1 Algebraic Fractions and Factoring Mar 2014 (No Calculators)

3 pts 1. Simplify: $\dfrac{3}{x+3}+\dfrac{2}{2x+1}-\dfrac{6x+8}{2x^2+7x+3}$ **Ans.** _____

4 pts 2. Find $7B-3A$, if $\dfrac{A}{2x-3}+\dfrac{B}{2x+3}=\dfrac{24x+6}{4x^2-9}$

Ans. _____

5 pts 3. Find all x such that $\dfrac{x-4}{x+3}-\dfrac{x-10}{x-3}=\dfrac{x-8}{2x+6}$

Ans. _____

---

2 Trigonometric Equations and Identities Mar 2014 (No Calculators)

3 pts 1. In right triangle ABC with $m\angle C = 90^\circ$, $\tan \angle B = 15/8$. Find $\sin \angle A$.

Ans. _____

4 pts 2. Find the value of $\tan 157^\circ 30'$.

Ans. _____

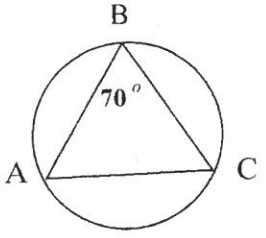
5 pts 3. Find all θ, where $0^\circ \le \theta < 360^\circ$, for which

$$\sqrt{3}\sec^2\theta - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1 = 0$$

Ans. _____

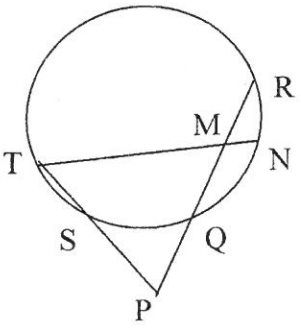
3 Circles and Spheres Mar 2014 (No Calculators)

3 pts 1. Find the measure of arc ABC in the figure.



Ans. _____

4 pts 2. In the figure at right PQ = 8, QR = 7, SP = MT, ST = 2, and MN = 1.
Find RM.



Ans. _____

5 pts 3. Point *P* is 9 cm. from a sphere whose diameter is 16 cm. Find the exact distance from *P* to the center of the circle made by the locus of points of tangency from *P* to the sphere.

Ans. _____

4 Conics Mar 2014 (No Calculators)

3 pts 1. Each of three different circles, centered at the origin, contains one of the following points (4, 7), (-3, 8) and (5, -6). What is the equation of the smallest circle?

Ans. _____

4 pts 2. Find the eccentricity of the ellipse whose equation is

$$x^2 + 16y^2 + 14x - 288y + 1329 = 0.$$

Ans. _____

5 pts 3. Find the coordinates of the foci of the hyperbola

$$16y^2 - 9x^2 + 72x + 160y + 832 = 0.$$

Ans. _____

---

5 Arithmetic with Statistics Mar 2014 (Calculators allowed)

3 pts 1. N is the least common multiple of 5, 6, and 7. Find the exact mean of the numbers one gets when dividing N by 5, then 6, then 7.

Ans. _____

4 pts 2. Convert $344_5 + 233_4 + 122_3$ to Base 6.

Ans. _____

5 pts 3. The mean, median and mode of a group of 8 positive integers are 8, 9, and 10 respectively. M is the largest of these integers. Find the sum of the largest and smallest possible values for M for all such groups.

Ans. _____

6 Team Mar 2014 (Calculators Allowed)

3 pts 1. Simplify $\dfrac{x}{x-\dfrac{1}{x-\dfrac{1}{x}}}$ (1) **Ans.** _____ **3 pts**

3 pts 2. The chord of a circle is perpendicular to the radius at its midpoint. If the radius of the circle is 12, exactly how long is the chord.

(2) **Ans.** _____ **3 pts**

3 pts 3. There are 1800 homes in Mammelville. The assessed value of them produced a mean and standard deviation of \$95,600 and \$6850. If the homes were normally distributed, how many of the homes had an assessed value ≥ \$81,900.

(3) **Ans.** _____ **3 pts**

4 pts 4. Express sec x – cos x in terms of sin in proper form.

(4) **Ans.** _____ **4 pts**

4 pts 5. The eccentricity of a hyperbola is 5/3. The vertices are (20, -3) and (-10, -3). Find its equation.

(5) **Ans.** _____ **4 pts**

4 pts 6. A circle of radius 4 is centered at the origin, and at time t_o its radius increases by 3 units every second. A second circle of radius 12 is centered at (30, 0), and its radius decreases 1 unit every second, also starting at t_o. How many seconds does it take for them to meet, and what is the ordered pair (x, y) for the point of tangency?

(6) **Ans.** _____ **4 pts**

5 pts 7. Find the exact values of x such that

$$\frac{x}{x+5}-\frac{x-2}{x+2}=\frac{x-6}{5x+1}$$

(7) **Ans.** _____ **5 pts**

5 pts 8. Find all positive angles less 180°, for which $8 \sin x \cos^5 x - 8 \sin^5 x \cos x = 1$.

(8) **Ans.** _____ **5 pts**

5 pts 9. A parabola opens upward. Three of its points are (3, -8), (4, 5) and (-2, 17). Find its exact focus.

(9) **Ans.** _____ **5 pts**

Solutions – Algebraic Fractions

1. $\dfrac{3(2x+1)}{(x+3)(2x+1)}+\dfrac{2(x+3)}{(2x+1)(x+3)}-\dfrac{(6x+8)}{(2x+1)(x+3}=\dfrac{2x+1}{(x+3)(2x+1)}=\dfrac{1}{x+3}$ **Ans.** $\dfrac{1}{x+3}$

2. A(2x + 3) + B(2x – 3) = 24x + 6 ➔ 2Ax + 3A + 2Bx – 3B = 24x + 6. Therefore 2A + 2B = 24, or (1) A + B = 12, and 3A – 3B = 6, or (2) A – B = 2. Adding (1) + (2): 2A = 14 or A = 7. In (1), (7) + B = 12, so B = 5. 7B – 3A = 7(5) – 3(7) = 14. **Ans. 14**

3. $\dfrac{x-4}{x+3}-\dfrac{x-10}{x-3}=\dfrac{x-8}{2(x+3)}$ ➔ (x - 4)(2x - 6) – (x - 10)(2x + 6) = (x - 8)(x - 3) ➔
$2(x^2 - 7x + 12) – (2x^2 - 14x – 60) = x^2 - 11x + 24$ or $0 = x^2 - 11x - 60$ ➔
$0 =(x – 15)(x + 4) = 0$. So x = 15 or -4. **Ans. 15 or – 4**

Trigonometric Equations and Identities

1. $\tan = \dfrac{opposite}{adjacent}$, so 15 is opp. ∠B and 8 is opp ∠A. Hyp = 17. $\sin A = \dfrac{8}{17}$. **Ans.** $\dfrac{8}{17}$

2. $\tan 157\tfrac{1}{2}^\circ = \tan\dfrac{315^\circ}{2} = -\sqrt{\dfrac{1-\cos 315^\circ}{1+\cos 315^\circ}} = -\sqrt{\dfrac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = -\sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}\cdot\dfrac{2-\sqrt{2}}{2-\sqrt{2}}} = -\dfrac{2-\sqrt{2}}{\sqrt{2}} = 1-\sqrt{2}$

Or $-\sqrt{\dfrac{4-4\sqrt{2}+2}{2}} = -\sqrt{3-2\sqrt{2}}$. **Ans.** $1-\sqrt{2}$ or $-\sqrt{3-2\sqrt{2}}$

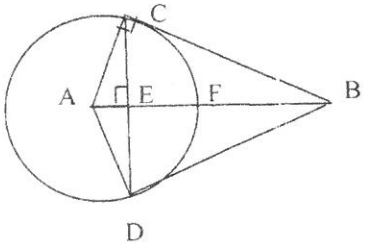
3. $\sqrt{3}\sec^2\theta - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1 = 0$ ➔ $\sqrt{3}(\tan^2\theta + 1) - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1 = 0$ ➔
$\sqrt{3}\tan^2\theta + \sqrt{3} - \tan\theta = \sqrt{3}\tan\theta + \sqrt{3} - 1$ ➔ $\sqrt{3}\tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + 1 = 0$ ➔
$\sqrt{3}\tan\theta(\tan\theta - 1) - (\tan\theta - 1) = 0$ ➔ $(\sqrt{3}\tan\theta - 1)(\tan\theta - 1) = 0$. So either $\tan\theta = \dfrac{1}{\sqrt{3}}$, in which case $\theta = 30^\circ$ or 210°; or $\tan\theta = 1$, in which case $\theta = 45^\circ$ or 225°.

Ans. 30°, 45°, 210°, 225°

Circles and Spheres

1. ∠ABC intercepts an arc of 140°. 360° – 140° = 220°. **Ans. 220°**

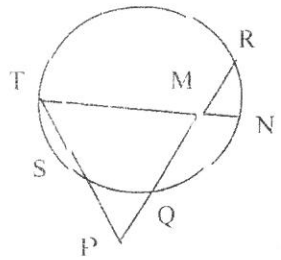
3. At right BF = 9, AF = AC = 8. So BC = 17. Since CD is The diameter of the circle, then CF is the radius. The right Triangles are all similar. So $\dfrac{AE}{8} = \dfrac{8}{17}$ ➔ $AE = \dfrac{64}{17} = 3\dfrac{13}{17}$.



$BE = 17 - 3\dfrac{13}{17} = 13\dfrac{4}{17}$. **Ans.** $13\dfrac{4}{17}$

NOTE: SOLUTIONS FOR QUESTION 3 AND 2 ARE SWITCHED IN THIS ROUND.

2. $PQ \cdot PR = PS \cdot PT$. Let PS = x, then $8 \cdot 15 = x(x+2)$. Then
$x^2 + 2x = 120 \rightarrow x^2 + 2x - 120 = 0 \rightarrow (x + 12)(x - 10) = 0$.
So PS = 10 = MT. $MT \cdot MN = QM \cdot MR$. Let MR = a, then
$10(1) = a(7 - a) \rightarrow a^2 - 7a - 10 = 0$ or $(a - 2)(a - 5) = 0$. **Ans. 2 or 5.**



Conics

1. The radius from (0, 0) to (-5, 6) is $\sqrt{(-5)^2 + (6)^2} = \sqrt{25+36} = \sqrt{61}$. **Ans.** $x^2 + y^2 = 61$

2. Completing the square for x and y for $x^2 + 16y^2 + 14x - 288y + 1329 = 0 \rightarrow$
$(x^2 + 14x + 49) + 16(y^2 - 18y + 81) = -1329 + 49 + 1296 = 16$. Thus $\frac{(x+7)^2}{16} + \frac{(x-9)^2}{1} = 1$
The eccentricity is $\sqrt{15}/4$. **Ans.** $\sqrt{15}/4$

3. Completing the square for $16y^2 - 9x^2 + 72x + 160y + 832 = 0$:
$16(y^2 + 10y + 25) - 9(x^2 - 8x + 16) = -832 + 400 - 144 = -576$. So $\frac{(x-4)^2}{64} - \frac{(y+5)^2}{36} = 1$.
The distance to the foci from the center is $\sqrt{64+36} = 10$. Center is (4, -5). Since the foci are horizontal, then they are $(4 \pm 10, -5)$. **Ans. (14, -5), (-6, -5)**

Arithmetic with Statistics

1. $N = 5 \cdot 6 \cdot 7$. $\frac{5 \cdot 6 \cdot 7}{5} = 42$, $\frac{5 \cdot 6 \cdot 7}{6} = 35$, $\frac{5 \cdot 6 \cdot 7}{7} = 30$. $(42+35+30)/3 = 35\frac{2}{3}$. **Ans.** $35\frac{2}{3}$

2. Converting each to base 10: $344_5 = 75 + 20 + 4 = 99$; $233_4 = 32 + 12 + 3 = 47$;
$122_3 = 9 + 6 + 2 = 17$. The sum is 163. Converting 163 to Base 6: there are 4 36's which is 144, with 19 left over. This is 3 6's with 1 left over. 431 Base 6. **Ans. 431 Base 6**

3. If the mean of eight numbers is 8, then their sum is 64. Since the median is 9 and the mode is 10 then the smallest possible value for the largest term is 10, the numbers being __ , __ , __ , 9 , 9 , 10, 10, 10. The sum of these numbers is 48, that leaves 16 for the other 3, which could be 4, 5. 7. The value for the largest term comes when the fourth and fifth are 8 and 10, the 6^{th} and 7^{th} are 10 and the first three terms are as small as possible. If the first three terms are 1, 1, and 2, then the first 7 numbers are 1, 1, 2, 8, 10, 10, 10. The sum of these is 42, making the 8^{th} term 22. 22 + 10 = 32. **Ans. 32**

Team

1. $\dfrac{x}{x-\dfrac{1}{x-\dfrac{1}{x}}} = \dfrac{x}{x-\dfrac{1}{\dfrac{x^2-1}{x}}} = \dfrac{x}{x-\dfrac{x}{x^2-1}} = \dfrac{x}{\dfrac{x^3-x-x}{x^2-1}} = \dfrac{x(x^2-1)}{x(x^2-2)} = \dfrac{x^2-1}{x^2-2}$. **Ans.** $\dfrac{x^2-1}{x^2-2}$

2. The radius drawn from the center to the endpoint of the chord is 12. From the center to the midpoint of the chord is 6. So $6\sqrt{3}$ is half the chord length. **Ans.** $12\sqrt{3}$

3. 95,600 – 6850 = 88750. 88750 – 6850 = 81,900. This is two standard deviations below the mean which is 34% + 13.5% plus the 50% above. 97.5% of 1800 = **Ans. 1755**

4. sec x – cos x ➔ $\dfrac{1}{\cos x}$ - cos x ➔ $\dfrac{1-\cos^2 x}{\cos x} = \dfrac{\sin^2 x}{\sqrt{1-\sin^2 x}} =$ **Ans.** $\dfrac{\sin^2 x\sqrt{1-\sin^2 x}}{1-\sin^2 x}$

5. The center of the ellipse is (5, -3). Since the eccentricity is $\dfrac{5}{3}$, and ecc = $\dfrac{f}{v}$, then $\dfrac{5}{3}=\dfrac{f}{15}$ and f = 25. 25^2 - 15^2 = b^2 in $\dfrac{(x-5)^2}{15^2}-\dfrac{(y+3)^2}{b^2}=1$ **Ans.** $\dfrac{(x-5)^2}{225}-\dfrac{(y+3)^2}{400}=1$

6. 4 + 3x + 12 – x = 30 ➔ 2x + 16 = 30, so x = 7. 4 + 3(7) = 25. **Ans. 7 seconds, (25, 0)**

7. $\dfrac{x}{x+5}-\dfrac{x-2}{x+2}=\dfrac{x-6}{5x+1}$ ➔ x(x + 2)(5x + 1) – (x – 2)(x + 5)(5x + 1) = (x – 6)(x + 5)(x + 2)
x(5x² + 11x + 2) – (x – 2)(5x² + 26x + 5) = (x – 6)(x² + 7x + 10) ➔
5x³ + 11x² + 2x – (5x³ + 16x² - 47x – 10) = x³ + x² - 32x – 60 ➔
0 = x³ + 6x² - 81x – 70. Synthetically:

7	1	6	-81	-70
		7	91	70
	1	13	10	0

So x^2 - 13x + 10 = 0 Solving by quadratic formula:

$x = \dfrac{-13 \pm \sqrt{169-40}}{2} = \dfrac{-13 \pm \sqrt{129}}{2}$. **Ans. 7 or** $\dfrac{-13 \pm \sqrt{129}}{2}$

8. 8 sin x $\cos^5$x – 8 $\sin^5$x cos x = 1 ➔ 8 sin x cos x ($\cos^4$x - $\sin^4$x) = 1 ➔
4 sin 2x ($\cos^2$x - $\sin^2$x)($\cos^2$x + $\sin^2$x) = 1 ➔ 4 sin 2x cos 2x = 1 ➔ 2 sin 4x = 1 ➔
sin 4x = ½. So 4x = 30° or 150°. Thus x = **Ans. 7½°, 37½°, 97½°, 127½°**

9. Equation form: $y = ax^2 + bx + c$. Using (3, -8): (1) -8 = 9a + 3b + c;
(4, 5): (2) 5 = 16a + 4b + c; (-2, 17): (3) 17 = 4a – 2b + c. (2) – (1): (4) 13 = 7a + b.
(3) – (1): 25 = -5a – 5b or (5) 5 = -a – b. (4) + (5): 18 = 6a, so a = 3. 5 = -3 – b, so b = -8
In (1): -8 = 9(3) + 3(-8) + c, so c = -11. $y = 3x^2 - 8x - 11$ ➔ $y + 11 + \dfrac{16}{3} = 3(x^2 - \dfrac{8}{3}x + \dfrac{16}{9})$
y + 16⅓ = 3(x - ⅔)². Focus is (⅔, - 16⅓ + ¾) = (⅔, -15 7/12). **Ans. (1⅓, -15 7/12)**

Answer Sheet Mar 2014

Algebraic Fractions and Factoring

1. $\frac{1}{x+3}$
2. 14
3. 15 or -4

Trigonometric Equations and Identities

1. 8/17
2. 1 - $\sqrt{2}$ or $-\sqrt{3-2\sqrt{2}}$
3. ~~30°, 45°, 210°, 225°~~ No correct answers

Circles and Spheres

1. 220° or $\frac{11\pi}{9}$
2. 2 or 5
3. $13\frac{4}{17}$ or 225/17

Conics

1. $x^2 + y^2 = 61$
2. $\sqrt{15}/4$
3. (14, -5), (-6, -5)

Arithmetic with Statistics

1. $35\frac{2}{3}$ or 107/3
2. 431 or 431_6 or 431 Base 6
3. 32

Team

1. $\frac{x^2-1}{x^2-2}$
2. $12\sqrt{3}$
3. 1755 ~~or~~ 1759
4. $\frac{\sin^2 x\sqrt{1-\sin^2 x}}{1-\sin^2 x}$ → $\frac{2\sin^3 x}{\sin 2x}$
5. $\frac{(x-5)^2}{225} - \frac{(y+3)^2}{400} = 1$
6. 7 seconds, (25, 0)
7. 7 or $\frac{-13 \pm \sqrt{129}}{2}$
8. 7½°, 37½°, 97½°, 127½°
9. $(1\frac{1}{3}, -15\frac{7}{12})$ or $(\frac{4}{3}, \frac{-187}{12})$