

**1. Algebraic Fractions with Factoring**

**November 1988**

1. Perform the indicated operations. Express your answer as a rational expression in lowest terms.

$$\frac{1}{x^2 + x} - \frac{4}{x^2 - 1} + \frac{1}{x^2 - x}$$

**Ans.** \_\_\_\_\_

2. Solve for  $x$ :  $\frac{3x + 2}{x - 1} + \frac{2x}{x + 1} = \frac{7x + 3}{x^2 - 1}$

**Ans.** \_\_\_\_\_

3. Find the smallest positive fraction  $\frac{x}{y}$ , in lowest terms with  $x$  and  $y$  as integers so that when 7 is added to both the numerator and the denominator of the fraction, the value of this new fraction is equal to the value of the fraction obtained by subtracting 2 from the numerator and 8 from the denominator of the original fraction.

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**November 1989**

1. Simplify:  $\frac{\frac{1}{w^2 - w} + \frac{w + 1}{w}}{\frac{w - 1}{w} - \frac{1}{w^2 - w}}$

**Ans.** \_\_\_\_\_

2. Find all the value(s) of  $X$ , where  $X$  is a real number and  $5 - \frac{4}{3 - \frac{2}{1 - \frac{1}{X}}} = 1 - \frac{8}{X - 3}$

**Ans.** \_\_\_\_\_

3. Find the value(s) of  $c$  if  $\frac{a + b}{b + c} = \frac{c + d}{d + a}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**November 1991**

1. Simplify:  $\frac{x^2 + x - 6}{x^2 - 4x - 5} \div \frac{x + 1}{x - 5} \cdot \frac{x^2 + x}{x^2 - 7x + 10}$

**Ans.** \_\_\_\_\_

2. Find all the solutions for  $a$  such that  $\frac{a + 5}{a + 3} + \frac{a + 6}{3 - a} = \frac{a^2 - 6a - 39}{a^2 - 9}$

**Ans.** \_\_\_\_\_

3. Find all real values of  $x$  such that  $4 = x + \frac{2 + \frac{3 + \frac{1}{x}}{x + 1}}{x + 1}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**November 1992**

1. Simplify the following to a reduced fraction. Leave answer in factored form.

$$\frac{x + 1}{x^2 - x - 6} - \frac{x + 4}{x^2 - 4x + 3} + \frac{x + 3}{x^2 + x - 2}$$

**Ans.** \_\_\_\_\_

2. A large water tank can be filled by an intake pipe in 12 hours and be emptied by a drain in 15 hours. If the tank is half full and both pipes are opened, how long will it take to fill the tank?

**Ans.** \_\_\_\_\_

3. Find  $A$ ,  $B$ , and  $C$ , if:  $\frac{2x - 3}{(x^2 - 4)(x + 1)} = \frac{A}{x - 2} + \frac{B}{x + 2} + \frac{C}{x + 1}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**November 1993**

1. Solve for  $x$ . Express answer(s) in exact form or to nearest tenth.  $x + 5 = \frac{1}{x+1} - 3x + 1$

**Ans.** \_\_\_\_\_

2. Simplify:  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$

**Ans.** \_\_\_\_\_

3. Find all value(s) of  $x$  such that:  $14x - \frac{3}{x^2 - 4} = \frac{6x}{x+2} - \frac{9}{x-2}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**March 1995**

1. For what values of  $A$ ,  $B$ , and  $C$  does the following equality hold?

$$\frac{2}{x+1} - \frac{Ax^2 + Bx + C}{(x+1)(x^2 - 2x + 3)} = \frac{x-1}{x^2 - 2x + 3}$$

**Ans.** \_\_\_\_\_

2. Luke can do a certain job 5 hours faster than Peter. Luke started the job early one morning. Two hours later Peter came to help. (He needed his beauty sleep.) They finished  $4\frac{4}{5}$  hours after Peter came to help. How long would it take Peter to do the job himself?

**Ans.** \_\_\_\_\_

3. Express in simplest form:  $\frac{3x+7}{x+2} - \frac{x^2-7}{x^2+x-2} + \frac{4x-1}{1-x}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**March 1996**

1. Express the following in simplest form:  $\frac{1 + \frac{1}{x}}{x - \frac{1}{x}}$

**Ans.** \_\_\_\_\_

2. Find the solution(s) for:  $\frac{1}{x^2 + 5x + 6} + \frac{2}{x + 3} = \frac{5x + 5}{3x + 6}$

**Ans.** \_\_\_\_\_

3. Determine  $A$ ,  $B$ , and  $C$  so that:  $\frac{11x^2 - 15x - 72}{x^3 - 9x} = \frac{A}{x + 3} - \frac{B}{x^2 - 3x} + \frac{C}{x}$

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**March 1997**

1. If  $(x - 2a)(x + 4) = x^2 - 7x + 3b$ , find the value of  $b$ .

**Ans.** \_\_\_\_\_

2. Solve for  $y$ :  $\frac{2y - 1}{y + 3} + \frac{1}{y^2 + y - 6} = \frac{y - 4}{2 - y}$

**Ans.** \_\_\_\_\_

3. Sudden Sam's rate on his second trial on the 12 km track was 2 km/min less than the first trial. The total time for both trials was  $3\frac{1}{2}$  minutes. Find the time in minutes for the first trial.

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**March 1998**

1. Simplify completely:  $\frac{20b^2}{a^2 - 4b^2} - \frac{2a + b}{a - 2b} + \frac{3a - b}{a + 2b}$

**Ans.** \_\_\_\_\_

2. Factor completely:  $x^2a^2 - 4a^2y^2 - 9x^2 + 36y^2$

**Ans.** \_\_\_\_\_

3. Ben and Alex can complete half of a certain task in 2 hours and 24 minutes. Ben takes 4 hours longer to complete the task alone than does Alex. Find how long it would take Ben to complete the task by himself.

**Ans.** \_\_\_\_\_

**1. Algebraic Fractions with Factoring**

**March 1999**

1. Simplify:  $\frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12}$

**Ans.** \_\_\_\_\_

2. The denominator of a fraction is one less than twice the numerator. If seven is added to both the numerator and the denominator, the resulting fraction has a value of seven-tenths. Find the original fraction.

**Ans.** \_\_\_\_\_

3. Solve:  $\frac{r + 2}{2r + 1} = \frac{r}{3} + \frac{3}{4r + 2}$

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1989**

1. Find the  $\cos 75^\circ$ . State your answer in simplest radical form.

**Ans.** \_\_\_\_\_

2. Find  $K$  where  $K$  is a rational number in lowest terms if  
 $[4(\sin(30 + A))][\sin(30 - A)] = 1 + K \sin^2 A$ .

**Ans.** \_\_\_\_\_

3. Solve for  $A$  if  $0 \leq A < 2\pi$  :  $\tan A + \sec A + 1 = 0$ .

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1990**

1. Reduce  $\frac{\tan X - \cot X}{\tan X + \cot X}$  to an expression whose trigonometry part involves only  $\sin X$ .

**Ans.** \_\_\_\_\_

2. Solve:  $\arcsin X + \arccos(1 - X) = 0$ .

**Ans.** \_\_\_\_\_

3. Find all values of  $X, 0 \leq X < 2\pi$  so that  $\sin X - \sqrt{3} \cos X = 1$ .

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1992**

1. If a ladder has to be placed such that the angle it makes with the ground is at most  $60^\circ$ , how high can a 30 foot ladder reach?

**Ans.** \_\_\_\_\_

2. Solve:  $\sin(x + 4)\csc(3x^2 + 2) = 1$

**Ans.** \_\_\_\_\_

3. Solve:  $\sin 2x = \sqrt{2} \cos x$  for  $0 \leq x \leq 2\pi$

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1993**

1. Find the exact value of the following. Express answer in simplest form with a rational denominator.  $\frac{\tan 210^\circ - \cos 300^\circ}{\csc 120^\circ - \cot 150^\circ}$

**Ans.** \_\_\_\_\_

2. In obtuse triangle  $WHY$ ,  $HY = 42$  and  $YW = 21\sqrt{6}$ . If  $m\angle W = 45^\circ$ , find all possible measure(s) of the exact value of  $m\angle H$ .

**Ans.** \_\_\_\_\_

3. Solve for  $x$ , where  $0^\circ \leq x < 360^\circ$ . Express answer(s) to the nearest minute.  
 $(\tan x)(1 - \tan x) = 4 \tan x - 3$

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1994**

1. Find the radian measure in terms of  $\pi$ , for which  $\sqrt{3}\csc x + 2 = 0$  and  $0 \leq x \leq 2\pi$ .

**Ans.** \_\_\_\_\_

2. Find all value(s) of  $x$  such that:  $0^\circ \leq x \leq 180^\circ$  and  $2\sin x \cos x + \cos 2x = 1$ .

**Ans.** \_\_\_\_\_

3. Determine all values of  $x$ , where  $0^\circ \leq x < 360^\circ$  for which  $\sin 2x \leq \tan x$ .

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1995**

1. Find all solutions of the equation  $\csc^2 x - 2 = 0$ , where  $0 \leq x < 2\pi$ . Express answer(s) in terms of  $\pi$ .

**Ans.** \_\_\_\_\_

2. Evaluate the expression  $\cos(\text{Arc tan } 1 + \text{Arc cos } x)$ . Express your answer as a single fraction in terms of  $x$ .

**Ans.** \_\_\_\_\_

3. Rewrite the following expression in simplest form so that it is not in fractional form and involves only sine and cosine.  $\frac{\sin^3 y}{1 - \cos y} + \frac{\cos^3 y}{1 + \sin y}$

**Ans.** \_\_\_\_\_



**2. Trigonometric Equations and Identities**

**March 1996**

1. Find all values of  $x$  such that  $0^\circ \leq x < 360^\circ$  for which  $\sin 70^\circ = \cos x$ .

**Ans.** \_\_\_\_\_

2. Express  $\frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x}$  in terms involving only  $\csc$ . Express in simplest possible form.

**Ans.** \_\_\_\_\_

3. Find all solutions for  $x$  which  $0^\circ \leq x < 360^\circ$  and  $2\csc^2 x - 3\cot x - 7 = 0$ . Find angles to the nearest minute.

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1997**

1. Rewrite  $\frac{\csc x + \sec x}{1 + \cot x}$  as an expression with a single trig function in simplest form, without a denominator.

**Ans.** \_\_\_\_\_

2. Find the sum of all solutions to the equation  $2\tan x - 4\sin x \cos x = 0$  for  $0^\circ \leq x < 360^\circ$ .

**Ans.** \_\_\_\_\_

3. Find all values of  $x$  such that  $x^3 + 27i = 0$ .

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1998**

1. Find all  $\theta$  for which  $\sin 2\theta - \cos \theta = 0$ , if  $0^\circ \leq \theta < 360^\circ$ .

**Ans.** \_\_\_\_\_

2. Change  $\frac{\sin \theta}{\csc \theta - 1} + \frac{\sin \theta}{\csc \theta + 1}$  into an expression with only  $\tan \theta$ . Express in simplest form.

**Ans.** \_\_\_\_\_

3. Find all  $x$ , where  $0^\circ \leq x < 360^\circ$  for which  $2 \cos x - 2 \sin x + 4 \sin x \cos x = 1$ .

**Ans.** \_\_\_\_\_

**2. Trigonometric Equations and Identities**

**March 1999**

1. Express  $\frac{1}{\sec x} (\tan x + \cot x)$  as a single trig function without a fraction in simplest form.

**Ans.** \_\_\_\_\_

2. If  $\sin \alpha = \frac{3\sqrt{10}}{10}$  where  $0 < \alpha < \pi/2$ , find  $\sin 2\alpha$ . Express answer in simplest form.

**Ans.** \_\_\_\_\_

3. Solve:  $\cos^4 2\theta - \sin^4 2\theta = 1$ .

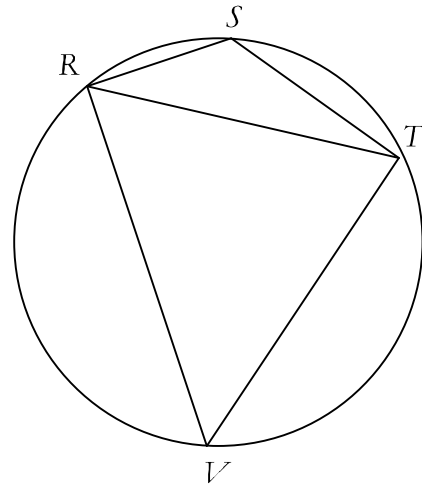
**Ans.** \_\_\_\_\_

3. *Circles and Spheres*

March 1989

1. Given:  $m\angle SRV = 85^\circ$   $m \text{ Arc } RS = 40^\circ$  Find:  $m\angle RTV$

Ans. \_\_\_\_\_



2. A hole 60 cm in diameter is cut in a sheet of plywood, and a sphere 80 cm in diameter is set in this hole. How many centimeters below the surface of the plywood will the sphere extend? Give your answer in simplest radical form.

Ans. \_\_\_\_\_

3. Find the radius, in centimeters, of the inscribed circle of  $\triangle ABC$  if  $m\overline{AC} = 36\text{cm}$ ,  $m\overline{BC} = 15\text{cm}$ ,  $m\overline{AB} = 39\text{cm}$ .

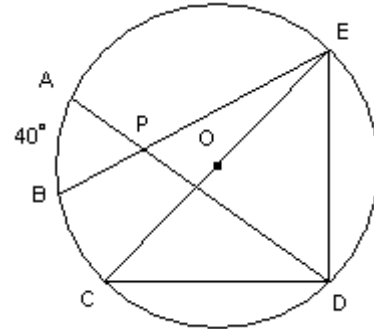
Ans. \_\_\_\_\_

3. *Circles and Spheres*

March 1990

1.  $O$  is the center of the circle.  $m\text{Arc}AB = 40^\circ$   $m\angle EPD = 70^\circ$ . Find  $m\angle CED$ .

Ans. \_\_\_\_\_

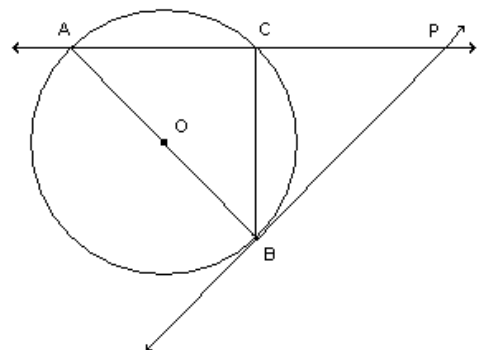


2. Two perpendicular chords intersect within a circle. The segments of one chord are six inches and eight inches, and the segments of the other are twelve inches and four inches. Find the area of the circle in square inches. Give your answer in terms of  $\pi$ .

Ans. \_\_\_\_\_

3.  $\overline{PA}$  is a secant,  $\overline{PB}$  is tangent to circle  $O$  at  $B$ .  $m\overline{PA} = 16$ ,  $m\overline{PB} = 12$ . Find the  $m\overline{BC}$ . State your answer in simplest radical form.

Ans. \_\_\_\_\_

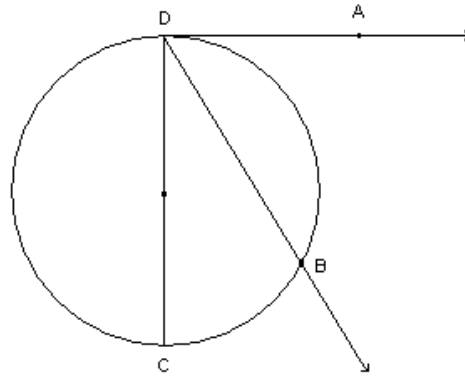


3. *Circles and Spheres*

March 1992

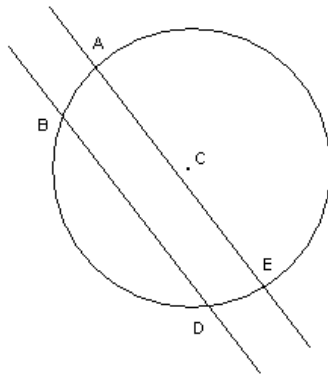
1.  $\overline{AD}$  is tangent to circle  $E$  at  $D$ .  $E$  is the center. If  $m\text{Arc}DB = 70^\circ$ , find  $m\angle EDB$ .

Ans. \_\_\_\_\_



2. Given: The circle centered at  $C$ .  
 $\overline{AE} \parallel \overline{BD}$ ,  $m\text{Arc}ABE = 40^\circ$   
 $m\angle BCE = 90^\circ$ . Find  $m\angle BCD$ .

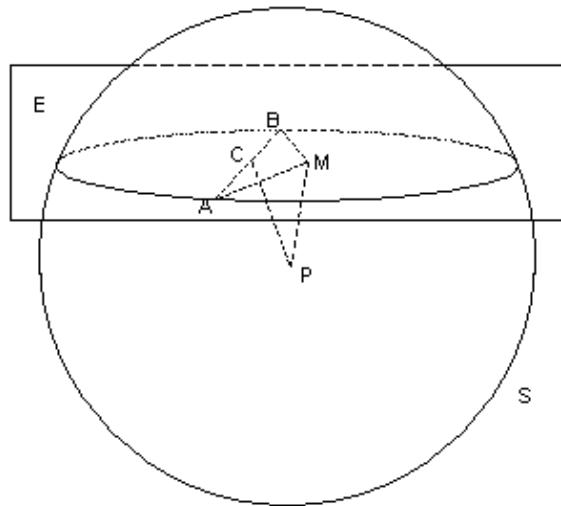
Ans. \_\_\_\_\_



3. Plane  $E$  intersects sphere  $S$ .  $P$  is the center of  $S$ .  $A, B, C, M$  are in  $E$ .  $A$  and  $B$  are in  $S$ .  
 $\overline{PM} \perp \overline{E}$ ,  $\overline{AM} \perp \overline{MB}$ ,  $AC = BC$   
 $AM = PM$ ,  $AB = 5$ .

Find the length of  $\overline{PC}$ .

Ans. \_\_\_\_\_



3. *Circles and Spheres*

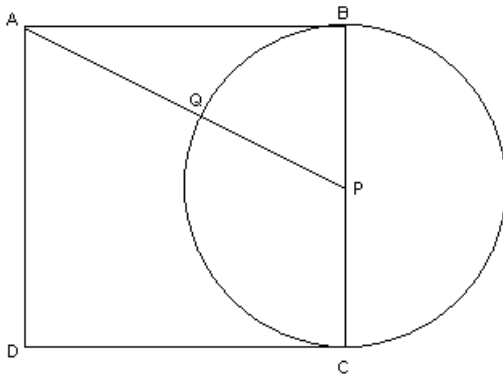
March 1993

1. Two concentric circles have diameters 42 cm and 70 cm. How long is a chord of the larger which is tangent to the smaller?

Ans. \_\_\_\_\_

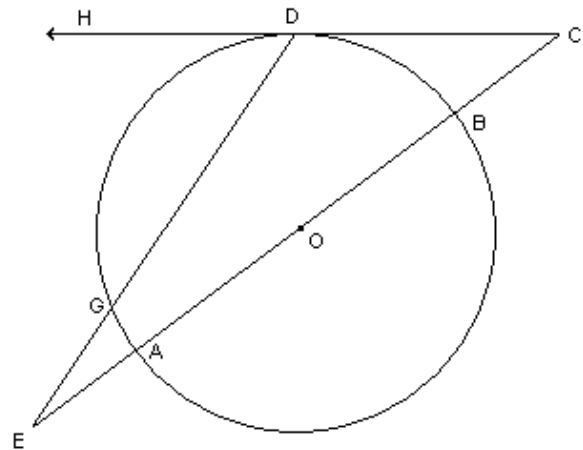
2. Side  $BC$  of square  $ABCD$  is a diameter of circle  $P$ .  $\overline{AP}$  intersects circle  $P$  at  $Q$ . If  $BC$  is 10, find the length of  $AQ$

Ans. \_\_\_\_\_



3.  $\overline{CD}$  is tangent to circle  $O$  at  $D$ . If  $m\angle CDE = 128^\circ$  and  $m\angle E = 20^\circ$ , Find the measure of arc  $AG$ .

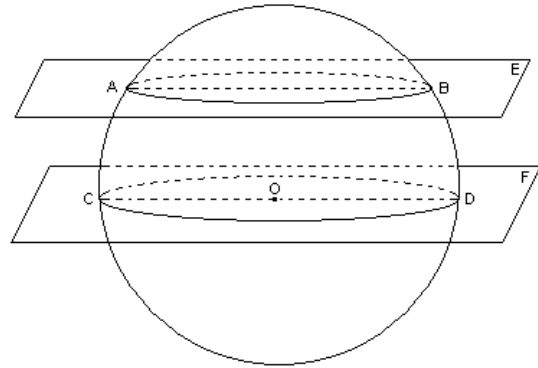
Ans. \_\_\_\_\_



3. *Circles and Spheres*

March 1994

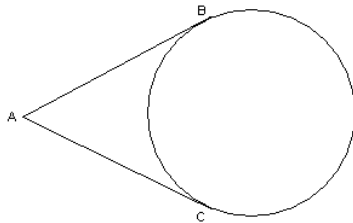
1. Parallel planes  $E$  and  $F$  intersect sphere  $O$ , plane  $F$  through the center  $O$ . Parallel diameters  $AB$  and  $CD$  intersect arcs  $BD$  and  $AC$  on the sphere, each measuring 60 degrees. If the diameter of the sphere is 12 cm long, find the length of  $AB$ .



Ans. \_\_\_\_\_

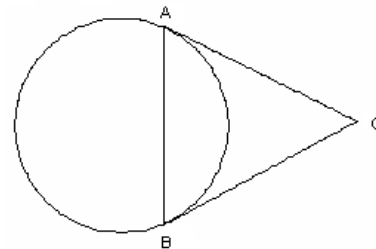
2. The segments  $AB$  and  $AC$  are tangent to the arc of a circle at points  $B$  and  $C$  as shown. If the radius of the circle is 8 and arc  $BC$  has a measure of 240 degrees, find the perimeter of the figure to the nearest 100th.

Ans. \_\_\_\_\_



3. Triangle  $ABC$  is a  $17 - 17 - 16$  triangle, where  $A$  and  $B$  are points of tangency. How far from the circle is point  $C$ ? Give answer in exact form as a fraction or as a decimal to the nearest 10th.

Ans. \_\_\_\_\_



3. *Circles and Spheres*

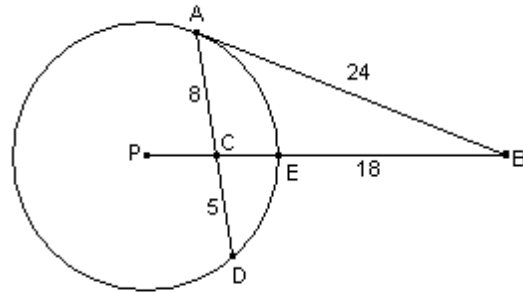
March 1995

- Two spheres, each having a radius of 13 cm. intersect so that their centers are 24 cm apart. Find the radius of their circle of intersection.

Ans. \_\_\_\_\_

- $BA$  is tangent to this circle at  $A$ .  $P$  is the center.  $AC = 8$ ,  $CD = 5$ ,  $BE = 18$  and  $AB = 24$ . Find the length of  $CE$ .

Ans. \_\_\_\_\_



- The diagonals of a 5 by 12 rectangle are drawn. Circles are inscribed in the two triangles formed by the diagonals and the sides of length 5. How far apart are the centers of the circles?

Ans. \_\_\_\_\_

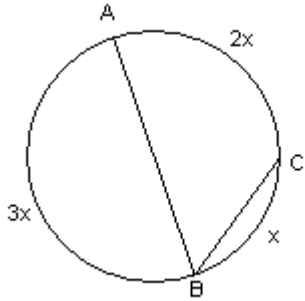


3. *Circles and Spheres*

March 1996

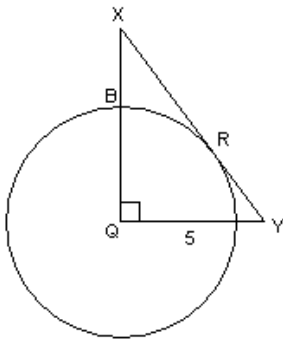
1. If the measure of arc  $BC = x$ , measure of arc  $AC = 2x$ , measure of arc  $AB = 3x$ , find  $m\angle B$ .

Ans. \_\_\_\_\_



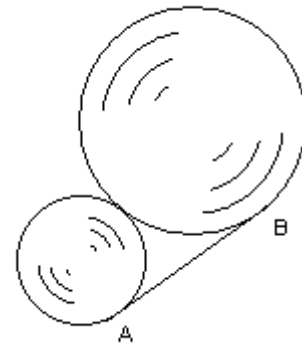
2. In circle  $Q$ ,  $\overline{XY}$  is tangent at  $R$ . The radius measures 4 cm.  $QY = 5$  cm. and  $m\angle Q = 90^\circ$ . Find the exact cm length of  $\overline{BX}$ .

Ans. \_\_\_\_\_



3. The two tangent circles at right have radii of 15 cm and 12 cm. Find the length of segment  $AB$  which is tangent to both circles. (in exact form)

Ans. \_\_\_\_\_



3. *Circles and Spheres*

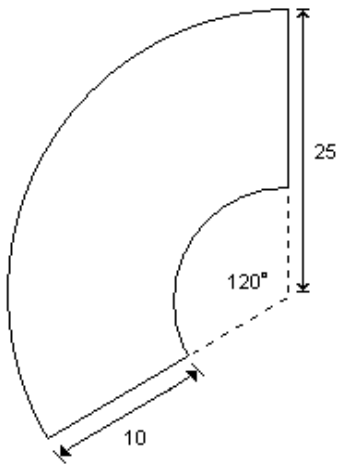
March 1997

1. A secant line and a tangent line intersect at an angle of  $51^\circ$  in the exterior of a circle. If the measures of the arcs intercepted by these lines are in a ratio of 5:2, find the measure of the third arc.

Ans. \_\_\_\_\_

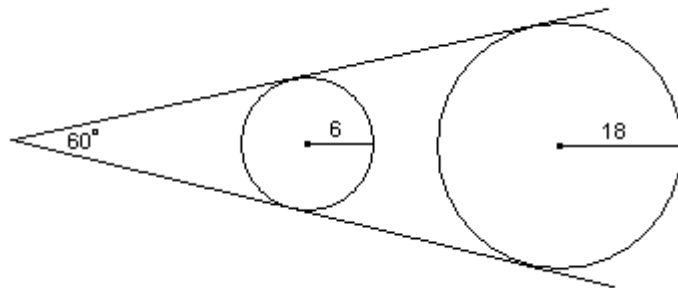
2. Find the area between the 25 in. concentric arcs at right. Give exact answer.

Ans. \_\_\_\_\_



3. The common external tangents to two circular pulleys of radii 6 inches and 18 inches respectively, intersect at an angle of  $60^\circ$ . What is the exact length of the belt required to fit and operate the pulleys?

Ans. \_\_\_\_\_

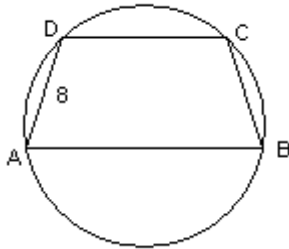


3. *Circles and Spheres*

March 1998

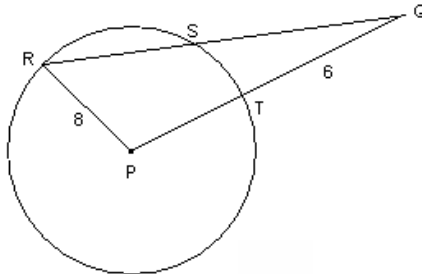
1.  $m\text{Arc}AD = m\text{Arc}DC = m\text{Arc}BC$   
 $m\text{Arc}AB = 3 \cdot m\text{Arc}AD$ ,  $AD = 8$ .  
 Find the perimeter of quadrilateral  $ABCD$ .

Ans. \_\_\_\_\_



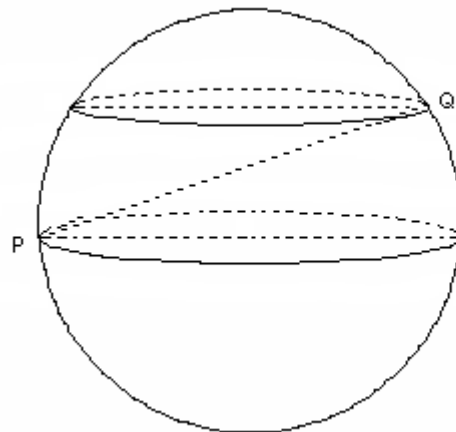
2. Given:  $PR = 8$ ,  $TQ = 6$ , and  $QS = 7$ .  $P$  is the center of the circle. Find the perimeter of  $\triangle PQR$  in exact form.

Ans. \_\_\_\_\_



3. In the sphere, the circles have radii of 9 and 15 and are 4 units apart. Determine the length of  $PQ$  which passes through the line of centers. Express answer in simplest radical form.

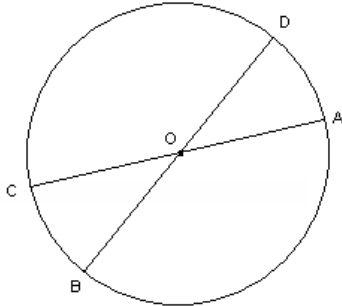
Ans. \_\_\_\_\_



3. *Circles and Spheres*

March 1999

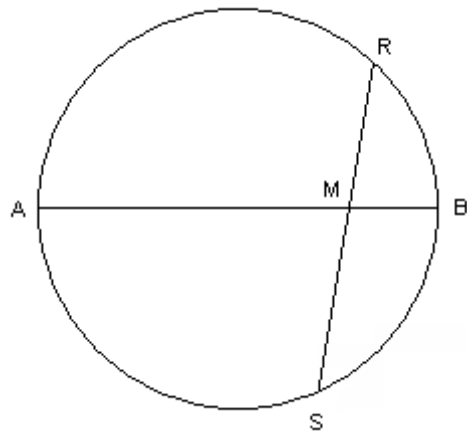
1. Find the length of arc  $AB$  given that the radius  $OA = 3$  inches and  $m\angle DOA = 80^\circ$ . Give exact answer or round to nearest hundredth.



Ans. \_\_\_\_\_

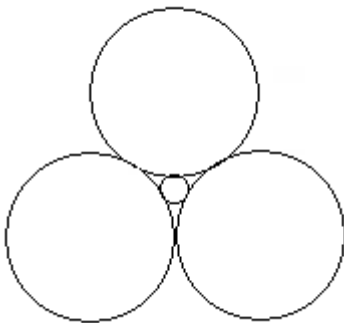
2. The diameter  $AB$  of circle  $O$  intersects chord  $RS$  of length 16 at its midpoint  $M$ . Find  $BM$  if  $AB = 20$ , and  $BM < AM$ .

Ans. \_\_\_\_\_



3. In the diagram, three circles of radius 1 cm are each tangent to each other, and a small circle of radius  $r$  is tangent to each of the large circles. Determine the exact value of  $r$ .

Ans. \_\_\_\_\_



**4. Conics**

**March 1989**

1. Find the equations of the asymptotes of the hyperbola whose equation is  $y^2 - 4x^2 - 24x - 52 = 0$ .

**Ans.** \_\_\_\_\_

2. Find the equation for the locus of all points  $P$  interior to the region bounded by  $x^2 + y^2 = 1$  such that the sum of the squares of the distances from  $P$  to the endpoints of the diameter is 3. State your answer in simplest form.

**Ans.** \_\_\_\_\_

3. Find the coordinates  $(x, y)$  for the points of intersection of the line  $L : \{(x, y) | 3x - 2y = 24\}$  and the parabola whose directrix has the equation  $y = -6$  and whose focus  $F$  is  $(2, 0)$ .

**Ans.** \_\_\_\_\_

**4. Conics**

**March 1990**

1. Find the distance between the foci of the conic whose equation is  $49x^2 + 9y^2 = 441$ . State your answer in simplest radical form.

**Ans.** \_\_\_\_\_

2. Find the equation of the hyperbola whose vertices are  $P(3, -3)$  and  $Q(3, 7)$  if the point  $(9, 15)$  lies on the hyperbola.

**Ans.** \_\_\_\_\_

3. Find the equation of the locus of the centers of all circles with a diameter of 5 units that are externally tangent to the curve whose equation is  $4x^2 + 4y^2 - 16x + 24y + 43 = 0$ . State your answer in  $(x - a)^2 + (y - b)^2 = k$  form.

**Ans.** \_\_\_\_\_

**4. Conics**

**March 1992**

1. Find the radius of the circle  $x^2 + y^2 - 4x - 6y = -9$ .

**Ans.** \_\_\_\_\_

2. Find the equations of the asymptotes of  $4x^2 - 9y^2 + 8x + 90y - 257 = 0$ . Leave answer in  $Ax + By + C = 0$ .

**Ans.** \_\_\_\_\_

3. The foci of the conic are  $(7, 3)$  and  $(7, -13)$ . Find the equation of the conic if its eccentricity is  $\frac{8}{17}$ .

**Ans.** \_\_\_\_\_

**4. Conics**

**March 1993**

1. Find the distance between the foci of the conic section whose equation is  $4x^2 - 9y^2 + 16x + 54y - 101 = 0$ .

**Ans.** \_\_\_\_\_

2. An ellipse is inscribed in a rectangle. The perimeter of the rectangle is 20 and the coordinates of one side of the rectangle are  $(-4, 3)$  and  $(-4, -1)$ . What are the possible equations of the ellipse?

**Ans.** \_\_\_\_\_

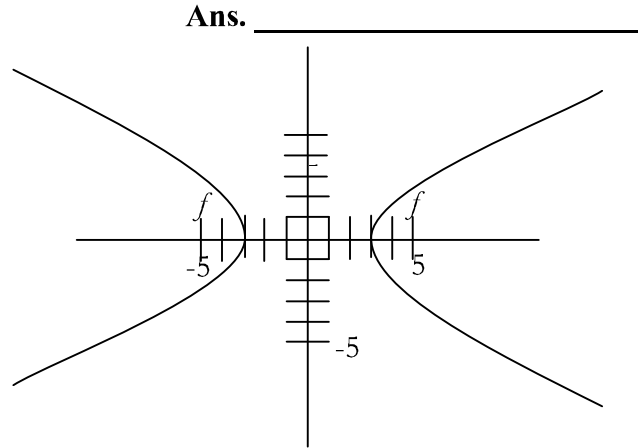
3. Write the equation of the line containing the longest chord of the conic section  $x^2 + y^2 - 8x + 6y - 15 = 0$ , if one point of intersection of the chord and the circle is  $(-2, -1)$ . Express your answer in  $y = mx + b$  form.

**Ans.** \_\_\_\_\_

4. Conics

March 1994

1. The standard form for the equation of all conics is  $Ax^2 + By^2 + Cx + Dy + E = 0$ . Find the sum of  $A + B + C + D + E$  for the conic section below, where  $A, B, C, D, E$  are relatively prime integers.



2. Find the equation of the parabola whose vertex is  $(-2, 3)$  and whose latus rectum has endpoints of  $(-\frac{1}{2}, 0)$  and  $(-\frac{1}{2}, 6)$ .

Ans. \_\_\_\_\_

3. The points of intersection of the equations  $y = x^2, y = -x^2, x = 2y^2$  and  $x = -2y^2$  are used to form a circle with its center. Find the equation of the circle. Express  $R^2$  to the nearest 1000th.

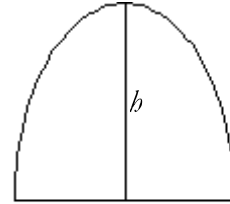
Ans. \_\_\_\_\_

4. Conics

March 1995

1. Each side of the arch at right is the arc of a circle that has radius 12 feet and center at the base of the opposite arch. Determine the height of the h arch, h, at the point of the arch directly above the center of the base. Express in exact form.

Ans. \_\_\_\_\_



2. Find the exact distance between the two points  $P_1$  and  $P_2$ , where  $P_1$  is the focus of the parabola  $y^2 = -6x$  and  $P_2$  is the center of the ellipse  $4x^2 + y^2 - 8y - 20 = 0$ .

Ans. \_\_\_\_\_

3. The endpoints of the conjugate axis of a hyperbola are the vertices of the parabolas  $y = x^2$  and  $y = 16 - x^2$ . The endpoints of the transverse axis are the points of intersection of the parabolas. Find the equation of the hyperbola.

Ans. \_\_\_\_\_



**4. Conics****March 1996**

1. Find the vertices of the hyperbola  $4x^2 - y^2 - 24x - 12y - 16 = 0$ .

**Ans.** \_\_\_\_\_

2. Find the equation of the parabola with axis of symmetry parallel to the y-axis, whose vertex is (3, -1) and contains the point (9, -4). Express answer in either of the normally accepted forms.

**Ans.** \_\_\_\_\_

3. An ellipse which is tangent to both the x and y-axis has an eccentricity of  $\frac{4}{5}$ . Find its equation in the form  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ , if one of its foci is the center of the  $x^2 + y^2 - 2x + 6y - 39 = 0$ .

**Ans.** \_\_\_\_\_**4. Conics****March 1997**

1. Find the distance between the vertices of the conic section whose equation is  $16y^2 - 64y - 9x^2 - 54x = 161$ .

**Ans.** \_\_\_\_\_

2. The ellipse  $4x^2 + 9y^2 = 36$  and the circle  $x^2 + (y-1)^2 = 9$  intersect at three points. Find the coordinates of the intersection with the smallest y-coordinates.

**Ans.** \_\_\_\_\_

3. The latus rectum of the parabola  $P$  is the horizontal axis of the circle  $x^2 + y^2 - 6x + 2y - 6 = 0$ . Find all possible equation for  $P$ .

**Ans.** \_\_\_\_\_

**4. Conics****March 1998**

1. Find the foci of the conic section  $4x^2 - 16y^2 = -256$ .

**Ans.** \_\_\_\_\_

2. Find the focus of the new parabola when the parabola  $x^2 - 8x - 8y = 0$  is rotated on its vertex  $90^\circ$  to open left.

**Ans.** \_\_\_\_\_

3. One of the endpoints of the diameter of a circle is (15, 18). The circle is tangent to the line  $3x + 4y = 17$  at the other endpoint. Determine the equation of the circle in the form  $x^2 + y^2 + ax + by + c = 0$ .

**Ans.** \_\_\_\_\_**4. Conics****March 1999**

1. Find the focus of the parabola  $x^2 - 6x - 4y + 1 = 0$ .

**Ans.** \_\_\_\_\_

2. Find the equation of the ellipse in  $Ax^2 + By^2 + Cx + Dy + E = 0$  form, if its major axis is 10 units long and its foci are (8, 2) and (0, 2).

**Ans.** \_\_\_\_\_

3. Find the locus of a point that moves so that its distance from the point (4, 0) is always twice its distance from the line whose equation is  $x = 1$ .

**Ans.** \_\_\_\_\_

5. *Arithmetic with Statistics*

*March 1997*

1. The ratio of the number of games won to the number of games lost by a certain team is 275%. To the nearest whole percent, what percent of its games did the team lose?

**Ans.** \_\_\_\_\_

2. 160 expressed as the sum of two primes  $a$  and  $b$ . Find  $a$  and  $b$  so that  $b - a$  is as small as possible.

**Ans.** \_\_\_\_\_

3. A communications channel can transmit 150 chunks per second. If each block consists of 512 chunks, how much time will it take to send 60 blocks of data? Give exact answer.

**Ans.** \_\_\_\_\_

5. *Arithmetic with Statistics*

*March 1999*

1. Meg and Peg both play on the basketball team. Meg made 38 out of 50 shots during the first half of the season, and Peg made 30 of 40 attempts. During the second half of the season, Meg made 42 of 70 attempts and Peg made 14 out of 24. Who had the better average during the entire season and by how much more? Give answer as a fraction in simplest form or as a decimal rounded to the nearest 1000<sup>th</sup>.

**Ans.** \_\_\_\_\_

2. A list consisting of seven positive, but not necessarily distinct, integers has an arithmetic mean of 8 and a median of 11. What was the largest value that the range can assume for this list? Where range is a difference between the largest and smallest number.

**Ans.** \_\_\_\_\_

3. Among 100 applicants for a technical position, 10 had never taken a course in chemistry or physics. Seventy-five had taken at least one chemistry course and 83 had taken at least one physics course. How many had taken both a chemistry and a physics course?

**Ans.** \_\_\_\_\_