1 Algebraic Fractions and Factoring Mar 2015 (No Calculators)

3 pts 1. Solve for x: $\frac{x+1}{6} = \frac{2}{x}$

Ans. _____

4 pts 2. In a certain city a birth occurs on the average every 24 minutes, and a death every half hour. A resident moves out of the city every 1.5 hours and a new person moves in every 4.5 hours. How long, in hours, does it take on average for the population to increase by one person?

Ans. _____

5 pts 3. An experienced plumber made $800 for working on a certain job. His apprentice, who makes $5 less per hour also made $800. However, the apprentice worked 8 hours more than the plumber. How much does the plumber make per hour?

Ans. _____

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2 Trigonometric Equations and Identities Mar 2015 (No Calculators)

3 pts 1. Find all values of x from 0^o to 360^o for which $\tan x = -\frac{1}{3}\sqrt{3}$.

Ans. _____

4 pts 2. Compute the value of the following:

$$\cos(\text{Arccos}\ \frac{3}{5} + \text{Arc sin}\ \frac{3}{5})$$

Ans. _____

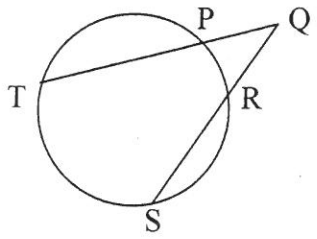
5 pts 3. What is the value of the sum:

$$\cos\frac{\pi}{6} + \cos\frac{\pi}{3} + \cos\frac{\pi}{2} + \ldots + \cos\frac{2017\pi}{6}\ ?$$

Ans. _____

3 Circles and Spheres Mar 2015 (No Calculators)

3 pts 1. In the figure QP = 6, QR = 8, and PT = 22. Find RS.

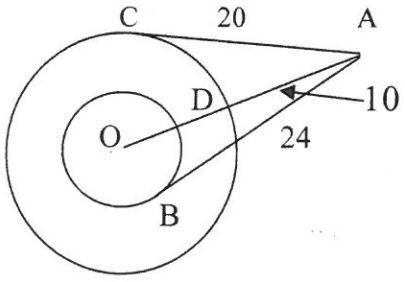


Ans. _____

4 pts 2. A circle is inscribed in a 40^{o} - 60^{o} - 80^{o} triangle. The points of tangency are connected to form a triangle inscribed in the circle. State the measures of the angles of the inscribed triangle in numerical order from least to greatest.

Ans. _____

5 pts 3. $\overline{AB}$ is tangent to the smaller of the two concentric circles at B shown. $\overline{AC}$ is tangent to the larger circle at C. O is the center of the smaller circle. $\overline{OA}$ intersects the larger circle at D. AC is 20, AB = 24, And AD = 10. Find the length of the radius of the smaller circle.



Ans. _____

4 Conics Mar 2015 (No Calculators)

3 pts 1. Find the center and the radius of the circle whose equations is

$$2x^2 + 2y^2 + 2x - 6y - 3 = 0$$

Ans. _____

4 pts 2. Find the equation of the conic whose center is (-5, 3), focus is (-5, -12) and whose vertex is (-5, 15).

Ans. _____

5 pts 3. Find the equation of the ellipse with foci (-4, 0) and (4, 0) that passes through the point $(3, \sqrt{15})$. Give answer in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

Ans. _____

5 Arithmetic with Statistics Mar 2015 (You may use calculators)

3 pts 1. Find a set of five positive integers with a mean of 4, a range of 6, a mode of 1 and a median of 5. List your numbers in increasing order.

Ans. _____

4 pts 2. Find all the distinct real numbers x, such that the median of the five numbers x, 4, 6, 1, 9 is equal to the mean of the same 5 numbers.

Ans. _____

5 pts 3. In a group of five people, the sums of the ages of each group of four of them are 124, 128, 130, 136, and 142. What is the age of the youngest?

Ans. _____

6 Team Mar 2015 (You may use calculators)

3 pts 1. On any individual round at this math meet, there were 3 point, 4 point and 5 point questions. How many different scores are possible for one student on one individual round?

(1) Ans. _____ **3 pts**

3 pts 2. Simplify: $\frac{2x^2-7x}{x-1}\cdot\frac{x^2-1}{2x^2-5x-7}$ **(2) Ans.** _____ **3 pts**

3 pts 3. If $180^\circ \le \alpha \le 270\alpha$ and $\cos\alpha = -\frac{15}{17}$,
find $\sin \alpha/2$. Give exact answer. **(3) Ans.** _____ **3 pts**

4 pts 4. Find the vertex in (x, y) form of $x = y^2 + 5y - 12$.

(4) Ans. _____ **4 pts**

4 pts 5. What is the value of the following:

$(\tan 5^\circ)(\tan 15^\circ)(\tan 25^\circ)(\tan 35^\circ)(\tan 45^\circ)(\tan 55^\circ)(\tan 65^\circ)(\tan 75)(\tan 85^\circ)$?

(5) Ans. _____ **4 pts**

4 pts 6. If $\frac{19x-8}{2x^2-x-21} = \frac{A}{2x-7}+\frac{B}{x+3}$, find the sum of the values of A and B.

(6) Ans. _____ **4 pts**

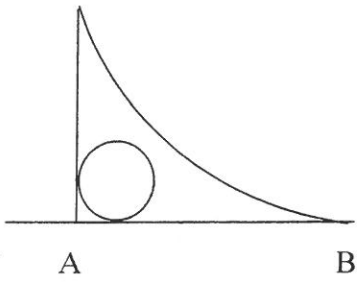
5 pts 7. Pentagon ACDEB is made up of equilateral triangle ABC on top of square BCDE each with side lengths of 2 cm. A circle passes through A, D, and E. Find the length of the radius.

(7) Ans. _____ **5 pts**

5 pts 8. Consider the points (x, y) whose distance from (8, 0) is twice their distance from the line x = 2. Find the equation of all points in the form
$Ax^2 + By^2 + Cx + Dy + E = 0$. **(8) Ans.** _____ **5 pts**

5 pts 9. The circle and the arc of a larger circle at right are a diagram of a traffic circle and off ramps of two freeways. The circle has a radius of 100 ft and is 20 ft from the arc of the larger circle. Both circles are tangent to both highways. Find the distance AB in feet.

(9) Ans. _____ **5 pts**



Solutions – Algebraic Fractions and Factoring

1. $\frac{x+1}{6}=\frac{2}{x}$ ➔ $x^2 + x = 12$ ➔ $x^2 + x - 12 = 0$ ➔ $(x + 4)(x - 3) = 0$. **Ans. 3 or -4**

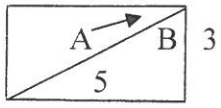
2. Let x = # minutes it takes for 1 person increase in population.
   Birth – Death – Out - In
   $\frac{x}{24} - \frac{x}{30} - \frac{x}{90} + \frac{x}{270} = 1$ ➔ $\frac{x}{24} - \frac{9x}{270} - \frac{3x}{270} + \frac{x}{270} = \frac{x}{24} - \frac{11x}{270} = 1$ ➔
   $270x - 264x = 24(270)$ ➔ $6x = 24(270)$ ➔ $x = 4(270)$. $4(270)/60 = 18$. **Ans. 18 hr**

3. Let R = Plumber's hourly rate, and T = Time plumber worked.
   (1) RT = 800, and (2) $(R - 5)(T + 8) = 800$ ➔ $RT - 5T + 8R - 40 = 800$ ➔
   $8R = 5T + 40$ ➔ $R = \frac{5T+40}{8}$. In (1) $(\frac{5T+40}{8})T = 800$ ➔ $5T^2 + 40T = 6400$ ➔
   $T^2 + 8T - 1280 = 0$ ➔ $(T - 32)(T + 40) = 0$. $T = 32$. In (1) $R(32) = 800$. **Ans. $25**

Trigonometric Equations and Identities

1. $\tan x = -\frac{1}{3}\sqrt{3} = -\frac{1}{\sqrt{3}}$, so related angle is 30°. **Ans. 150° or 330°**

2. Mechanically putting the two angles at the same vertex, shows angle sum to be 90° (at right). Alternate solution: Let A = Arccos 3/5 and let B = Arc sin 3/5, then cos (A + B) = cos A cos B – sin A sin B:
   $\frac{3}{5}\cdot\frac{4}{5} - \frac{4}{5}\cdot\frac{3}{5} = 0$.



Ans. 0

3. The sum of the cosines of all these angles to $2\pi = 0$. $\frac{2017\pi}{6} = 336\frac{1}{6}\pi$. Many 2π's Plus $\frac{1}{6}\pi$, $\cos\frac{1}{6}\pi = \frac{\sqrt{3}}{2}$. **Ans. $\frac{\sqrt{3}}{2}$**

Circles and Spheres

1. $QP \cdot QT = QR \cdot QS = 6 \cdot 28 = 8(8 + RS)$ ➔ $168 = 64 + 8RS$ ➔ $104 = 8RS$. **Ans. 13**

2. The measure of the angle formed by tangents to the circle from the same point is supplementary to the measure of the arc on the circle it intercepts. The measure of an inscribed angle is half the arc it intercepts. So the angles of the circumscribed triangle are 40°, 60°, 80°. Their corresponding supplements 140°, 120°, 100°. Their corresponding inscribed angles: 70°, 60°, 50°. **Ans. 50°, 60°, 70°**

3. Connect O to C and O to B. OC = OD. Let each = x, then $x^2 + 20^2 = (x + 10)^2$ ➔
   $x^2 + 400 = x^2 + 20x + 100$ ➔ $300 = 20x$, So $x = 15$, OA = 25, thus OB = 7. **Ans. 7**

Conics

1. $2x^2 + 2y^2 + 2x - 6y - 3 = 0 \rightarrow (x^2 + x + \frac{1}{4}) + (y^2 - 3y + \frac{9}{4}) = \frac{3}{2} + \frac{1}{4} + \frac{9}{4} = 4.$

$(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = 4.$ **Ans. Center $(-\frac{1}{2}, \frac{3}{2})$, radius 2**

2. The vertex is 12 units away from the center. The focus is 15 units from the center. This must be a hyperbola. The endpoint of the conjugate axis is 9 units from the center, since it involves a 9 – 12 – 15 right triangle. The hyperbola opens up and down thus the equation: $\frac{(y+5)^2}{144} - \frac{(x-3)^2}{81} = 1$ $\frac{(y-3)^2}{144} - \frac{(x+5)^2}{81} = 1$ **Ans.** $\frac{(y+5)^2}{144} - \frac{(x-3)^2}{81} = 1$

3. Center is at (0, 0), thus the equation takes on the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $a^2 - b^2 = 16 \rightarrow$

$a^2 = b^2 + 16$. Plugging in the point $(3, \sqrt{15})$: $\frac{9}{a^2} + \frac{15}{b^2} = 1, \rightarrow \frac{9}{b^2+16} + \frac{15}{b^2} = 1 \rightarrow$

$9b^2 + 15b^2 + 240 = b^4 + 16b^2 \rightarrow b^4 - 8b^2 - 240 = 0 \rightarrow (b^2 - 20)(b^2 + 12) = 0.$

So $b^2 = 20$ and $a^2 = 36$. **Ans.** $\frac{x^2}{36} + \frac{y^2}{20} = 1$

Arithmetic with Statistics

1. 1, 1, 5, x, 7; mean 4 implies $x + 14 = 20$, so $x = 6$. **Ans. 1, 1, 5, 6, 7**

2. Since the order of the known numbers is 1, 4, 6, 9 the median must be 4 , 6, or the only number in between, 5. If 4, then x is 0. If 5, then $x = 5$. If 6, then $x = 10$. **Ans. 0, 5, 10**

3. Let a, b, c, d, e be the ages of the people from youngest to oldest. Then a+b+c+d = 124, a+b+c+e = 128, a+b+d+e=130,a+c+d+e = 136, and b+c+d+e = 142. Adding all 5 equations: 4(a+b+c+d+e) = 660 or a+b+c+d+e = 165 → a + 142 = 165, a = 23. **Ans. 23**

Team

1. Possible scores: 0, 3, 4, 5, 7, 8, 9, 12. There are 8. **Ans. 8**

2. $\frac{2x^2 - 7x}{x-1} \cdot \frac{x^2 - 1}{2x^2 - 5x - 7} \rightarrow \frac{x(2x-7)}{x-1} \cdot \frac{(x-1)(x+1)}{(2x-7)(x+1)} = x.$ **Ans. x**

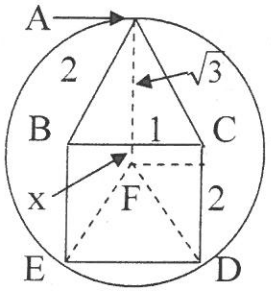
3. $\alpha/2$ must be between 90° and 135°, a second quadrant angle, so sin of the angle is positive. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(\frac{-15}{17}\right)}{2}} = \sqrt{\frac{17+15}{34}} = \sqrt{\frac{32}{34}} = \sqrt{\frac{16}{17}} = \frac{4\sqrt{17}}{17}.$ **Ans.** $\frac{4\sqrt{17}}{17}$

4. $(y^2 + 5y + \frac{25}{4}) = x + 12 + \frac{25}{4} \rightarrow (y + \frac{5}{2})^2 = x + \frac{73}{4}.$ **Ans.** $\left(-\frac{73}{4}, -\frac{5}{2}\right)$

5. $(\tan 5^\circ, \tan 85^\circ)$, $(\tan 15^\circ, \tan 75^\circ)$, etc are reciprocals. Tan $45^\circ = 1$. **Ans. 1**

6. $\frac{19x-8}{2x^2-x-21} = \frac{A}{2x-7} + \frac{B}{x+3}$ ➔ A(x + 3) + B(2x – 7) = 19x – 8 ➔ (1): A + 2B = 19 and (2): 3A – 7B = -8. -3(1) + (2): (-3A - 6B = -57) + (3A – 7B = -8): -13B = -65, so B = 5. In (1): A + (10) = 19, so A = 9. 9 + 5 = 14. **Ans. 14**

7. At right, radius AF = $\sqrt{3}$ + x and radius DF = $\sqrt{1^2 + (2-x)^2}$. Thus $\sqrt{3}$ + x = $\sqrt{5-4x+x^2}$. Squaring: $3 + 2\sqrt{3}x + x^2 = 5 - 4x - x^2$ or $(4 + 2\sqrt{3})x = 2$ ➔ $x = \frac{2}{4+2\sqrt{3}} = \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2 - \sqrt{3}$. So AF = 2.



Ans. 2

8. $\sqrt{(x-8)^2 + y^2} = 2|x-2|$. Squaring: $x^2 - 16x + 64 + y^2 = 4(x^2 - 4x + 4)$ ➔ $x^2 - 16x + 64 + y^2 = 4x^2 - 16x + 16$ ➔ $0 = 3x^2 - y^2 - 48$. **Ans. $3x^2 - y^2 - 48 = 0$**

9. Let O be the center of the small circle and P the center of the large circle. The line bisecting the angle at point A passes through O and P. OA = $100\sqrt{2}$. Let R = the radius of the larger circle PB which also = AB. $\sqrt{2}R = 100\sqrt{2} + 100 + 20 + R$ ➔ $R(\sqrt{2} - 1) = 100\sqrt{2} + 120$ ➔ $R = \frac{100\sqrt{2}+120}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = 200 + 100\sqrt{2} + 120\sqrt{2} + 120 = 320 + 220\sqrt{2}$.

Ans. $320 + 220\sqrt{2}$

Answer Sheet Mar 2015

Algebraic Fractions and Factoring

1. -4 or 3
2. 18 hrs.
3. $25 or 25

Trigonometric Equations and Identities

1. $150^\circ, 330^\circ$
2. 0
3. $\sqrt{3}/2$

Circles and Spheres

1. 13
2. $50^\circ, 60^\circ, 70^\circ$
3. 7

Conics

1. Center $(-\frac{1}{2}, \frac{3}{2})$, radius 2
2. $\frac{(y+5)^2}{144} - \frac{(x+5)^2}{81} = 1$ (y-3)
3. $\frac{x^2}{36} + \frac{y^2}{20} = 1$

Arithmetic with Statistics

1. 1, 1, 5, 6, 7
2. 0, 5, 10
3. 23

Team

1. 8
2. x
3. $\frac{4\sqrt{17}}{17}$
4. $\left(-\frac{73}{4}, -\frac{5}{2}\right)$ or (-18¼, -2½) or (-18.25, -2.5)
5. 1
6. 14
7. 2
8. $3x^2 - y^2 - 48 = 0$
9. $320 + 220\sqrt{2}$ or 631.1270