts 3. Consider an interesting five-digit number A. If 1 is placed at the end of A to make a six-digit number, it is 3 times the number made by placing 1 in front of A to make a six-digit number. Find A.

Ans. ____________________

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2 Logs and Logarithmic Equations Feb 2014 (No Calculators)

3 pts 1. Find the sum: $\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27$.

Ans. ____________________

4 pts 2. Simplify completely: $8^{\log_2 5}$

Ans. ____________________

5 pts 3. Solve for x: $3\log_8(9x+5) - 2\log_4(x^2 - 1) = 2$.

Ans. ____________________

3 Linear Coordinate Geometry Feb 2014 (No Calculators)

3 pts 1. Find the y-intercept in (x, y) form of the line passing through (-3, 6) and (5, 10).

Ans. ____________________

4 pts 2. Find the next highest point (x, y) on the line $5x - 12y = -6$, which is beyond the point (6, 3) that also has integral values for both x and y.

Ans. ____________________

5 pts 3. Line m passing through the point (–3, 15) is perpendicular to line p, whose equation is $3x - 4y = -27$. How far is the y-intercept of m from line p?

Ans. ____________________

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4 Functions Feb 2014 (No Calculators)

3 pts 1. If $f(x) = x^2 - 5x - 8$, find all values of x such that $f(x) = 6$.

Ans. ____________________

4 pts 2. If $f(x) = \frac{x-3}{x+2}$, find $f^{-1}(2)$.

Ans. ____________________

5 pts 3. $f(x) = \frac{2x-3}{3x+2}$. $g(x) = \frac{5x+2}{4x-3}$. Find the domain of fog(x).

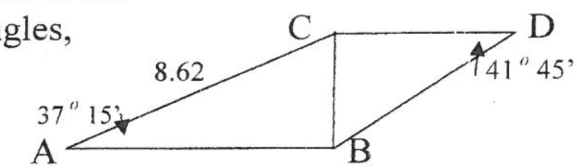
Ans. ____________________

5 **Trigonometric Mechanics Feb 2014 (Calculators allowed)**

3 pts 1. A water balloon is dropped from point C and hit the ground at point B, 59 ft from point A. If A is 74 ft from C, find the measure of angle ACB to the nearest 10th of a degree. (Assume the ground is flat)

Ans. ____________

4 pts 2. In the figure at right, find the length of CD to the nearest 100th. Angles ABC and BCD are right angles, AC = 8.62, m∠A = 37° 15' and m∠D = 41° 45'.



Ans. ____________

5 pts 3. A hot-air balloon is 2400 ft directly above Interstate 80 in Nebraska which extends for miles in a straight line. The angle of depression to a truck on Interstate 80 is 16° 42'. Directly ahead of the truck at an angle of depression of 7° 24' is a tall statue on the side of the road of I 80. To the nearest 10 ft., how far from the statue is the truck?

Ans. ____________

6 Team Feb 2014 (Calculators allowed)

3 pts 1. $\log_A B = C$, $C = 1.5$ and $A = 4x^2 + 4x + 1$. Find B.

(1) **Ans.** ______________________ **3 pts**

3 pts 2. Determine the domain of the function f, if $f(x) = \dfrac{4x^4 - 12x + 9}{4x^2 - 9}$.

(2) **Ans.** ______________________ **3 pts**

3 pts 3. If $g(y) = y^2 - 1$, find $\dfrac{g(g(y))}{g(y) - 1}$ (3) **Ans.** ______________________ **3 pts**

4 pts 4. The number $(2^{16} - 1)$ is divisible by four prime numbers. Find the sum of these numbers.

(4) **Ans.** ______________________ **4 pts**

4 pts 5. Find the area of the triangle bounded by: $y = 3x - 1$, $x = y - 1$, and $x + y = 7$.

(5) **Ans.** ______________________ **4 pts**

4 pts 6. If $\log_{10} x = 1.125$, $\log_5 y = 2.375$ and $\log_2 z = 3.625$, find log xyz. Round answer to nearest 1000^{th}.

(6) **Ans.** ______________________ **4 pts**

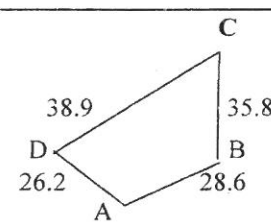
5 pts 7. Find the exact range of f, if $f(x) = \sqrt{6x - x^2} + \sqrt{12x - x^2 - 32}$

(7) **Ans.** ______________________ **5 pts**

5 pts 8. Find the exact coordinates (x, y) of the orthocenter of triangle ABC, where A(-3, -2), B(9, 8) and C(5, 12).

(8) **Ans.** ______________________ **5 pts**

5 pts 9. Find the measure of angle B to the nearest minute in quadrilateral ABCD at right. The measure of angle A is $110^\circ 37'$.



Solutions – Arithmetic with Literal Equations

1. S - 2LW = H(2L + 2W) ➔ $H = \frac{S-2LW}{2L+2W}$ **Ans.** $\frac{S-2LW}{2L+2W}$

2. Since this is the solution for x in $ax^2 + bx + c = 0$, then $c = -ax^2 - bx$. **Ans.** $-ax^2 - bx$

3. Let A = the number. Then 10A + 1 = 3(A + 100,000) ➔ 10A + 1 = 3A + 300,000 ➔ 7A = 299,999 ➔ A = 42,857. **Ans. 42,857**

Logs and Log Equations

1. $\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27 = 3 + 3/2 + 1 + 3/4 + 3/5 =$

   4 + (30 + 15 + 12)/20 = 4 + 57/20 = 6 17/20 or 137/20. **Ans.** $6\frac{17}{20}$ **or 137/20**

2. $8^{\log_2 5} = (2^3)^{\log_2 5} = 2^{3\log_2 5} = 2^{\log_2 125} = 125$. **Ans. 125**

3. $3\log_8(9x+5) - 2\log_4(x^2-1) = 2$. $3\log_8(9x+5) = \log_8(9x+5)^3 = \log_2(9x+5)$
   $2\log_4(x^2-1) = \log_4(x^2-1)^2 = \log_2(x^2-1)$. Therefore
   $\log_2(9x+5) - \log_2(x^2-1) = 2$ ➔ $\log_2 \frac{9x+5}{x^2-1} = 2$ ➔ $\frac{9x+5}{x^2-1} = 4$ ➔ $9x + 5 = 4x^2 - 4$ ➔
   $4x^2 - 9x - 9 = 0$ ➔ $(4x-3)(x-3) = 0$. So x = 3 or - ¾. – ¾ cannot be used. **Ans. 3**

Linear Coordinate Geometry

1. $m = \frac{6-10}{-3-5} = 1/2$. $y = \frac{1}{2}x + b$ ➔ $10 = \frac{1}{2}(5) + b$ ➔ b = 7½. **Ans. (0, 7½)**

2. The slope is 5/12, so the next higher integral values above (6, 3) are (6 + 12, 3 + 5) = (18, 8). **Ans. (18, 8)**

3. The line perpendicular to 3x – 4y = -27 has the form 4x + 3y =, plugging in the point (-3, 15) makes 4x + 3y = 4(-3) + 3(15) = 33. The y-intercept of this line is (0, 11). The line parallel to 3x – 4y = -27 passing through (0, 11) is 3x + 4y = -44. The distance between these two parallel lines is $\frac{|-44-(-27)|}{5} = \frac{17}{5} = 3\frac{2}{5} = 3.4$. **Ans.** $3\frac{2}{5}$ **or 3.4**

Functions

1. $6 = x^2 - 5x - 8$ ➔ $0 = x^2 - 5x - 14$ ➔ $0 = (x-7)(x+2)$. So x = 7 or -2. **Ans. 7 or -2**

2. $f(x) = \frac{x-3}{x+2}$, to find $f^{-1}(2)$, simply set $\frac{x-3}{x+2} = 2$ and solve for x: $x - 3 = 2x + 4$ ➔ $-7 = x$. The alternative is to find $f^{-1}(x)$ and plug in 2 for x. To find $f^{-1}(x)$ we switch the x for y and y for x and solve for y: $x = \frac{y-3}{y+2}$ ➔ $x(y + 2) = y - 3$ ➔ $xy + 2x = y - 3$ ➔ $xy - y = -2x - 3$ ➔ $y(x - 1) = -2x - 3$ ➔ $y = \frac{-2x-3}{x-1}$ ➔ $f^{-1}(x) = \frac{-2x-3}{x-1}$, $f^{-1}(2) = \frac{-7}{1}$

Ans. -7

3. $f(x) = \frac{2x-3}{3x+2}$, $g(x) = \frac{5x+2}{4x-3}$. $fog(x) = \frac{2\left(\frac{5x+2}{4x-3}\right)-3}{3\left(\frac{5x+2}{4x-3}\right)+2} = \frac{2(5x+2)-3(4x-3)}{3(5x+2)+2(4x-3)} =$

$\frac{10x+4-12x+9}{15x+6+8x-6} = \frac{-2x+13}{23x}$. The domain is:

Ans. All reals $\neq$ -2/3, 3/4, or 0

Trigonometric Mechanics

1. $\sin \angle C = 59/74$. $C = 52.9^\circ$.

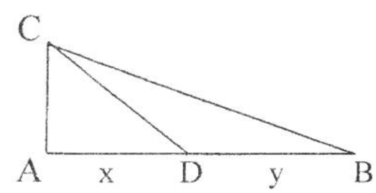
Ans. 52.9°

2. $\sin 37^\circ 15' = \frac{BC}{8.62}$, so $BC = 8.62 \sin 37^\circ 15'$. $\tan 41^\circ 45' = \frac{BC}{CD}$.

$CD = \frac{8.62 \sin 37^\circ 15'}{\tan 41^\circ 45'} = 5.8458$.

Ans. 5.85

3. In the figure, x + y is the distance from a point A directly below the balloon on I 80 to the statue at B. y is the distance from the truck D to the statue and x is the distance from A to D.



$m\angle B = 7^\circ 24'$, so $\tan 7^\circ 24' = \frac{2400}{x+y}$ ➔ $x + y = \frac{2400}{\tan 7^\circ 24'}$.

$m \angle B = 16^\circ 42'$, so $\tan \angle B = 16^\circ 42' = \frac{2400}{x}$ ➔ $x = \frac{2400}{\tan 16^\circ 42'}$.

$x + y - x = \frac{2400}{\tan 7^\circ 24'} - \frac{2400}{\tan 16^\circ 42'} = 10{,}479$ = Rounded: 10, 480 ft.

Ans. 10,480 ft

Team

1. $\log_A B = C$, $C = 1.5$ and $A = 4x^2 + 4x + 1$ so $A = \left((2x+1)^2\right)^{3/2} = (2x + 1)^3 =$ $(4x^2 + 4x + 1)(2x + 1) = 8x^3 + 8x^2 + 6x + 1$.

Ans. $8x^3 + 12x^2 + 6x + 1$

2. $f(x) = \frac{4x^4 - 12x + 9}{4x^2 - 9} = \frac{(2x-3)(2x-3)}{(2x-3)(2x+3)}$. Although the $2x - 3$ can cancel, in the function f(x) it still exists.

Ans. All reals $\neq$ 3/2 or -3/2

3. $\frac{g(g(y))}{g(y)-1} = \frac{(y^2-1)^2-1}{y^2-1-1} = \frac{y^4-2y^2+1-1}{y^2-2} = \frac{y^2(y^2-2)}{y^2-2} = y^2$ **Ans. y^2**

4. $2^{16}-1=(2^8-1)(2^8+1) = (2^8+1)(2^4-1)(2^4+1) = (2^8+1)(2^4+1)(2^2+1)(2+1)(2-1)$.
$(2^8+1)+(2^4+1)+(2^2+1)+(2^2-1) = 257+17+5+3$. **Ans. 282**

5. The lines y = 3x – 1 and x = y – 1 intersect at (3, 4). The lines and y = 3x – 1 and x + y = 7 intersect at (2, 5). x = y – 1 and x + y = 7 intersect at (3, 4). Using determinants, the area is $\frac{1}{2}\left\|\begin{matrix}3 & 4 & 1\\ 1 & 2 & 1\\ 2 & 5 & 1\end{matrix}\right\| = \frac{1}{2}|(4+4+15)+(-6-5-8)| = \frac{1}{2}(23-19)=2$. **Ans. 2**

6. (1) $\log_{10} x = 1.125$. $\log_5 y = 2.375$, so $y = 5^{2.375}$. Taking log base 10 of both sides: $\log_{10} y = \log_{10} 5^{2.375} = 2.375 \log_{10} 5$. $\log_2 z = 3.625$, so $z = 2^{3.625}$, thus $\log_{10} z = 3.625 \log_{10} 2$. $\log_{10} x + \log_{10} y + \log_2 z = \log_{10} xyz = \log xyz = 1.125 + 2.375 \log_{10} 5 + 3.625 \log_{10} 2 =$ 3.8762.
Ans. 3.876

7. $f(x) = \sqrt{6x-x^2} + \sqrt{12x-x^2-32}$. Let $y = \sqrt{6x-x^2}$, so $y^2 = 6x - x^2$ ➔ $y^2 + (x^2 - 6x + 9) = 9$, a circle with center at (3, 0) and radius of 3. Let $y = \sqrt{12x-x^2-32}$, then $y^2 = 12x - x^2 - 32$ or $y^2 + (x^2 - 12x + 36) = 4$, a circle with center at (6, 0) radius 2. Graphing both on the same axis and adding ordinates to find the highest point. We need to find where $\sqrt{6x-x^2} = \sqrt{12x-x^2-32}$, $6x - x^2 = 12x - x^2 - 32$ ➔ $6x = 32$, $x = 16/3$.
Plugging this in: $f(16/3) = \sqrt{6(16/3)-(16/3)^2} = \sqrt{32-256/9} = \sqrt{32/9} = \frac{4\sqrt{2}}{3}$. Doubling this makes $\frac{8\sqrt{2}}{3}$, which is greater than 3. **Ans. $0 \le x \le \frac{8\sqrt{2}}{3}$**

8. A(-3, -2), B(9, 8) and C(5, 12). Slope of $\overline{BC} = \frac{8-12}{9-5} = -1$. Equation form: x + y =, the line perpendicular: y – x = , plugging in A(-3, -2) this altitude is (1) x – y = -1. The slope of $\overline{AB}$ is $\frac{-2-8}{-3-9} = \frac{5}{6}$. Equation form: $y = \frac{5}{6}x$ or 5x – 6y = , the line perpendicular has form 6x + 5y = , plugging in C(5, 12) this altitude is (2) 6x + 5y = 90. Multiplying (1) by 5 ➔ 5x – 5y = -5. Adding this to (2): 11x = 85, x = 85/11 = $7\frac{8}{11}$, so y = $8\frac{8}{11}$.
Ans. ($7\frac{8}{11}$, $8\frac{8}{11}$) or **(85/11, 96/11)**

9. Connect B to D. $BD^2 = 26.2^2 + 28.6^2 - 2(26.2)(28.6)\cos 110°37' = 45.08$
$\frac{\sin\angle 1}{26.2} = \frac{\sin 110°37'}{BD}$ ➔ $m\angle 1 = 32°57'19''$. $38.9^2 = 35.8^2 + BD^2 - 2(35.8)(BD)\cos\angle 2$.
$m\angle 2 = 56°05'35''$. So $m\angle B = 89°02'54''$. **Ans. 89° 03'**

Answer Sheet – Feb 2014

Arithmetic with Literal Equations

1. $\frac{S-2LW}{2L+2W}$
2. $-ax^2 - bx$
3. 42,857

Logs and Log Equations

1. 137/20 or $6\frac{17}{20}$ or 6.85
2. 125
3. 3

Linear Coordinate Geometry

1. $(0, 7\frac{1}{2})$
2. (18, 8)
3. $3\frac{2}{5}$ or 17/5 or 3.4

Functions

1. 7 or -2
2. -7
3. All reals $\neq$ -2/3, 3/4, or 0

Trigonometric Mechanics

1. 52.9°
2. 5.85
3. 10,480 ft or 10,480

Team

1. $8x^3 + 12x^2 + 6x + 1$
2. All reals $\neq 3/2$ or $-3/2$
3. y^2
4. 282
5. 2
6. 3.876
7. $0 \le x \le \frac{8\sqrt{2}}{3}$
8. $(7\frac{8}{11}, 8\frac{8}{11})$ or (85/11, 96/11)
9. $89^\circ 03'$