1 Algebraic Fractions and Factoring Mar 2016 (No Calculators)

3 pts 1. Simplify: $\frac{1}{x-1}-\frac{1}{x+1}+\frac{2x}{x^2-1}$

Ans. _____

4 pts 2. Factor completely: abc – abz + acy – ayz – bcx + bxz – cxy + xyz

Ans. _____

5 pts 3. Solve the following equation where $x \neq 0$: $\frac{x+1}{x^2}-\frac{1}{x^3}=\frac{x+3}{4x}$

Ans. _____

2 Trigonometric Equations and Identities Mar 2016 (No Calculators)

3 pts 1. Find all values of x, where $0^o \le x < 360^o$ and tan x = 1.

Ans. _____

4 pts 2. Express $\frac{\sin\theta-1}{\cos\theta}-\frac{\cos\theta}{\sin\theta+1}$ in simplest form.

Ans. _____

5 pts 3. Find all values of θ, where $0^o \le \theta < 360^o$, if $\sqrt{3}\sec^2\theta+2\tan\theta=2\sqrt{3}$.

Ans. _____

3 Circles and Spheres Mar 2016 (No Calculators)

3 pts 1. Find the measure of the arc of a circle which is subtended by a central angle of 150°, if the radius of the circle is 42 cm.

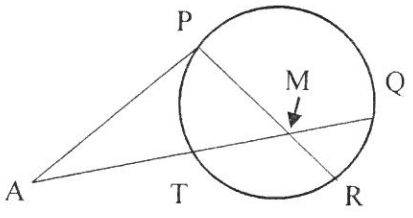
Ans. _____

4 pt 2. Two parallel planes each intersect the same sphere. One passes through the center of the sphere and the other bisects the radius. Find the ratio of the area of the larger circle of intersection to that of the smaller circle of intersection of the planes and the sphere.

Ans. _____

5 pts 3. In the figure, P is the point of tangency of segment AP.

$AT = 3x - 6$, $MR = 5x - 12$, $MQ = 4x - 4$, $PM = 2x + 1$ and $MT = x + 2$. Find AP.



Ans. _____

4 Conics Mar 2016 (No Calculators)

3 pts 1. Find the center in the form (x, y) and the radius of the circle whose equation is

$x^2 + y^2 + 6x - 6y + 14 = 0$.

Ans. _____

4 pts 2. Find the equation of the parabola whose directrix is y = - 4 and focus is (-2, 2).

Write the equation in the form $y = ax^2 + bx + c$ or $x = ay^2 + by + c$.

Ans. _____

5 pts 3. The endpoints of the major axis of an ellipse are (-7, 5) and (1, 5). The eccentricity is $\frac{\sqrt{3}}{2}$. What is the equation of the ellipse in the form $\frac{(x-h)}{a^2} + \frac{(y-k)}{b^2} = 1$?

Ans. _____

5 Arithmetic with Statistics Mar 2016 (You may use calculators)

3 pts 1. Find the absolute value of the difference between the median and the mean of the following data set: {4, 0, -1, 7, 6, 3}

Ans. _____

4 pts 2. Find the sum of the mean, median, mode and range of the following data.

size	1	2	3	4	5
frequency	5	15	5	10	5

Ans. _____

5 pts 3. Seven data points are each one-digit numbers. Range of the data points is 5. There are two modes which have a difference of 2 and each mode occurs twice. The median is one of the modes. Six of the seven data points are prime numbers. Find the sum of all possible data points that satisfy these conditions.

Ans. _____

6 Team Mar 2016 (You may use calculators)

3 pts 1. The launch-angles of a water-propelled rocket were 45°, 60°, 60°, 30°, 45° and 60°. Find the exact value of (the mean minus the median) of the sines of the angles.

(1) Ans. _____ **3 pts**

3 pts 2. Simplify $\left(-x^{-1}\right)^{-1}+\left(x^{-1}-y^{-1}\right)^{-1}$,

where x ≠ y and neither is equal to 0. **(2) Ans.** _____ **3 pts**

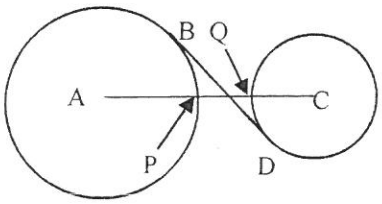
3 pts 3. Given right triangle ABC. Find the largest possible value of

sin A + sin B + sin C. **(3) Ans.** _____ **3 pts**

4 pts 4. Circle O has equation $x^2+y^2-8x-6y+9=0$. Circle P has equation $x^2+y^2-8x-6y+\frac{75}{4}=0$ Circle Q is tangent to both circles. Find the radii of all possible circles Q.

(4) Ans. _____ **4 pts**

4 pts 5. The distance between the centers A and C of these two circles is 25. AP = 9 and CQ = 6. Find the Length of internal tangent segment BD.



(5) Ans. _____ **4pts**

4 pts 6. A sphere has a radius of 25. The Cartesian Coordinate system intersects the sphere in a circle whose equation is $x^2+y^2-40x+20y+100=0$. How far from the center of the circle is the closest point of the sphere? **(6) Ans.** _____ **4 pts**

5 pts 7. Find the x-coordinate of the vertex of the parabola passing through the points

(1, 1), (4, -1) and (8, 3), which opens upward?

(7) Ans. _____ **5 pts**

5 pts 8. Simplify completely in terms of csc x.

$$\frac{\dfrac{\sin 2x}{\sin x\cos x}+\dfrac{1-\sin^2 x}{(1+\tan^2 x)^{-1}}}{\dfrac{\sec^2 x-\tan^2 x}{\csc x}}$$

(8) Ans. _____ **5 pts**

5 pts 9. The vertices of a hyperbola are the endpoints of the minor axis of the ellipse $16x^2+25y^2-64x-250y+289=0$. The eccentricity of the hyperbola is 1.25. Exactly how far apart are the 2 points on the hyperbola which have an x-coordinate of 4?

(9) Ans. _____ **5 pts**

Solutions – Algebraic Fractions with Factoring

1. $\frac{1}{x-1}-\frac{1}{x+1}+\frac{2x}{x^2-1}=\frac{x+1-x+1+2x}{(x+1)(x-1)}=\frac{2(x+1)}{(x+1)(x-1)}=\frac{2}{x-1}$. **Ans.** $\frac{2}{x-1}$

2. $abc - abz + acy - ayz - bcx + bxz - cxy + xyz = (c - z)(ab + ay - bx - xy) =$

$(c - z)[a(b + y) - x(b + y)] = (c - z)(a - x)(b + y)$. **Ans. (c – z)(a – x)(b + y)**

3. $\frac{x+1}{x^2}-\frac{1}{x^3}=\frac{x+3}{4x}$ ➔ $4x(x + 1) - 4 = x^2(x + 3)$ ➔ $4x^2 + 4x - 4 = x^3 + 3x^2$ ➔

$x^3 - x^2 + 4x - 4 = 0$ ➔ $x^2(x - 1) - 4(x - 1) = 0$ ➔ $(x^2 - 4)(x - 1) = 0$. **Ans. 1, 2, -2**

Trigonometric Equations and Identities

1. $\tan x = 1$ at 45° and 225°. **Ans. 45° or 225°**

2. $\frac{\sin\theta-1}{\cos\theta}-\frac{\cos\theta}{\sin\theta+1}$ ➔ $\frac{(\sin\theta-1)(\sin\theta+1)-\cos^2\theta}{\cos(\sin\theta+1)}$ ➔ $\frac{\sin^2\theta-1-(1-\sin^2\theta)}{\cos\theta(\sin\theta+1)}$ ➔ $\frac{2\sin^2\theta-2}{\cos\theta(\sin\theta+1)}$ ➔

$\frac{2(\sin^2\theta-1)}{\cos\theta(\sin\theta+1)}$ ➔ $\frac{2(\sin\theta-1)(\sin\theta+1)}{\cos\theta(\sin\theta+1)}$ ➔ $\frac{2(\sin\theta-1)}{\cos\theta}$ ➔ $\frac{2\sin\theta-2}{\cos\theta}$. **Ans.** $\frac{2\sin\theta-2}{\cos\theta}$

Working with cos instead of sin produces $\frac{-2\cos\theta}{\sin\theta+1}$, which is equivalent to above answer.

3. $\sqrt{3}\sec^2\theta+2\tan\theta=2\sqrt{3}$ ➔ $\sqrt{3}(\tan^2\theta+1)+2\tan\theta=2\sqrt{3}$ ➔ $\sqrt{3}\tan^2\theta+\sqrt{3}+2\tan\theta=2\sqrt{3}$

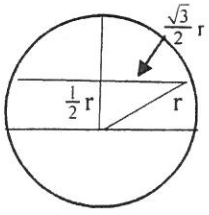
$\sqrt{3}\tan^2\theta+2\tan\theta-\sqrt{3}=0$ ➔ $(\sqrt{3}\tan\theta-1)(\tan\theta+\sqrt{3})=0$, thus (1) $\tan\theta=\frac{1}{\sqrt{3}}$ or (2) $\tan\theta = -\sqrt{3}$.

In (1): $\theta = 30°$ or $210°$. In (2): $\theta = 120°$ or $300°$. **Ans. 30°, 120°, 210°, 300°**

Circles and Spheres

1. $\frac{150}{360}(84\pi)=\frac{5}{12}(84\pi)=5(7\pi)=35\pi$. **Ans.** 35π

2. In the figure at right the radius is r. The distance from the center of the sphere to the other circle of intersection is 1/2 r. Therefore the radius of the other circle is $\frac{\sqrt{3}}{2}$r. Ratio of areas: $\frac{\pi r^2}{\pi\left(\frac{\sqrt{3}}{2}r\right)^2}=\frac{\pi r^2}{\pi\left(\frac{3}{4}\right)r^2}=\frac{1}{\frac{3}{4}}=\frac{4}{3}$



Ans. 4/3

3. The chords produce: PM•MR = TM•MQ ➔ (2x + 1)(5x – 12) = (x + 2)(4x – 4) ➔

$10x^2 - 19x - 12 = 4x^2 + 4x - 8$ ➔ $6x^2 - 23x - 4 = 0$ ➔ (6x + 1)(x – 4) = 0, so x = 4.

Thus AT = 3(4) – 6 = 6, TQ = TM + MQ = (4) + 2 + 4(4) – 4 = 18. $(AP)^2$ = AT•AQ = 6•24

$(AP)^2 = 144$, thus AP = 12. **Ans. 12**

Conics

1. $x^2 + y^2 + 6x - 6y + 14 = 0$ ➔ $(x^2 + 6x + 9) + (y^2 - 6y + 9) = -14 + 18 = 4$. **Ans. C(-3, 3), r = 2**

2. The vertex of the parabola is halfway from the focus (-2, 2) to the directrix y = -4. The vertex is (-2, -1) and the parabola opens up, so it has the form $(x - h)^2 = 4p(y - k)$.

$(x + 2)^2 = 4(3)(y + 1)$ ➔ $x^2 + 4x + 4 = 12y + 12$ ➔ $12y = x^2 + 4x - 8$. **Ans.** $y = \frac{1}{12}x^2 + \frac{1}{3}x - \frac{2}{3}$

3. The midpoint of (-7, 5) and (1, 5) is (-3, 5) which is the center of the hyperbola. $\frac{f}{v} = \frac{\sqrt{3}}{2} = \frac{f}{4}$

$f = 2\sqrt{3}$. For $a^2 + b^2 = c^2$ ➔ $16 + b^2 = 12$, $b^2 = 4$. **Ans.** $\frac{(x+3)^2}{16} + \frac{(y-5)^2}{4} = 1$

Statistics

1. Median = 3.5. Mean = 19/6 = $3\frac{1}{6}$. Median – Mean: $3\frac{1}{2} - 3\frac{1}{6} = \frac{1}{3}$. **Ans. 1/3**

2. Mode is 2. Range is 4. Median = 2.5. Mean = 115/40 = $2\frac{7}{8}$. Sum = $11\frac{3}{8}$. **Ans.** $11\frac{3}{8}$

3. Possibilities for modes are either 3and 5 or 5 and 7. Primes are 2, 3, 5, 7. Non-primes are 0, 1, 4, 6, 8, and 9. Only one non-prime can be used. Possibilities: 0, 2, 3, 3, 5, 5, 7, out of range; 0, 2, 3, 5, 5, 7, 7, out of range; 1, 2, 3, 3, 5, 5, 7, out of range; 1, 2, 3, 5, 5, 7, 7, out of range; 2, 3, 3, 4, 5, 5, 7, median not mode. 2, 3, 3, 5, 5, 6, 7, works (1); 2, 3, 3, 5, 5, 8, out of range; 2, 3, 4, 5, 5, 7, 7, works (2); 2, 3, 5, 5, 6, 7, 7, works (3); 2, 3, 5, 5, 7, 7, 8, out of range. (1) + (2) + (3) = 31 + 33 + 35 = 99. **Ans. 99**

Team

1. $\sin 45^\circ = \frac{\sqrt{2}}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$. Mean = $\frac{2\sqrt{2} + 3\sqrt{3} + 1}{2} \div 6 = \frac{2\sqrt{2} + 3\sqrt{3} + 1}{12}$.

Median = $\frac{1}{2}(\sin 45^\circ + \sin 60^\circ) = \frac{1}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} + \sqrt{3}}{4}$. Mean – Median = $\frac{3\sqrt{3} + 2\sqrt{2} + 1 - 3\sqrt{3} - 3\sqrt{2}}{12}$

Ans. $\frac{1 - \sqrt{2}}{12}$

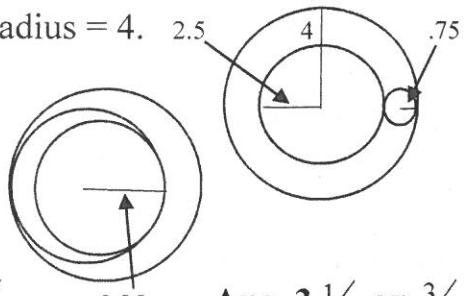
2. $\left(-x^{-1}\right)^{-1}+\left(x^{-1}-y^{-1}\right)^{-1} = -x + \left(\frac{1}{x}-\frac{1}{y}\right)^{-1} = -x + \left(\frac{y-x}{xy}\right)^{-1} = -x + \frac{xy}{y-x} = \frac{-xy+x^2+xy}{y-x}$ **Ans.** $\frac{x^2}{y-x}$

3. If y = sin x and y = cos x are graphed on the same graph, they intersect at 45° and by the method of adding ordinates, you will find that it is the maximum of their sum from 0° to 90°.

So sin 45° + 45° + sin 90° = $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+1 = \sqrt{2}+1$. **Ans.** $\sqrt{2}+1$

4. Circle O: $(x^2 - 8x + 16) + (y^2 - 6y + 9) = -9 + 16 + 9 = 16$. Radius = 4.

Circle P: $(x^2 - 8x + 16) + (y^2 - 6y + 9) = -\frac{75}{4} + 16 + 9 = \frac{25}{4}$.



There are two tangent-circle types shown at right.

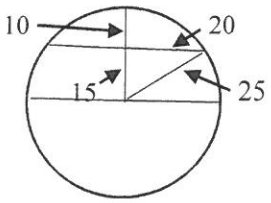
The radii of the two circle types are $\frac{4\pm\frac{5}{2}}{2} = \frac{6\frac{1}{2} \text{ or } \frac{3}{2}}{2} = 3\,\frac{1}{4}$ or $\frac{3}{4}$ **Ans. 3 1/4 or 3/4**

5. Connecting A to B and C to D makes two similar right triangles.

AB = 9 and CD = 6. $\frac{9}{AE}=\frac{6}{CE}$. If AE = x, then $\frac{9}{x}=\frac{6}{25-x}$ ➔ 225 – 9x = 6x ➔ 225 = 15x, so

x = 15 = AE. By th Pyth. Thm BE = 12 and DE = 8. Thus BD = 20. **Ans. 20**

6. $(x^2 - 40x + 400) + (y^2 + 20y + 100) = -100 + 400 + 100 = 400$.



The radius of the circle is 20. The figure at right shows the right triangle in the sphere with hypotenuse 25, radius circle 20, and thus 15 for the third side, which extended is also radius and 10 is the shortest distance from the center of the circle to the sphere. **Ans. 10**

7. The form of the equation is $y = ax^2 + bx + c$. (1, 1): 1 = a + b + c; (4, -1): -1 = 16a + 4b + c

(8, 3): 64a + 8b + c = b. Negating the first equation and adding it to each of the others produces: (1) 15a + 3b = -2 and (2) 63a + 7b = 2. 7(1) – 3(2) = 105a – 189a = -14 – 6 ➔

-84a = -20, So a = 20/84 = 5/21. 15(5/21) + 3b = -2 ➔ 25/7 + 3b = -2 ➔ $3b = -2 - 3\frac{4}{7} = -5\frac{4}{7} =$

$-\frac{39}{7}$, so $b = -\frac{13}{7}$. The x-coordinate of the vertex is $\frac{-b}{2a}=\frac{-\left(\frac{-13}{7}\right)}{2\left(\frac{5}{21}\right)}=\frac{13}{7}\cdot\frac{21}{10}=\frac{39}{10}$. **Ans.** $\frac{39}{10}$

8. $$\frac{\dfrac{\sin 2x}{\sin x\cos x}+\dfrac{1-\sin^2 x}{(1+\tan^2 x)^{-1}}}{\dfrac{\sec^2 x-\tan^2 x}{\csc x}}=\frac{\dfrac{2\sin x\cos x}{\sin x\cos x}+\dfrac{\cos^2 x}{(\sec^2 x)^{-1}}}{\dfrac{1}{\csc x}}=\frac{2+\dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{1}{\csc x}}=3\csc x.$$ **Ans. 3 csc x**

9. $16x^2+25y^2-64x-250y+289=0 \rightarrow 16(x^2-4x+4)+25(y^2-1y+25)=-289+64+625=400$

The ellipse has form $\frac{(x-2)^2}{25}+\frac{(y-5)^2}{16}=1$. The center is the same as the ellipse the transverse axis is 8 units long. The eccentricity is 1.25 or 5/4, so the conjugate axis is 6 units long. The equation of the hyperbola is $\frac{(y-5)^2}{16}-\frac{(x-2)^2}{9}=1$. If x = 4, then $\frac{(y-5)^2}{16}-\frac{(4-2)^2}{9}=1 \rightarrow$

$\frac{(y-5)^2}{16}=\frac{13}{9} \rightarrow (y-5)^2=\frac{13\cdot 16}{9} \rightarrow y-5=\pm\frac{4\sqrt{13}}{3} \rightarrow y=5\pm\frac{4\sqrt{13}}{3}$. **Ans.** $\frac{8\sqrt{13}}{3}$

Answer Sheet – Mar 2016

Algebraic Fractions and Factoring

1. $\frac{2}{x-1}$

2. $(c-z)(b+y)(a-x)$

3. 1, 2, -2

Trigonometric Equations and Identities

1. 45° or 225°

2. $\frac{2\sin\theta-2}{\cos\theta}$ or $\frac{-2\cos\theta}{\sin\theta+1}$ or $2\tan\theta-\sec\theta$

3. 30°, 120°, 210°, 300°

Circles and Spheres

1. $35\pi \rightarrow$ or 180° or $\frac{5\pi}{6}$

2. 4/3 → 4:3 or $1 < r \leq 4/3$

3. 12

Conics

1. Center (-3, 3) Radius = 2

2. $y=\frac{1}{12}x^2+\frac{1}{3}x-\frac{2}{3}$

3. $\frac{(x+3)^2}{16}+\frac{(y-5)^2}{4}=1$

4^2 2^2

Statistics

1. 1/3 or .3333 or $.\bar{3}$

2. $11\frac{3}{8}$ or 11.375 or $\frac{91}{8}$

3. 99

Team

1. $\frac{1-\sqrt{2}}{12}$

2. $\frac{x^2}{y-x}$

3. $\sqrt{2}+1$ or 2.4142

4. $\frac{3}{4}$ or $3\frac{1}{4}$

5. 20

6. 10

7. 39/10 or $3\frac{9}{10}$ or 3.9

8. 3 csc x

9. $\frac{8\sqrt{13}}{3}$