1 Arithmetic with Literal Equations Feb 2015 (No Calculators)

3 pts 1. If none of the variables in the equation $A - P = \frac{AB}{C}$ are equal to zero, solve for A as a single fraction in simplest form.

Ans. _____

4 pts 2. Find the remainder when the product of 342(130)(234) is divided by 7.

Ans. _____

5 pts 3. The surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$. Solve the equation for the radius (r). Express your answer as a single fraction in simplest form.

Ans. _____

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2 Logs and Log Equations Feb 2015 (No Calculators)

3 pts 1. If $6 \log_x x = 2n$, solve for n.

Ans. _____

4 pts 2. Solve for x, if $\log_6(x - 2) + \log_6(x + 3) = 2$.

Ans. _____

5 pts 3. $\log A = 2x + 1$, $\log B = 2x - 1$. Find all value(s) of x such that

$$\log_B A - \log_A B = \frac{A}{B}.$$

Ans. _____

3 Linear Coordinate Geometry Feb 2015 (No Calculators)

3 pts 1. If (6, 9) and (10, 3) are the coordinates of two opposite vertices of a square, what is the equation of the line that contains the other diagonal? State your answer in the form $y = mx + b$.

Ans. _____

4 pts 2. Consider line L which contains the points (-3,8) and (6, -4). What is the length of the hypotenuse of the right triangle formed by L, the x-axis and the y-axis?

Ans. _____

5 pts 3. A(0, 0), B(2, 6) and C(4, -2) are the coordinates of Triangle ABC. Find the point (x, y) which is the intersection of the median to side AB and perpendicular bisector of side BC.

Ans. _____

4 Functions Feb 2015 (No Calculators)

pts 1. If $f(x) = 5x + 3$ and $g(x) = 7 - 2x$. Find $\frac{f(x) - g(x)}{f(g(x))}$. Express answer as a single fraction in simplest form.

Ans. _____

4 pts 2. $f(x) = 5x + 2$, $g(x) = x^2 - 1$, find all value(s) of x such that

$$f \circ g(x) + g \circ f(x) = 10$$

Ans. _____

5 pts 3. Suppose that f(x) is a linear function such that $3\,f(x) + 2\,f(1 - x) = 2x + 9$ for every real number x. What is the value of f(2)?

Ans. _____

5 Trigonometric Mechanics Feb 2015 (You may use Calculators)

pts 1. The legs of a right triangle are 2 and $\sqrt{5}$ units long. Let α be the smallest angle of the triangle. To the nearest degree, what is the measure of α?

Ans. _____

4 pts 2. Two ladders, one twice as long as the other and each having one end resting on the floor, have their opposite ends reaching the same vertical height along a wall. The shorter ladder makes a 60 degree angle with the floor. What angle (to the nearest degree) does the longer ladder make with the floor?

Ans. _____

5 pts 3. From the top of a building, the angle of elevation to the top of the building next to it is $23^{\circ}46'$. The angle of depression to the base of the building is $54^{\circ}27'$. If the taller building is 60 feet high, what is the distance between the buildings? Give answer correct to 4 decimal places.

Ans. _____

6 Team Feb 2015 (You may use calculators)

pts 1. Find the only whole number between 16,100,000 and 16,300,000 that is the perfect cube of an integer.

(1)**Ans.** _____ **3 pts**

3 pts 2. If log A = .3 and log B = .4, find $\log \frac{A}{B} + \log AB$. **(2)Ans.** _____ **3 pts**

3 pts 3. An inlet pipe can fill an empty tank in 2 hrs. When the tank is full, an outlet pipe can empty the tank in 5 hours. If the tank is half full, and both pipes are opened, how soon will the tank be filled? Express answer in hours and minutes, rounded to the nearest minute.

(3) Ans. _____ **3 pts**

4 pts 4. What is the value of $\frac{\log_2 3 \cdot \log_4 5 \cdot \log_6 7}{\log_4 3 \cdot \log_6 5 \cdot \log_8 7}$? **(4) Ans.** _____ **4 pts**

4 pts 5. At 11:30 am the bearing of a Carnival Cruise ship was $52^\circ 40'$ from the Mayflower Lighthouse. The ship was 2.6 km from the lighthouse on a course of 180°. At 12:15 pm on the same day, the bearing of the ship from the lighthouse was $142^\circ 40'$. If the ship was traveling at a constant rate, to the nearest hundredth, what was its speed in km/hr?

(5) Ans. _____ **4 pts**

4 pts 6. If $f(2x + 1) = 4x^2 + 14x$, then find the sum of the roots of $f(x) = 0$.

(6) Ans. _____ **4 pts**

5 pts 7. From an airplane 2400 ft above the ground, the angle of depression to a building straight ahead of the plane is $8^\circ 42'$. At that same moment, the angle of depression to a second building is $9^\circ 25'$, but this building is at 90° to the direction of the plane. To the nearest hundredth of a mile, how far apart are the two buildings? The buildings are on level ground and the angles of depression are to the base of each building.

(7) Ans. _____ **5 pts**

5 pts 8. What is the area of the region bounded by the graph of $|x+y|+|x-y| = 4$?

(8) Ans. _____ **5 pts**

5 pts 9. Quadrilateral ABCD is a "kite", with $\overline{AC}$ bisecting $\overline{BD}$. Find the exact value for the coordinates of D, if A(-2, -4), B(-4, 1) and C(3, 8).

(9) Ans. _____ **5 pts**

Solutions – Arithmetic with Literal Equations

1. $AC - PC = AB \rightarrow AC - AB = PC \rightarrow A(C - B) = PC$. $A = \frac{CP}{C-B}$. **Ans.** $\frac{CP}{C-B}$

2. Dividing 342, 130, and 234 by 7yields respective remainders 6,4,3. $6(4)(3) = 72$. Dividing by 7 gives remainder of 2. **Ans. 2**

3. $A = 2\pi r^2 + 2\pi rh \rightarrow 0 = 2\pi r^2 + 2\pi rh - A$. Using the quadratic formula:

$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 - 4(2\pi)(-A)}}{4\pi} = \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi A}}{4\pi}$. Since the radius cannot be negative,

then $r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$. **Ans.** $\frac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$

Logs and Log Equations

1. $\log_x x = 1$, so $6 \log_x x = 2n \rightarrow 6(1) = 2n$. $n = 3$. **Ans. 3**

2. $\log_6(x - 2) + \log_6(x + 3) = 2 \rightarrow x^2 + x - 6 = 36 \rightarrow x^2 + x - 42 = 0 \rightarrow (x + 7)(x - 6) = 0$. So $x = -7$ or 6. But x cannot be -7. **Ans. 6**

3. Since $\log A = 2x + 1$, then $A = 10^{2x+1}$. Likewise $\log B = 2x - 1$, then $B = 10^{2x-1}$.

$\log_B A - \log_A B = \frac{A}{B} \rightarrow \frac{\log A}{\log B} - \frac{\log B}{\log A} = \frac{10^{2x+1}}{10^{2x-1}} \rightarrow \frac{2x+1}{2x-1} - \frac{2x-1}{2x+1} = 10^2 \rightarrow$

$\frac{(2x+1)^2 - (2x-1)^2}{4x^2 - 1} = 100 \rightarrow 8x = 400x^2 - 100 \rightarrow 0 = 400x^2 - 8x - 100 \rightarrow 0 = 100x^2 - 2x - 25$.

$x = \frac{2 \pm \sqrt{4 - 4(-2500)}}{200} = \frac{1 \pm \sqrt{2501}}{100}$. **Ans.** $\frac{1 \pm \sqrt{2501}}{100}$

Linear Coordinate Geometry

1. The midpoint of (6, 9) and (10, 3) = (8, 6). The slope of the line through (6, 9) and (10, 3) is $\frac{9-3}{6-10} = -\frac{3}{2}$. So the other diagonal has slope $\frac{2}{3}$. $y = \frac{2}{3}x + b \rightarrow 6 = \frac{2}{3}(8) + b \rightarrow$

$6 = 5\frac{1}{3} + b \rightarrow b = \frac{2}{3}$. **Ans.** $y = \frac{2}{3}x + \frac{2}{3}$

2. Slope of L: $\frac{8-(-4)}{-3-6} = -\frac{4}{3}$. Line L is: $8 = -\frac{4}{3}(-3) + b \rightarrow 8 = 4 + b$, $b = 4$. $y = -\frac{4}{3}x + 4$. In standard form, $4x + 3y = 12$. x-i = 3 and y-i = 4. A basic 3-4-5 Δ. **Ans. 5**

3. Median to $\overline{AB}$: mdpt of $\overline{AB}$: (1,3) through C(4, -2): $y = \frac{5}{-3}x \rightarrow 5x + 3y = 14$ (1)

$\perp$ bis. of $\overline{BC}$: mdpt of $\overline{BC}$: (3, 2); slope of $\overline{BC}$: $\frac{8}{-2}$; $y = \frac{1}{4}x \rightarrow x - 4y = -5$ (2)

5(2) – (1): $-23y = -39$, so $y = \frac{39}{23}$. $3(2) + 4(1)$: $23x = 41$, so $x = \frac{41}{23}$. **Ans.** $\left(\frac{41}{23}, \frac{39}{23}\right)$

Functions

1. $\dfrac{f(x)-g(x)}{f(g(x))} = \dfrac{5x+3-(7-2x)}{5(7-2x)+3} = \dfrac{7x-4}{35-10x+3} = \dfrac{7x-4}{38-10x}$. **Ans.** $\dfrac{7x-4}{38-10x}$

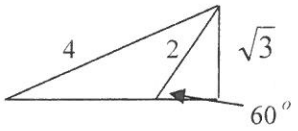
2. f o g (x) + g o f (x) = 10 ➔ $5(x^2 - 1) + 2 + (5x + 2)^2 - 1 = 10$ ➔
$5x^2 - 3 + 25x^2 + 20x + 3 = 10$ ➔ $30x^2 + 20x - 10 = 0$ ➔ $3x^2 + 2x - 1 = 0$ ➔
$(3x - 1)(x + 1) = 0$, so x = -1 or 1/3. **Ans. -1 or 1/3**

3. f(x) must be a linear function, thus f(x) = ax + b. $3(ax + b) + 2(a(1 - x) + b) = 2x + 9$.
$3ax + 3b + 2a - 2ax + 2b = 2x + 9$. $3a - 2a = 2$, so a = 2. $3b + 2a + 2b = 9$ ➔ 5b = 5, so b = 1.
Thus f(x) = 2x + 1, and f(2) = 2(2) + 1 = 5. **Ans. 5**

Trigonometric Mechanics

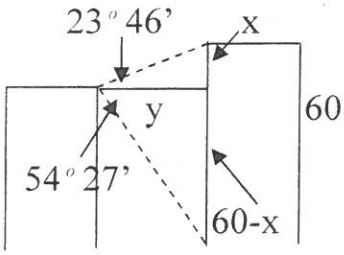
1. $2^2 + (\sqrt{5})^2 = 9$. So hyp. = 3. Smallest angle is opposite 2. $\cos\alpha = \dfrac{\sqrt{5}}{3}$. **Ans. 42°**

2. Use figure at right: $\sin y = \dfrac{\sqrt{3}}{4}$, so $y = 26°$.



Ans. 26°

3. In figure at right: $\tan 23°46' = \dfrac{x}{y}$, so $y = \dfrac{x}{\tan 23°46'}$.



$\tan 54°27' = \dfrac{60-x}{y}$, so $y = \dfrac{60-x}{\tan 54°27'}$. $\dfrac{x}{\tan 23°46'} = \dfrac{60-x}{\tan 54°27'}$ ➔

$x \tan 54°27' = 60 \tan 23°46' - x \tan 23°46'$. Therefore

$x = \dfrac{60\tan 23°46'}{\tan 54°27' + \tan 23°46'} = 14.36167$. $y = \dfrac{x}{\tan 23°46'} = 32.61363$ **Ans. 32.6136**

Team

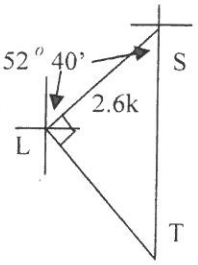
1. Using a calculator $16{,}100{,}000^{1/3} = 252.508$. $253^3 = 16{,}194{,}277$. **Ans. 16,194,277**

2. $\log \dfrac{A}{B} + \log A \cdot B = \log A - \log B + \log A + \log B = 2 \log A = 2(.3) = .6$. **Ans. .6**

3. $\dfrac{x}{2} - \dfrac{x}{5} = \dfrac{1}{2}$, $5x - 2x = 5$, x = 5/3 = 1hr 40 min. **Ans. 1hr 40 min**

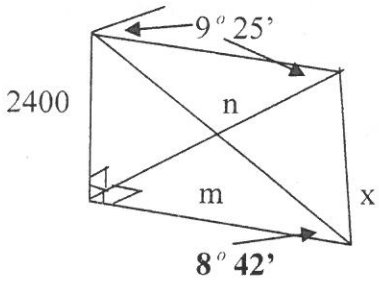
4. $\dfrac{\log_2 3 \cdot \log_4 5 \cdot \log_6 7}{\log_4 3 \cdot \log_6 5 \cdot \log_8 7} = \dfrac{\frac{\log 3}{\log 2}\cdot\frac{\log 5}{\log 4}\cdot\frac{\log 7}{\log 6}}{\frac{\log 3}{\log 4}\cdot\frac{\log 5}{\log 6}\cdot\frac{\log 7}{\log 8}} = \dfrac{\log 3}{\log 2}\cdot\dfrac{\log 5}{\log 4}\cdot\dfrac{\log 7}{\log 6}\cdot\dfrac{\log 4}{\log 3}\cdot\dfrac{\log 6}{\log 5}\cdot\dfrac{\log 8}{\log 7} = \dfrac{\log 8}{\log 2}$ **Ans. 3**

5. At the lighthouse L, the $52^\circ 40'$ bearing angle to the ship is the same as the angle made by the ship and its course bearing, angle S. The angle SLT has a measure of 90°. Since LS = 2.6, then $\cos 52^\circ 40 = 2.6/ST$. Therefore ST = 2.6/(cos $52^\circ 40'$) = 4.287238. Since it took ¾ hr to cover this distance, Then 4.287238/.75 = 5.71631 km/hr. **Ans. 5.72kph**



6. $f(2x + 1) = 4x^2 + 14x$, f(x) must have the form $f(x) = ax^2 + bx + c$.
$f(2x + 1) = a(2x + 1)^2 + b(2x + 1) + c = a(4x^2 + 4x + 1) + 2bx + b + c =$
$4ax^2 + 4ax + a + 2bx + b + c = 4x^2 + 14x$. Thus 4a = 4, so a = 1. 4a + 2b = 14 ➔ 4 + 2b = 14
b = 5. a + b + c = 0 ➔ 1 + 5 + c = 0, so c = -6. $f(x) = x^2 + 5x - 6$. Sum = -5. **Ans. -5**

7. $\tan 9^\circ 25' = 2400/n$, $n = 2400/\tan 9^\circ 25'$. $\tan 8^\circ 42' = 2400/m$,
$m = 2400/\tan 8^\circ 42'$. $x = \sqrt{\left(\dfrac{2400}{\tan 9^\circ 25'}\right)^2 + \left(\dfrac{2400}{\tan 8^\circ 42'}\right)^2} =$
21,340. $\dfrac{21{,}340}{5{,}280} = 4.04169$. **Ans. 4.04**



8. Ordered pairs: (2, 2), (2, -2), (2, 2), (-2, -2); (2, 1), (2, -1), (-2, 1), (-2, -1); (2, 0), (-2, 0), (0, 2), (0, -2). As you can see, it forms a square 4 units on a side. Area = 16. **Ans. 16**

9. Slope of $\overline{AC}$ = 12/5. Slope of $\overline{BD}$ = -5/12. Equation of $\overline{BD}$: $y = -\dfrac{5}{12}x$ ➔ 5x + 12y = -8.
Distance from B to $\overline{AC}$: Line $\overline{AC}$ ➔ $y = \dfrac{12}{5}x$, 12x – 5y = -4; Line through B parallel to $\overline{AC}$:
12x – 5y = -53. Distance = $\dfrac{|F|}{\sqrt{a^2 + b^2}}$, where F is the difference between the constants, and a and b are the coefficients of ax + by = c. $\dfrac{|-53-(-4)|}{\sqrt{12^2+5^2}} = \dfrac{49}{13}$. This is the same distance that D is from $\overline{AC}$. $|c+4| = 49$, then c = -53 or 45. The line through D parallel to $\overline{AC}$ is 12x - 5y = 45.
We need to find where the lines through $\overline{BD}$ and the line through D which is parallel to $\overline{AC}$ intersect: (1) 5x + 12y = -8, (2) 12x – 5y = 45 ➔ 12(1) -5(2): 169y = -96 – 225 = -321,
So y = -321/169. 5(1) + 12(2): 169x = -40 +540 = 500, so x = 500/169. **Ans.** $\left(\dfrac{500}{169}, \dfrac{-321}{169}\right)$

Answer Sheet Feb 2015

Arithmetic with Literal Equations

1. $\frac{CP}{C-B}$
2. 2
3. $\frac{-\pi h+\sqrt{\pi^2 h^2+2\pi A}}{2\pi}$

Logs and Logarithmic Equations

1. 3
2. 6
3. $\frac{1\pm\sqrt{2501}}{100}$

Linear Coordinate Geometry

1. $y = \frac{2}{3}x + \frac{2}{3}$
2. 5
3. $\left(\frac{41}{23},\frac{39}{23}\right)$

Functions

1. $\frac{7x-4}{38-10x}$
2. 1/3 or -1
3. 5

Trigonometric Mechanics

1. 42^o
2. 26^o
3. 32.6136

Team

1. 16,194,277
2. .6
3. 1 hr 40 min
4. 3
5. 5.72 k/h
6. -5
7. 4.04 mi
8. 16
9. $\left(\frac{500}{169},\frac{-321}{169}\right)$