1 Arithmetic with Literal Equations Feb 2016 (No Calculators)

3 pts 1. Solve the following for T_1: $\frac{R_1}{T_1}+\frac{R_2}{T_2}=1$.

Ans. _____

4 pts 2. Find the smallest integer k such that $11k$ divided by 19 has a remainder of 10.

Ans. _____

5 pts 3. In general, S = 1.2F – 24 and H = $\frac{1}{6}$(S + 1), where S = shoe size, F = foot length in centimeters, and H = height in meters. If a person is 1.9 meters tall, how long is his foot in centimeters according to the above formulas? Round answer to nearest tenth.

Ans. _____

2 Logs and Log Equations Feb 2016 (No Calculators)

3 pts 1. Simplify the following: $\log_{27} 9\sqrt[4]{27}$.

Ans. _____

4 pts 2. Solve for x, if $\log_3(5x^2-8x-4)-\log_3(x^2-6x+8)=3$

Ans. _____

5 pts 3. Solve for x, if $\frac{\log_2 3\cdot\log_4 5\cdot\log_6 7}{\log_4 3\cdot\log_6 5\cdot\log_8 7}=\log_{10}\left(\frac{1}{x}\right)$

Ans. _____

3 Linear Coordinate Geometry Feb 2016 (No Calculators)

3 pts 1. Line m is parallel to the line $y = \frac{3}{5}x + 7$ and passes through the point (8, -5). What is the y-intercept of m?

Ans. _____

4 pts 2. The coordinates of A and B of right triangle ABC are (4, 1) and (9, 8) respectively. If angle B is the right angle, and point C has coordinates $(a, -2)$, find a.

Ans. _____

5 pts 3. (10, -2) and (6, -4) are the vertices of the base of an isosceles triangle. The third vertex of the triangle is 10 units from the base of the triangle and is in the first quadrant. Find the coordinates of the third vertex.

Ans. _____

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4 Functions Feb 2016 (No Calculators)

3 pts 1. If $f(x) = 5x - 3$, $g(x) = 6x - 4$ and $h(x) = 7x - 5$, find the value of

$$\frac{f(7) - g(5) + h(6) - 7}{f(5) - g(4) + h(3) - 6}.$$

Ans. _____

4 pts 2. $f(x) = 1 - x^2$, $g(x) = \frac{1}{x+1}$ and $k(x) = \frac{x}{x-1}$. Find the domain of

g(x) divided by $\frac{k(x)}{f(x)+3}$.

Ans. _____

5 pts 3. If $f(n) = 2f(n-1) - 3$ and $f(2) = 1$, find the value of $f(11) - f(12)$.

Ans. _____

5 Trig Mechanics Feb 2016 (You may use calculators)

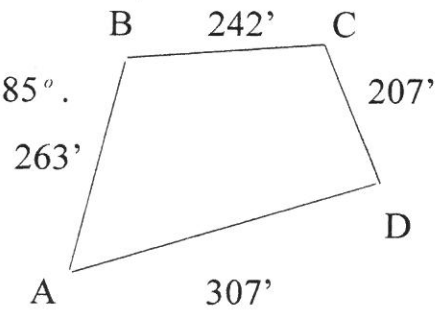
3 pts 1. A tree snapped in high winds and bent over so that its top made an angle of 37° with the ground. If the top touched the ground 26 feet from the center of its base, how tall was the tree before it snapped and bent over? Give answer to nearest tenth of a foot.

Ans. _____

4 pts 2. Two buildings of the same height are 72 feet apart. From a window in one of the buildings the angle of depression to the base of the other is $13^\circ 26'$ and the angle of elevation to the top is $11^\circ 33'$. How tall is each building to the nearest 100^{th} of a foot.

Ans. _____

5 pts 3. The plot of land at right has measurements as indicated. Find the measure of $\angle ABC$ to the nearest minute, if $m\angle BAD = 85^\circ$.



Ans. _____

6 Team Feb 2016 (You may use calculators)

3 pts 1. The point (2, -1) is rotated 90° counterclockwise about the origin. Express the coordinates of the destination of the point in (x, y) form.

(1) Ans. _____ 3 pts

3 pts 2. In right triangle ABC, hypotenuse AC = 10

and $\sin A = \sqrt{3}/3$. Find tan C. **(2) Ans. _____ 3 pts**

3 pts 3. Given $C = \frac{5}{9}(F - 32)$, find the value of F at which

C has half the value of F. **(3) Ans. _____ 3 pts**

4 pts 4. The xy-coordinate plane is to be divided into regions using 2 lines and a parabola.

Let L be the least number of regions that can be created and G the greatest number of regions that can be created. Find L + G.

(4) Ans. _____ 4 pts

4 pts 5. After the January Midterm, Mr. Snowy thought the math final was too hard. Looking at the grades, he decided to use a linear function to scale them. He let the low score of 16 be 40 and the high score of 80 be 100. What would the grade of 64 be scaled to?

(5) Ans. _____ 4 pts

4 pts 6. Solve for *e*, all variables are positive: $(m^e)^a \cdot (n^a)^e = mn^e$

(6) Ans. _____ 4 pts

5 pts 7. Find the radius of the circle which passes through A(7, 5), B(1, -7) and C(9, -1).

(7) Ans. _____ 5 pts

5 pts 8. The line y = x + 5 is rotated 15° about the point (2, 7) producing lines m and n. To 4 decimal places, find the sum of the y-intercepts of new lines m and n.

(8) Ans. _____ 5 pts

5 pts 9. The function $f(ax^n)$ is defined as $f(ax^n) = anx^{n-1}$, where a and n are nonzero real numbers. For example $f(x^2) = 2x^{2-1} = 2x$. If $f(f(f(f(f(ax^n))))) = 480$, find the sum of a and n.

(9) Ans. _____ 5 pts

Solutions – Arithmetic with Literal Equations

1. $\frac{R_1}{T_1}+\frac{R_2}{T_2}=1 \rightarrow \frac{R_1T_2+R_2T_1}{T_1T_2}=1 \rightarrow R_1T_2+R_2T_1=T_1T_2 \rightarrow R_1T_2=T_1(T_2-R_2)$ **Ans.** $\frac{R_1T_2}{T_2-R_2}$

2. Basically 11k = 19n + 10. So we plug in positive integers for n until we find one that is divisible by 11: 1 → 29, 2 → 48, 3 → 67. Notice the ten's digit increases by 2 and the umit's digit decreases by 1, thus 86, 105, 124, 143. 143/11 = 13. **Ans. 13**

3. $1.9=\frac{1}{6}(S+1) \rightarrow 11.4 = S + 1$, S = 10.4. 10.4 = 1.2F – 24, 1.2F = 34.4, F = 344/12 =

172/6 = 86/3 = $28\frac{2}{3}$ = 28.7 to nearest 10th. **Ans. 28.7**

Logs and Log Equations

1. $\log_{27} 9\sqrt[4]{27} = \log_{27} 9+\frac{1}{4}\log_{27} 27=\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$. **Ans. 11/12**

2. $\log_3(5x^2-8x-4)-\log_3(x^2-6x+8)=3 \rightarrow \log_3\frac{5x^2-8x-4}{x^2-6x+8}=3 \rightarrow 5x^2 = 8x - 4 = 27x^2 - 162x + 216$

$0 = 22x^2 – 154x + 220 = 22(x^2 – 7x + 10) \rightarrow (x – 5)(x – 2) = 0$. Only 5 works.

Alt. Sol: If students factor : $\frac{5x^2-8x-4}{x^2-6x+8}=\frac{(5x+2)(x-2)}{(x-4)(x-2)}$, then $\frac{5x+2}{x-4}=27 \rightarrow 22x = 110$ **Ans. 5**

3. $\frac{\log_2 3\cdot\log_4 5\cdot\log_6 7}{\log_4 3\cdot\log_6 5\cdot\log_8 7}=\log_{10}\left(\frac{1}{x}\right) \rightarrow \frac{\log 3}{\log 2}\cdot\frac{\log 5}{\log 4}\cdot\frac{\log 7}{\log 6}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 6}{\log 5}\cdot\frac{\log 8}{\log 7}=\frac{\log 8}{\log 2}=3=\log_{10}\left(\frac{1}{x}\right)$.

So $10^3 = 1/x$, thus x = 1/1000. **Ans. 1/1000**

Linear Coordinate Geometry

1. m takes same slope, so $y=\frac{3}{5}x + b \rightarrow -5=\frac{3}{5}(8) + b \rightarrow -5 - 4\frac{4}{5}= b.$ **Ans.** $-9\frac{4}{5}$

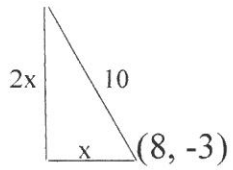
2. The slope of $\overline{AB} = \frac{8-1}{9-4}=\frac{7}{5}$. The slope of $\overline{BC}$ is $-\frac{5}{7}$, thus taking the form $y = -\frac{5}{7}x$ or

5x + 7y = c. Plugging in B(9,8): 5(9) + 7(8) = 45 + 56 = 101, thus line BC is 5x + 7y = 101.

Plugging in the point C(a, -2): 5a + -14 = 101 → 5a = 115, so a = 23. **Ans. 23**

3. The slope of the base is: $\frac{-4+2}{6-10}=\frac{1}{2}$. The midpoint of the base is: $\left(\frac{10+6}{2},\frac{-2-4}{2}\right)=(8,-3)$.

So the slope of the perpendicular bisector of the base is -2. Using the right triangle at right to find the coordinates of the vertex angle: $(2x)^2 + x^2 = 10^2$. $5x^2 = 100$, so $x = 2\sqrt{5}$. The x coordinate is $8 - 2\sqrt{5}$ and the y coordinate is $-3 + 4\sqrt{5}$. Thus $(8 - 2\sqrt{5}, -3 + 4\sqrt{5})$.



Ans. $(8 - 2\sqrt{5}, -3 + 4\sqrt{5})$

Functions

1. $\frac{f(7)-g(5)+h(6)-7}{f(5)-g(4)+h(3)-6}=\frac{32-26+37-7}{22-20+16-6}=\frac{69-33}{38-26}=\frac{36}{12}=3.$ **Ans. 3**

2. $f(x) + 3 = 4 - x^2$, thus $x \neq 2$ or -2. $k(x)=\frac{x}{x-1}$, $x \neq 1$ or 0. $G(x)=\frac{1}{x+1}$, thus $x \neq -1$.

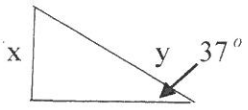
Ans. All reals except -2, -1, 0, 1, 2

3. $f(3) = 2f(2) - 3 = 2(1) - 3 = -1$. $f(4) = 2f(3) - 3 = 2(-1) - 3 = -5$. $f(5) = 2f(4) - 3 = 2(-5) - 3 = -13$, $f(5) = 2(-13) - 3 = -29$. $f(2) - f(3) = 1 - (-1) = 2$. $f(3) - f(4) = -1 - (-5) = 4$. $f(4) - f(5) = -5 - (-13) = 8$. Notice the pattern:

$f(2) - f(3) = 2^1$, $f(3) - f(4) = 2^2$, $f(4) - f(5) = 2^3$, so $f(11) - f(12) = 2^{10} = 1024$. **Ans. 1024**

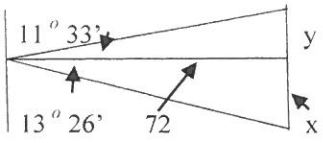
Trig Mechanics

1. We need x + y in the figure. $\tan 37^\circ = \frac{x}{26}$, so $x = 26 \tan 37^\circ = 19.592$



$\cos 37^\circ = \frac{26}{y}$, so $y = 26/\cos 37^\circ = 32.556$. $x + y = 52.148$. Rounded: 52.1 **Ans. 52.1**

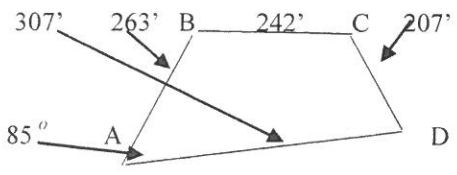
2. In the figure at right, $\tan 11^\circ 33' = \frac{y}{72}$, and $\tan 13^\circ 26' = \frac{x}{72}$.



$x + y = 72(\tan 11^\circ 33' + \tan 13^\circ 26') = 31.91$.

3. Insert $\overline{BD}$. Using Cosine Law: $(BD) = \sqrt{263^2 + 307^2 - 2(263)(307)\cos 85} = 386.450$.

$m\angle ABC = m\angle ABD + m\angle CBD$. $\frac{\sin \angle ABD}{307}=\frac{\sin 85}{386.45}$ ➔



$m\angle ABD = 52^\circ 18' 55''$. $\cos \angle CBD = \frac{207^2 - 242^2 - BD^2}{-2(242)BD}$.

$m\angle CBD = 28^\circ 03' 30''$. $m\angle ABC = 80^\circ 22' 25''$. **Ans. $80^\circ 22'$**

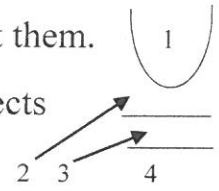
Team

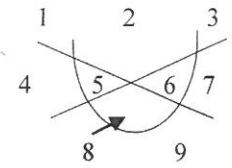
1. Rotating through 90° counterclockwise puts the 3rd quadrant into the first. **Ans. (1, 2)**

2. The hypotenuse 10 can be used to find the answer, but since $\sin A = \frac{\sqrt{3}}{3}$ which was $\frac{1}{\sqrt{3}}$ before expressed properly is all that is needed. The opposite is 1, hypotenuse is $\sqrt{3}$, and adjacent is $\sqrt{2}$. So $\tan C = \sqrt{2}$. **Ans.** $\sqrt{2}$

3. $\frac{1}{2}F = \frac{5}{9}(F - 32)$ ➔ $9F = 10F - 320$ ➔ $F = 320$. **Ans. 320**

4. If the two lines are parallel and the parabola does not intersect them. There are 4 regions. If the lines intersect and the parabola intersects the lines, there are 9 regions.





Ans. 13

5. First we'll find the slope of the line through (16, 40) and (80, 100): $\frac{100-40}{80-16} = \frac{60}{64} = \frac{15}{16}$

The linear function: $f(x) = \frac{15}{16}x + b$ ➔ $40 = \frac{15}{16}(16) + b$, $b = 25$. So $f(x) = \frac{15}{16}x + 25$.

$f(64) = \frac{15}{16}(64) + 25 = 60 + 25 = 85$. **Ans. 85**

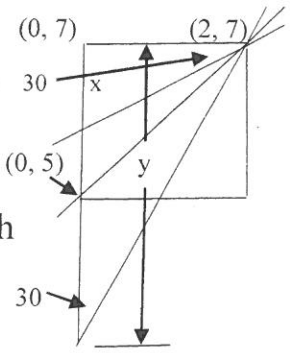
6. $(m^e)^a \cdot (n^a)^e = mn^e$ ➔ $(mn)^{ae} = mn^e$ ➔ ae log mn = log m + e log n ➔

ae log mn – e log n = log m ➔ e(a log mn – log n) = log m ➔ **Ans.** $\frac{\log m}{a \log mn - \log n}$

7. We need to find the center. The perpendicular bisectors of two segments will meet at the center. For $\overline{AB}$: slope: $\frac{-7-5}{1-7} = \frac{-12}{-6} = 2$, mdpt = (4, -1), $y = -\frac{1}{2}x$ ➔ (1) x + 2y = 2.

For $\overline{BC}$: slope: $\frac{-1+7}{9-1} = \frac{6}{8} = \frac{3}{4}$, mdpt = (5, -4), $y = -\frac{4}{3}x$ ➔ (2) 4x + 3y = 8. (2) – 4(1) = -5y = 0, so y = 0 and in (1) x +2(0) = 2, so x = 2. The distance from (2, 0) to (7, 5) = **Ans.** $5\sqrt{2}$

8. From the y-intercept (0, 5) to the point of the line (2, 7) is shown at left. The line makes a 45° angle, so the rotation one way makes a line with a 60° angle and the other with a 30° angle. The 60° angle line makes a 30° angle with the horizontal to the point (0, 7). Tan 30° = x/2, so x = 1.154700,which means that the y-i of the line with a 60° angle is 7 – x = 5.845299. The 30° line makes an angle of 60° with the same segment of length 2.



Tan 60° = y/2, so y = 3.464101 and the y –i = 7 – y = 3.535898. Adding the two: **Ans. 9.3812**

9. Work backwards: 480 ==> 480x ==> 240x^2 ==> 80x^3 ==> 20x^4 ==>4x^5, so a = 4 and n = 5. The sum is 9.

Ans. 9

Answer Sheet – 2015-2016 Feb Meet

Arithmetic with Literal Equations

1. $\frac{R_1 T_2}{T_2 - R_2}$
2. 13
3. 28.7

Logs and Log Equations

1. 11/12
2. 5
3. 1/1000

Linear Coordinate Geometry

1. $-9\frac{4}{5}$ or -49/5 or $(0, -9\frac{4}{5})$ or (0, -49/5)
2. 23
3. $(8 - 2\sqrt{5}, -3 + 4\sqrt{5})$

Functions

1. 3
2. All reals except -2, -1, 0, 1, 2
3. 1024

Trigonometric Mechanics

1. 52.1
2. 31.91
3. 80°22'

Team

1. (1, 2)
2. $\sqrt{2}$
3. 320
4. 13 or 12
5. 85
6. $\frac{\log m}{a \log mn - \log n}$
7. $5\sqrt{2}$
8. 9.3812
9. 9