What Did You Learn?

Key Terms

infinite sequence, *p. 580*
finite sequence, *p. 580*
recursively defined sequence, *p. 582*
n factorial, *p. 582*
summation notation, *p. 584*
series, *p. 585*
arithmetic sequence, *p. 592*
common difference, *p. 592*
geometric sequence, *p. 601*
common ratio, *p. 601*
first differences, *p. 616*
second differences, *p. 616*
binomial coefficients, *p. 619*
Pascal's Triangle, *p. 623*
Fundamental Counting Principle, *p. 628*
sample space, *p. 637*
mutually exclusive events, *p. 641*

Key Concepts

8.1 ■ Find the sum of an infinite sequence

Consider the infinite sequence $a_1, a_2, a_3, \ldots a_i, \ldots$.

1. The sum of the first n terms of the sequence is the finite series or partial sum of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

where i is the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation.

2. The sum of all terms of the infinite sequence is called an infinite series and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_i + \cdots = \sum_{i=1}^{\infty} a_i.$$

8.2 ■ Find the *n*th term and the *n*th partial sum of an arithmetic sequence

1. The nth term of an arithmetic sequence is $a_n = dn + c$, where d is the common difference between consecutive terms and $c = a_1 - d$.

2. The sum of a finite arithmetic sequence with n terms is given by $S_n = (n/2)(a_1 + a_n)$.

8.3 ■ Find the *n*th term and the *n*th partial sum of a geometric sequence

1. The nth term of a geometric sequence is $a_n = a_1 r^{n-1}$, where r is the common ratio of consecutive terms.

2. The sum of a finite geometric sequence $a_1, a_1 r, a_1 r^2, a_1 r^3, a_1 r^4, \ldots, a_1 r^{n-1}$ with common ratio $r \neq 1$ is $S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1 - r^n}{1 - r}\right)$.

8.3 ■ Find the sum of an infinite geometric series

If $|r| < 1$, then the infinite geometric series $a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} + \cdots$ has the sum

$$S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1 - r}.$$

8.4 ■ Use mathematical induction

Let P_n be a statement with the positive integer n. If P_1 is true, and the truth of P_k implies the truth of P_{k+1} for every positive integer k, then P_n must be true for all positive integers n.

8.5 ■ Use the Binomial Theorem to expand a binomial

In the expansion of $(x + y)^n = x^n + nx^{n-1}y + \cdots + {}_nC_r x^{n-r}y^r + \cdots + nxy^{n-1} + y^n$, the coefficient of $x^{n-r}y^r$ is ${}_nC_r = n!/[(n - r)!r!]$.

8.6 ■ Solve counting problems

1. If one event can occur in m_1 different ways and a second event can occur in m_2 different ways, then the number of ways that the two events can occur is $m_1 \cdot m_2$.

2. The number of permutations of n elements is $n!$.

3. The number of permutations of n elements taken r at a time is given by ${}_nP_r = n!/(n - r)!$

4. The number of distinguishable permutations of n objects is given by $n!/(n_1! \cdot n_2! \cdot n_3! \cdots n_k!)$.

5. The number of combinations of n elements taken r at a time is given by ${}_nC_r = n!/[(n - r)!r!]$.

8.7 ■ Find probabilities of events

1. If an event E has $n(E)$ equally likely outcomes and its sample space S has $n(S)$ equally likely outcomes, the probability of event E is $P(E) = n(E)/n(S)$.

2. If A and B are events in the same sample space, the probability of A or B occurring is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

3. If A and B are independent events, the probability that A and B will occur is $P(A \text{ and } B) = P(A) \cdot P(B)$.

4. Let A be an event and let A' be its complement. If the probability of A is $P(A)$, then the probability of the complement is $P(A') = 1 - P(A)$.