## 1 Algebraic Fractions and Fractional Equations Mar 2013 (No Calculators)

**3 pts 1.** Find, in simplest form, as a single fraction with no parentheses, for x > 0:

$$\frac{x^4+2x^3+x^2}{4} \div \frac{2x^3+3x^2+x}{6}$$

**Ans.** ____________________

**4 pts 2.** Simplify and write as a binomial without parentheses:

$$\frac{8x^4-24x^3-32x^2+96x}{8x-\frac{32}{x}}$$

**Ans.** ____________________

**5pts 3.** Find all values of x such that $\frac{x-2}{2x-3}+\frac{2x-7}{1-x}=\frac{x^2-3x-3}{2x^2-5x+3}$.

**Ans.** ____________________

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## 2 Trigonometric Equations and Identities Mar 2013 (No Calculators)

**3 pts 1.** If $0^\circ \le \theta < 360^\circ$, find all values of $\theta$ satisfying:

$$\frac{\sin\theta}{5}-\frac{\sin\theta}{3}=\frac{\sqrt{3}}{15}$$

**Ans.** ____________________

**4 pts 2.** Let $A = \sec x$. Find both values of $A$ for which $\cot x = 2$.

**Ans.** ____________________

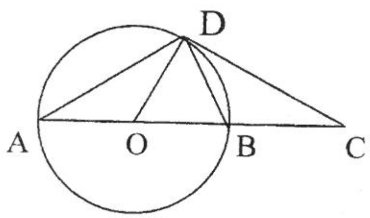
**5 pts 3.** On the domain $0 \le x < \pi$, find all values of $x$ satisfying

$$4\sin^2 x + 6\cos x = 8\sin^2 x\cos x + 3$$

**Ans.** ____________________

 **Circles and Spheres Mar 2013 (No Calculators)**

**3 pts 1.** Points A, B, and D lie on the circle O. O is the center. Line segment AC passes through points O and B. Line segment CD is tangent to O at point D. Line segment BD is chord of circle O. If angle DCB measures $34^\circ$, find the measure of angle OBD.

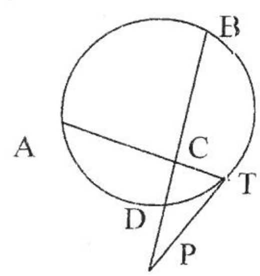



**Ans.** ______________________

**4 pts 2.** Two balls are rolled the length of a 10 foot table. The larger ball makes 10 revolutions and the smaller makes 12 revolutions. How many inches greater is the radius of the larger ball than the radius of the smaller ball?

**Ans.** ______________

**5 pts 3.** In the drawing, $\overline{PT}$ is a tangent with length 6. $\overline{PB}$ is a secant passing through point D on the circle. $\overline{AT}$ is a chord intersecting with $\overline{PB}$ at C. If PD = 3, DC = 4, and CT = $\sqrt{13}$, find the distance from $\overline{BD}$ to the center of the circle.




**Ans.** ______________________

## 4 Conics Mar 2013 (No Calculators)

**3 pts 1.** Find the distance from the origin, (0, 0), to the center of the circle whose equation is

$$x^2 + 8x + y^2 - 6y + 21 = 0.$$

**Ans.** ____________________

**4 pts 2.** An ellipse with major axis parallel to the x-axis is inscribed in a rectangle with vertices (0, 15), (0, 17), (8, 17) and (8, 15). Find *d* if the equation of the ellipse is written in the form $x^2 + ay^2 + bx + cy = d$.

**Ans.** ____________________

**5 pts 3.** The vertices of a hyperbola are (-7, 5) and (1, 5). The slopes of the asymptotes are $\pm\frac{1}{2}$. Find the equation of the hyperbola.

**Ans.** ____________________

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## 5 Arithmetic with Statistics Mar 2013 (You may use calculators)

**3 pts 1.** A data set of three samples of positive integers has a sum of 93. One of the samples is the mean of the data set and the range is 8. Find the largest sample of the set.

**Ans.** ____________________

**4 pts 2.** The following chart shows a student's quiz grades on all math quizzes so far through his high school career. He has two quizzes left to take.

| Grade | 70 | 75 | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|---|---|
| Number | 2 | 5 | 8 | 11 | 19 | 16 | 18 |

What must he average on the last two quizzes in order to finish high school with exactly a 90 math quiz average?

**Ans.** ____________________

**5 pts 3.** For a data set, define the *MS Index* ("Makes Sense") as $\frac{|Median - Mean|}{Range}$. Find the value of the smallest *MS Index* among the following data sets:

{1, 2, 9} {1, 2, 3, 12} {1, 17, 24} {1, 19, 19, 21} {1, 2, 14, 15, 23}

**Ans.** ____________________

## 6 Team Mar 2013 (You may use calculators)

**3 pts 1.** Simplify the following: $\frac{x^4 - y^4}{x^3 + y^3} \cdot \frac{x + y}{x^2 - y^2}$. (1) **Ans.** ____________ **3 pts**

**3 pts 2.** Simplify: $\frac{\csc\theta - \sin\theta}{\cot^2\theta}$. (2) **Ans.** ____________ **3 pts**

**3 pts 3.** There are four ten-inch diameter basketballs placed very tightly in the bottom of a square box. How far is it from the very top of one of the basketballs to the center of the ball diagonally across from it?

(3) **Ans.** ____________ **3 pts**

**4 pts 4.** In how many different combinations of 3-point shots, 2-point shots, and 1-point free-throws can a team score 10 points?

(4) **Ans.** ____________ **4 pts**

**4 pts 5.** While doing his trig homework, Elmer thought to himself: "This stuff's too easy. I can't see why I shouldn't invent a new trig function to add some fun. I herby define the *Elmersine* as the ratio between the opposite side and the altitude to the hypotenuse for an acute angle in a right triangle." By what more common name do we know the *Elmersine*?

(5) **Ans.** ____________ **4 pts**

**4 pts 6.** A Ferris wheel has a radius of 8 meters and rotates 12 degrees/second. At time t = 0, a seat is at its lowest point, which is 2 meters above the ground. Determine how high above the ground the seat is at t = 40 seconds.

(6) **Ans.** ____________ **4 pts**

**5 pts 7.** Find all the ordered triples $(a, b, c)$ of positive integers which satisfy the following equations: $ab + bc = 44$ and $ac + bc = 23$.

(7) **Ans.** ____________ **5 pts**

**5 pts 8.** Let the focus F be (0, 3) and the directrix have the equation y = 12. Find the equation of the set of points whose ratio of distance from the focus F and the distance from the directrix y = 12 is 2 to 1.

(8) **Ans.** ____________ **5 pts**

**5 pts 9.** A parabola is formed of the points equidistant from the line y = x – 10 and the point (-2, 0). The parabola crosses the y axis at points A and B, Find the distance between A and B.

(9) **Ans.** ____________ **5 pts**

## Solutions - Algebraic Fractions and Factoring

(1) $\frac{x^4+2x^3+x^2}{4} \div \frac{2x^3+3x^2+x}{6} = \frac{x^2(x+1)^2}{4} \cdot \frac{6}{x(2x+1)(x+1)} = \frac{3x(x+1)}{2(2x+1)}$. **Ans.** $\frac{3x^2+3x}{4x+2}$

(2) $\frac{8x^4-24x^3-32x^2+96x}{8x-\frac{32}{x}} = \frac{8x(x^3-3x^2-4x+12)}{8\left(\frac{x^2-4}{x}\right)} = \frac{x^2(x^2[x-3]-4[x-3])}{x^2-4} = x^2(x-3) =$

$x^3 - 3x^2$, without parentheses. **Ans. $x^3 - 3x^2$**

(3) $\frac{x-2}{2x-3} + \frac{2x-7}{1-x} = \frac{x^2-3x-3}{2x^2-5x+3}$ ➔ $\frac{x-2}{2x-3} - \frac{2x-7}{x-1} = \frac{x^2-3x-3}{2x^2-5x+3}$ ➔

$(x-2)(x-1) - (2x-7)(2x-3) = x^2 - 3x - 3$ ➔ $x^2 - 3x + 2 - 4x^2 + 20x - 21 = x^2 - 3x - 3$

$0 = 4x^2 - 20x + 16$ ➔ $0 = x^2 - 5x + 4 = (x-4)(x-1)$. x = 4 or 1, but x can't be 1. **Ans. 4**

## Trigonometric Equations and Identities

(1) $\frac{\sin\theta}{5} - \frac{\sin\theta}{3} = \frac{\sqrt{3}}{15}$ ➔ $3\sin\theta - 5\sin\theta = \sqrt{3}$ ➔ $\sin\theta = -\frac{\sqrt{3}}{2}$. **Ans. 240° or 300°**

(2) $\cot x = 2 = \frac{2}{1}$, where 2 is the adjacent side and 1 is the opposite side of a right triangle.

Therefore the hypotenuse is $\sqrt{5}$. $\sec x = \frac{\sqrt{5}}{2}$. Since there are no restrictions on x, then x

could be a first or a third quadrant angle, thus $\sec x = \pm\frac{\sqrt{5}}{2}$. **Ans.** $\pm\frac{\sqrt{5}}{2}$

(3) $4\sin^2 x + 6\cos x = 8\sin^2 x\cos x + 3$ ➔ $8\sin^2 x\cos x - 4\sin^2 x - 6\cos x + 3 = 0$

$4\sin^2 x(2\cos x - 1) - 3(2\cos x - 1) = 0$ ➔ $(4\sin^2 x - 3)(2\cos x - 1) = 0$. Therefore

$\sin x = \pm\frac{\sqrt{3}}{2}$ or $\cos x = 1/2$. $x = \pi/3$ or $2\pi/3$. **Ans. $\pi/3$ or $2\pi/3$**

## Circles and Spheres

(1) There are several ways to find the measure of angle OBD. Since $\overline{CD}$ is tangent at D. Then angle ODC is a right angle. Thus m∠COD = 56. From here, (1) since OD = OB, then m∠OBD = ½(180 - 56) = 62, (2) since m∠AOD = 124, then arc AD = 124 and inscribed angle ABD = 62, (3) since ∠ADB is a right angle because it is inscribed in a semicircle and AO = OD, then m∠DAO = 28, and its compliment ABD = 62. **Ans. 62°**

(2) 10 feet = 120 inches. Since $C = 2\pi r$, then Large ball: $C = \frac{120in}{10} = 12$ in. for the Small ball: $C = \frac{120in}{12} = 10$ in. For large ball: $r = \frac{12}{2\pi} = \frac{6}{\pi}$. For small: $r = \frac{10}{2\pi} = \frac{5}{\pi}$.
The difference is $1/\pi$ inches. **Ans. $1/\pi$**

(3) Since $6^2 + (\sqrt{13})^2 = 7^2$, then $\Delta$TCP is a right triangle, $\angle$CTP is a right angle and thus AT is a diameter. Applying thr Power of a Point Theorem twice:
(1) $3(7 + BC) = 6^2$, thus BC = 5. (2) $4(5) = \sqrt{13}(AC)$, thus $AC = 20/\sqrt{13}$. The length of the diameter is: $20/\sqrt{13} + \sqrt{13} = \frac{33\sqrt{13}}{13}$. The radius is $\frac{33\sqrt{13}}{26}$. Let x = the distance from $\overline{BD}$ to the center. Then $x^2 + \left(\frac{9}{2}\right)^2 = \left(\frac{33\sqrt{13}}{26}\right)^2$. $x^2 = \left(\frac{33^2(13)}{2^2\cdot 13^2}\right) - \frac{81}{4} = \left(\frac{9\cdot 11^2\cdot 13 - 81\cdot 13^2}{2^2\cdot 13^2}\right) =$

$\left(\frac{9(121-9\cdot 13)}{4\cdot 13}\right) = \frac{9\cdot 4}{4\cdot 13}$. So $x = \frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$.

**Ans.** $\frac{3\sqrt{13}}{13}$

**Conics**

(1) The center of the circle is (-4, 3). It is 5 units from the origin.

**Ans. 5**

(2) The center of the ellipse is (4, 16). The semi-major axis is 4 and the semi-minor is 1.
Equation: $\frac{(x-4)^2}{16} + \frac{(y-16)^2}{1} = 1 \Rightarrow x^2 - 8x + 16 + 16y^2 - 512y + 4096 = 16 \Rightarrow$
$x^2 + y^2 - 8x - 512y = -4096$.

**Ans. -4096**

(3) Since vertices are (-7, 5) and (1, 5), then center is at (-3, 5) and semi transverse axis is 4. Slopes of asymptotes are ½, so $\frac{1}{2} = \frac{b}{4}$, thus b = 2(the semi-conjugate axis).

**Ans.** $\frac{(x+3)^2}{16} - \frac{(y-5)^2}{4} = 1$

**Arithmetic with Statistics**

(1) Since mean (31) is a sample and range is 8, let x = largest sample. x - 8 + x = 93 – 31
2x = 70, x = 35.

**Ans. 35**

(2) 90(81) = 2a + 2(70) + 5(75) + 8(80) + 11(85) + 19(90) + 16(95) + 18(100)
7290 = 2a + 7120. 2a = 170, so a = 85.

**Ans. 85**

(3)

| | {1, 2, 9} | {1, 2, 3, 12} | {1, 17, 24} | {1, 19, 19, 21} | {1, 2, 14, 15, 23} |
|---|---|---|---|---|---|
| Median | 2 | 2.5 | 17 | 19 | 14 |
| Mean | 4 | 4.5 | 14 | 15 | 11 |
| Range | 8 | 11 | 23 | 20 | 22 |
| MS Index | 1/4 | 2/11 | 3/23 | 1/5 | 3/22 |

**Ans. 3/23**

**Team**

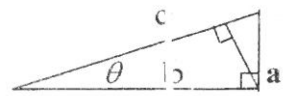
(1) $\frac{x^4 - y^4}{x^3 + y^3}\cdot\frac{x+y}{x^2 - y^2} = \frac{(x^2 - y^2)(x^2 + y^2)}{(x+y)(x^2 - xy + y^2)}\cdot\frac{x+y}{x^2 - y^2} = \frac{x^2 + y^2}{x^2 - xy + y^2}$. **Ans.** $\frac{x^2 + y^2}{x^2 - xy + y^2}$

(2) $\dfrac{\csc\theta-\sin\theta}{\cot^2\theta} = \dfrac{\dfrac{1}{\sin\theta}-\dfrac{\sin^2\theta}{\sin\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta}} = \dfrac{\cos^2\theta}{\sin\theta}\cdot\dfrac{\sin^2\theta}{\cos^2\theta} = \sin\theta.$ **Ans. sin $\theta$**

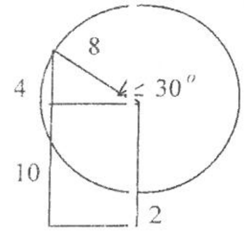
(3) The distance between the centers of adjacent basketballs is 10. The distance between diagonal basketballs is $10\sqrt{2}$. From the center to the top of a ball is 5. Using the Pyth. Thm. for these distances: $d^2 = 5^2 + (10\sqrt{2})^2 = 225$. So d = 15. **Ans. 15 in.**

| (4) | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 pointers | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 2-point shots | 0 | 2 | 1 | 0 | 3 | 2 | 1 | 0 | 5 | 4 | 3 | 2 | 1 | 0 | |
| Free-throws | 1 | 0 | 2 | 4 | 1 | 3 | 5 | 7 | 0 | 2 | 4 | 6 | 8 | 10 | **Ans. 14** |

(5) In any right triangle, the length of the altitude through area is ab/c. Elmersine $\theta = \dfrac{a}{ab/c} = a\cdot\dfrac{c}{ab} = \dfrac{c}{b}$ = secant $\theta$. **Ans. secant**




(6) The figure at right shows the height of the seat at t = 40 seconds. 40(12) = 480, so we are working with a 120 degree rotation from t = 0. The central angle made by The horizontal at the center of the wheel and the radius makes A 30 degree angle. Since the radius is 8, the side opposite the 30 degree angle is 4. Thus 14 meters high. **Ans. 14 m**




(7) (1) $ab + bc = 44$; (2) $ac + bc = 23$. In (1) b(a + c) = 44; in (2) c(a + b) = 23, here c can only be 1 (23 is too large). From (1) b(a + 1) = 44 and from (2) a + b = 23 or b = 23 – a. In (1): (23 – a)(a + 1) = 44 ➔ $23a + 23 - a^2 - a = 44$ ➔ $a^2 - 22a + 21 = 0$ ➔ (a – 21)(a – 1) = 0 ➔ a = 21, a = 1. Thus: (21, 2, 1), (1, 22, 1) **Ans. (21, 2, 1), (1, 22, 1)**

(8) Let (x, y) be the point. The distance from (x, y) to (0, 3) is $\sqrt{(x-0)^2+(y-3)^2}$. The distance from (x, y) to y = 12 is from (x, y) to the point (x, 12): $\sqrt{(x-x)^2+(12-y)^2}$. For the ratio and squaring: $4(x^2 + y^2 - 6y + 9) = 144 - 24y + y^2$ ➔ $4x^2 + 4y^2 - 24y + 36 = 144 - 24y + y^2$ ➔ $4x^2 + 3y^2 = 108$. **Ans. $4x^2 + 3y^2 = 108$**

(9) Let (x, y) be the points equidistant from (-2, 0) and the line y = x – 10. The distance between the points is: $\sqrt{(x+2)^2+(y-0)^2}$. The distance from the point to the line x – y = 10 is $\dfrac{|x-y-10|}{\sqrt{2}}$. Setting these equal to each other and squaring both sides we get: $2(x^2 + 4x + 4 + y^2) = x^2 - xy - 10x - xy + y^2 + 10y - 10x + 10y + 100$. Since we are looking for the y-intercepts, we set x = 0 and solve: $8 + 2y^2 = y^2 + 20y + 100$ ➔ $y^2 - 20y - 92 = 0$ ➔ $y^2 - 20y + 100 = 192$ or $(y - 10)^2 = 192$. The distance between the intercepts is $2\sqrt{192} = 16\sqrt{3}$. **Ans. $16\sqrt{3}$**

# Answer Sheet Mar 2013

## Algebraic Fractions

1. $\frac{3x^2+3x}{4x+2}$
2. $x^3 - 3x^2$
3. 4 or x = 4

## Trigonometric Equations and Identities

1. 240° or 300° ~~or~~ 240 or 300
2. $\pm\frac{\sqrt{5}}{2}$
3. $\pi/3$ or $2\pi/3$

## Circles and Spheres

1. 62 or 62°
2. $\frac{1}{\pi}$
3. $\frac{3\sqrt{13}}{13}$

## Conics

1. 5
2. – 4096
3. $\frac{(x+3)^2}{16}-\frac{(y-5)^2}{4}=1$

or

$x^2 - 4y^2 + 6x + 40y - 107 = 0$

## Arithmetic with Statistics

1. 35
2. 85
3. 3/23 or 0.1304

## Team

1. $\frac{x^2+y^2}{x^2-xy+y^2}$
2. $\sin\theta$
3. 15 or 15 inches
4. 14
5. secant
6. 14 or 14 meters
7. (1, 22, 1), (21, 2, 1)
8. $4x^2 + 3y^2 = 108$

   or

   $\frac{x^2}{27}+\frac{y^2}{36}=1$

   $x^2 - 3y^2 + 90y - 567 = 0$

   or

   $\frac{(y-15)^2}{36} - \frac{x^2}{108} = 1$
9. $16\sqrt{3}$ or 27.7128