## 1. Arithmetic with Literal Equations

February 2010 (No calculators)

1. If $a x=\frac{a^{2} b x+a b p}{a x+p}$, where $a>0$ and $a x \neq-p$, find $x$ in simplest form.

Ans. $\qquad$
2. A grocery store had a sale on eggs, selling thirteen eggs for the usual price of a dozen eggs. As a result, the price of the eggs was reduced by 4 cents a dozen. What was the original price for a dozen eggs?

Ans. $\qquad$
3. If $x=\frac{1}{4-y}$, compute $\frac{1}{x}+4 x+y-y x-1$.

## 1. Arithmetic with Literal Equations

February 2011 (No calculators)

1. Find the sum of the natural number divisors of 108 .

Ans. $\qquad$

Ans. $\qquad$
2. If $A$ and $B$ are negative integers, $\frac{A}{B}(D)=2 C, A C=B(D+4)$ and $D=-8$, find C.

Ans. $\qquad$
3. If $a=b$, find all values for $a, b, c$, and $d$ that make the following true. $\frac{a(c+5)}{d-1}=a-b$

Ans. $\qquad$

## 1. Arithmetic with Literal Equations

January 2012 (No calculators)

1. Solve for a , assuming $\mathrm{b} \neq-1 / 2: 2 \mathrm{x}=2 \mathrm{a}+4 \mathrm{ab}-2$.

Ans. $\qquad$
2. Given that $\mathrm{D}=\mathrm{rt}$ and $\mathrm{r}=2 \mathrm{x}^{2}-1$, find x if $\mathrm{D}=7$ and $\mathrm{t}=18$. Express your answer as a fraction.
3. Solve $a=\frac{1}{a}-e p^{D}$ for $D$.

Ans. $\qquad$

Ans. $\qquad$

## 1. Arithmetic with Literal Equations

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1. Solve $\mathrm{L}=\mathrm{a}+(n-1) \mathrm{d}$ for $n$. Express your answer as a single fraction in simplest form.

Ans. $\qquad$
2. P is increased by $20 \%$. The new value of P is then decreased by $30 \%$. What percent of P is the final result?

Ans. $\qquad$
3. If $\frac{a+2}{m-1}+\frac{b-2}{m+1}=\frac{a-2 b+8}{m^{2}-1}$, where $\mathrm{m} \neq 1$ or -1 , solve for $m$. Express your answer as a single fraction in simplest form.

Ans. $\qquad$

## 1. Arithmetic with Literal Equations

1. Solve the following for $H: \quad \mathrm{S}=2 \mathrm{LW}+2 \mathrm{~L} H+2 \mathrm{~W} H$.

Ans.
2. Solve the following for $\mathrm{c}: \quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

Ans.
3. Consider an interesting five-digit number $A$. If 1 is placed at the end of $A$ to make a sixdigit number, it is 3 times the number made by placing 1 in front of A to make a six-digit number. Find A.

Ans.

1. Find $c$, if $\log _{c} 100=10$. Express answer in radical form.

Ans.
2. Solve the following: $\log _{2}(2 x+3)-\log _{2}(3 x+2)=1$

Ans. $\qquad$
3. Find all values of x such that $2 \log _{5}(12+x)=\log _{5} 3+\log _{5}(15-4 x)$

Ans.

## 2. Logs and Log Equations

February 2011 (No calculators)

1. If $\log a=.21$ and $\log b=.09$, find $\log a^{2} b$.

Ans. $\qquad$
2. $\ln a=3$ and $\ln b=2$. Find $\ln \left(\frac{a^{2}}{\sqrt{b e}}\right)$

Ans.
3. If $\log \sqrt{2 x-1}+\log \sqrt{x-9}=1$, solve for $x$.

Ans.

1. Find the value of the $k$, if $\log _{2} 8=k \log _{8} 2$.

Ans.
2. Find the value of $\left(\log _{7} 8\right)\left(\log _{6} 7\right)\left(\log _{5} 6\right)\left(\log _{4} 5\right)$.

Ans. $\qquad$
3. Solve for x if: $8\left(\log _{4} x\right)^{2}+5=7 \log _{4} x^{2}$

## 2. Logs and Log Equations

January 2013 (No calculators)

1. Find $x$, if $\log _{x} 128=7 / 3$.

Ans. $\qquad$
2. If $\log _{4} P=\frac{2}{3} \log _{4} 27-\frac{3}{2} \log _{4} 25+4 \log _{4} \sqrt{15}-6 \log _{4} \sqrt{6}$, find P in simplest form.

Ans. $\qquad$
3. Find all values of $x$ such that

$$
\log _{5}\left(2 x^{2}-5\right)+\log _{5}(x-1)=\log _{5}\left(13 x^{2}-42 x+35\right)
$$

Ans.

1. Find the sum: $\log _{3} 27+\log _{9} 27+\log _{27} 27+\log _{81} 27+\log _{243} 27$.

Ans. $\qquad$
2. Simplify completely: $8^{\log _{2} 5}$

Ans.
3. Solve for $\mathrm{x}: \quad 3 \log _{8}(9 x+5)-2 \log _{4}\left(x^{2}-1\right)=2$.

Ans.

## 3. Linear Coordinate Geometry

1. $(3,6),(-2,3)$ and $(13, k)$ are collinear. Find $k$.

Ans. $\qquad$
2. Line segment $A B$ has endpoints $A(3,-1)$ and $B(13,-5)$. Find the point of intersection of the line $3 x-4 y=8$ and the perpendicular bisector of segment $A B$.

Ans. $\qquad$
3. Triangle ABC has vertices $\mathrm{A}(1,2), \mathrm{B}(7,8)$ and $\mathrm{C}(11,-6)$. find the coordinates of the centroid, which is the point of concurrency of the medians of the triangle.

Ans. $\qquad$

## 3. Linear Coordinate Geometry

1. Find the equation of the line in standard form $(\mathrm{Ax}+\mathrm{By}=\mathrm{C}$, where $\mathrm{A}, \mathrm{B}$, and C are relatively prime integers and $A>0$ ) passing through $(-2,5)$ and parallel to the line $3 y+$ $2 x=5$.

Ans. $\qquad$
2. Find the equation in standard form in terms of $a$ and $b$ for the line which is the perpendicular bisector of the line segment joining $(2 a, 0)$ and $(0,2 b)$.

Ans. $\qquad$
3. Determine $k$ so that the line with equation $3 x-4 y=k$ is $6 / 5$ units away from the point $(2,3)$.

Ans.

## 3. Linear Coordinate Geometry

1. Find the value of the slope of the line: $2 x-y=3 y-x+2$.

Ans.
2. $\frac{2}{3} x+\frac{1}{4} y=\frac{1}{2}$ is reflected across the main diagonal $\mathrm{x}=\mathrm{y}$ to form a line with equation Ax $+\mathrm{By}=\mathrm{C}$ where $\mathrm{A}, \mathrm{B}$, and C are relatively prime integers and $\mathrm{A}>0$. Find the sum $\mathrm{A}+\mathrm{B}$ +C .

Ans. $\qquad$
3. Find the equations of the lines A and B in slope-intercept form such that:

- the sum of the values of their slopes is 4 .
- the y-intercept of line A subtracted from the y-intercept of line B equals 2.
- line A intersects line B at $(-1,3)$.

Ans. $\qquad$

## 3. Linear Coordinate Geometry

January 2013 (No calculators)

1. Two lines pass through the point $(-4,-7)$ in the $x y$-coordinate plane. One line has slope $m_{1}$ and a $y$-intercept at $(0,9)$. The other line has slope $m_{2}$ and a $y$-intercept at $(0,10)$. Find the value of $m_{2}-m_{1}$.

Ans. $\qquad$
2. Line segment $L$ has endpoints at $(a, 2 a)$ and $(5 a, 4 a)$ for some $a \neq 0$. In terms of $a$, find the point at which the perpendicular bisector of $L$ has its y-intercept.

Ans. $\qquad$
3. The coordinates of $\triangle \mathrm{ABC}$ are $\mathrm{A}(6,11), \mathrm{B}(-2,1)$ and $\mathrm{C}(10,-1) . \overline{A D}$ is the median from A to side BC . Point E on line AD has an x -coordinate of 8 . Find the x -intercept of the line through E perpendicular to line AD.

Ans.

1. Find the $y$-intercept in $(x, y)$ form of the line passing through $(-3,6)$ and $(5,10)$.

Ans.
2. Find the next highest point $(x, y)$ on the line $5 x-12 y=-6$ which is beyond the point $(6,3)$ that also has integral values for both $x$ and $y$.

Ans.
3. Line $m$ passing through the point $(-3,15)$ is perpendicular to the line $p$, whose equation is $3 \mathrm{x}-4 \mathrm{y}=-27$. How far is the y -intercept of m from line $p$ ?

Ans.

## 4. Functions

1. If $f(x+1)=2 x-3$, where $f$ is a linear function, find $f(x+3)$.

Ans. $\qquad$
2. $f(x)=\frac{x-2}{x+3}-\frac{x+3}{x-2}$, find the domain of f .

Ans. $\qquad$
3. The number $r$ is said to be a fixed point of the function of $f$, if $f(r)=r$. Find all ordered pairs $(\mathrm{a}, \mathrm{b})$ for which the function $f(x)=x^{2}+a x+b$ has exactly one fixed point.

Ans. $\qquad$

## 4. Functions

February 2011 (No calculators)

1. Suppose $f(x)=\frac{x}{2}$ and $g(x)=x-3$. Find $f(g(-5))$.

Ans. $\qquad$
2. A function $f(n)$ defined for all positive integers has the property that $f(m)+f(n)=f(m n)$ for any two positive integers $m$ and $n$. If $f(2)=7$ and $f(3)=10$, then calculate $f(12)$.

Ans. $\qquad$
3. Let $\mathrm{f}(\mathrm{x})$ be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=x$ for all x not equal to 0 or 1 . Find the exact value of $f(2)$.

Ans. $\qquad$

## 4. Functions

January 2012 (No calculators)

1. If $f(x)=x^{2}-6 x+2$, find $f(x+2)$.

Ans. $\qquad$
2. If $f(2 x-3)=6 x-14$, find $f(3 x+2)$.

Ans. $\qquad$
3. Function $f(t)$ is defined recursively as follows:
$4^{f(t)}=f(t-1)$ and $f(0)=4^{\left(2^{512}\right)}$.
If the range of $f(t)$ is limited to real numbers, how many whole numbers are in the domain of $f(t)$ ?

Ans. $\qquad$

## 4. Functions

January 2013 (No calculators)

1. $f(x)=x^{2}-3 x+7$. If $x>0$ and $f(x)=11$, find $x$.

Ans. $\qquad$
2. When Alice gets a number, she doubles it and tells the result to Bill. Bill adds 7 and tells the result to Carol. Carol subtracts what she gets from 100 and tells the result to Don. Don adds 65 and says the result. Call these functions $\mathrm{A}(\mathrm{x}), \mathrm{B}(\mathrm{x}), \mathrm{C}(\mathrm{x}), \mathrm{D}(\mathrm{x})$, respectively and let $\mathrm{E}(\mathrm{x})=\mathrm{D}(\mathrm{C}(\mathrm{B}(\mathrm{A}(\mathrm{x}))))$. Find $\mathrm{E}^{-1}(52)$.

Ans.
3. Let $f$ be a real-valued function such that $f(x)+2 f(2002 / x)=3 x$. Find $f(2)$ in simplest form.

Ans.

1. If $f(x)=x^{2}-5 x-8$, find all values of $x$ such that $f(x)=6$.

Ans.
2. If $f(x)=\frac{x-3}{x+2}$, find $\mathrm{f}^{-1}(2)$.

Ans.
3. $f(x)=\frac{2 x-3}{3 x+2} . g(x)=\frac{5 x+2}{4 x-3}$. Find the domain of $f \circ g(x)$.

Ans.

## 5. Trig Mechanics

February 2010 (You may use a calculator)

1. The lengths of the sides of an isosceles triangle are 23, 23, and 36. To the nearest degree find the measure of a base angle.

Ans.
2. Find the measure of angle P. Express answer to the nearest minute.

3. From a lighthouse on a small island in the ocean, an observer sighted a buoy at an angle of depression of $11^{\circ} 38^{\prime}$, due north. He then turned due east and sighted a second buoy at an angle of depression of $7^{\circ} 24^{\prime}$. If the sighting takes place 204 feet above sea level, how far apart are the two buoys? Give answer to nearest foot.

Ans. $\qquad$

1. A rhombus has sides of 25 units long and its height is 24 units. Find the measure of its obtuse angle to the nearest hundredth of a degree.

Ans.
2. The side of the beach shelter shown below is made of four $30^{\circ}-60^{\circ}-90^{\circ}$ triangles. Find the dimensions marked $x$. Give exact answer or give answer rounded to the nearest thousandth. The side AB is 2 meters long.


Ans. $\qquad$
3. A vertical pole 30 meters tall standing on a $10^{\circ}$ slope is braced by two cables extending from the top of the pole to two points on the ground, one point 25 meters directly up the slope and the other point 25 meters directly down the slope. To the nearest hundredth find the sum of the lengths of the two cables.

Ans.

1. In right triangle $\mathrm{ABC}, \sin \mathrm{A}=\frac{7}{20}$. Find the cosine of angle A in simplest radical form.

Ans. $\qquad$
2. The sides of a triangle are 9,13 , and 19 . What is the sum of the two smallest angles? Give answer to nearest minute.

Ans. $\qquad$
3. In triangle $\mathrm{ABC}, \mathrm{AC}=20, \mathrm{AB}=17$, and $\mathrm{m} \angle \mathrm{C}=30^{\circ}$. There are two possible triangles which exist under these conditions. Find the difference between the area of the larger triangle and the area of the smaller triangle. Round answer to nearest $100^{\text {th }}$.

Ans. $\qquad$

1. Find the perimeter of triangle ABC . Round to the nearest unit.


Ans. $\qquad$
2. At time $t=0$, ship $A$ is 40 nm west of ship $B$ and is traveling due east at 10 knots. Ship $B$ is heading due north at 8 knots. Find the value of $t$ in hours when ship $B$ will be 50 degrees north of east as measured from ship A. Round answer to nearest $100^{\text {th }}$ of an hour.

Ans. $\qquad$
3. The ground in a field slopes upward $3^{\circ}$ over the horizontal in the direction from a kite string holder to her kite. The kite is flying on 400 feet of straight string and its shadow is 327 ft from the holder (measured along the ground) when the sun is directly vertical. Because the wind is so strong, she is standing on the spool of string so the kite won't blow away. Measured perpendicular to the ground, how high is the kite in feet? Round answer to nearest tenth of a foot.

Ans.

1. A water balloon is dropped from point C and hit the ground at point $\mathrm{B}, 59 \mathrm{ft}$ from point A. If A is 74 ft from C , find the measure of angle ACB to the nearest $10^{\text {th }}$ of a degree. (Assume the ground is flat.)

Ans. $\qquad$
2. In the figure at right, find the length of CD to the nearest $100^{\text {th }}$. Angles ABC and BCD are right angles, $\mathrm{AC}=8.62, \mathrm{~m} \angle \mathrm{~A}=$ $37^{\circ} 15^{\prime}$ and $\mathrm{m} \angle \mathrm{D}=41^{\circ} 45^{\prime}$.


Ans. $\qquad$
3. A hot-air balloon is 2400 ft directly above Interstate 80 in Nebraska which extends for miles in a straight line. The angle of depression to a truck on Interstate 80 is $16^{\circ} 42^{\prime}$. Directly ahead of the truck at an angle of depression of $7^{\circ} 24^{\prime}$ is a tall statue on the side of the road of I 80 . To the nearest 10 ft ., how far from the statue is the truck?

Ans. $\qquad$

