Individuals Round 1 (No Calculators) States 2009

3 pts 1. Find the sum of the mean, median and mode for:

3, 5, -2, 18, 6, 2, 3

Ans. ____________________

4 pts 2. Find $x + y + z$ if:

$$x + y - z = 8$$
$$x - y - 2z = 0$$
$$2x + y + z = 12$$

Ans. ____________________

5 pts 3. If $\frac{-1+3i}{-2+i} \div \frac{-1+i}{1+i} = a + bi$, find a + b. **Ans.** ____________________

Individuals Round 2 (No Calculators) States 2009

3 pts 1. In $\triangle ABC$, C is the complement of ½ B and A + C + 20 = B. Find the degree measure of angle A.

Ans. ____________________

4 pts 2. Find the distance from (3, -1) to the line 3x – 4y = 12 in simplest form.

Ans. ________________

5 pts 3. Find the area in terms of π of the ellipse

$$4x^2 + y^2 - 8x - 4y = 28.$$

Ans. ________________

Individuals Round 3 (No Calculators) States 2009

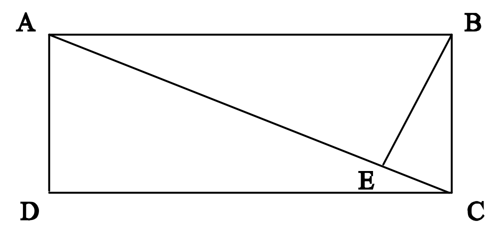
3 pts 1. The volume of a rectangular solid A is 20 cm^3. If each of the dimensions is increased by 20% to form rectangular solid B, determine the volume of B. Express your answer as a decimal.

Ans. ____________________

4 pts 2. If $8^{1/2} \cdot 4^{1/3} = \sqrt[a]{b^c}$ where *a* and *c* are relatively prime integers, and *b* is a prime number, find the sum of a + b + c.

Ans. ____________________

5 pts 3. ABCD is a rectangle with AB = 2BC. $\overline{BE} \perp \overline{AC}$. Find the ratio of the area of $\triangle$BEC to the area of quadrilateral ABCD.



Ans. ____________________________

Individuals Round 4 (No Calculators) States 2009

3 pts 1. Each of the sides of a regular hexagon is 6. Find its area.

Ans. ____________________

4 pts 2. How many distinguishable rearrangements of the letters of the word *gagegele* are possible if all the words begin with the letter "*e*"?

Ans. ____________________

5 pts 3. There are 10 DVD's in a case. Mike reads the front of the case and realizes that 3 of the DVD's are ones he likes. If he selects 4 at random, what is the probability that he gets exactly two of the ones he likes?

Ans. ____________________

Individuals Round 5 (No Calculators) States 2009

3 pts 1. Find the values of x, such that $\frac{3}{2}x - \frac{1}{2}x^2 < 1$.

Ans. ____________________

4 pts 2. Quadrilateral ABCD has vertices A(0, 0), B(1, 1), C(3, 1) and (4, 0). Find k so that $y = x + k$ divides ABCD into two equal areas.

Ans. ____________________

5 pts 3. Find the leading coefficient of a cubic function, with decreasing powers of the variable, that has zeroes of ½ , 2 and 3, which has a y-intercept of 10.

Ans. ____________________

Individuals Round 6 (No Calculators) States 2009

3 pts 1. Evaluate $\sum_{k=3}^{5} \frac{k}{k+2}$. Express in the form $\frac{a}{b}$ in simplest form.

Ans. ____________________

4 pts 2. The first 4 terms of an arithmetic sequence are 2, 5, 8, and 11. Which term is 242?

Ans. ____________________

5 pts 3. Let x, y and z be three positive real numbers whose sum is 1(one). If no one of these numbers is more than twice any other, then find the minimum value for the product xyz.

Ans. ____________________

Team Round 1 (You may use calculators) States 2009

4 pts 1. Find the fractional solution for $\dfrac{2}{2-\dfrac{1}{1-x}}=6$ in simplest form.

Ans. ____________

4 pts 2. Find the range of the function f, if $f(x) = 3|x-2|+4$.

Ans. ____________________________

6 pts 3. Find the vertex of the parabola $y = 2x^2 + \sqrt{3}x + 5$. Give answer as an ordered pair (x, y) in fraction form.

Ans. ____________

6 pts 4. Find the sum of the solutions of $2x^2 + 2x\sqrt{3} - 3 = 2x$.

Ans. _________________

6 pts 5. Find all the angles in radian measure for which $2\sin^2 x + \cos x - 1 = 0$ and $0 \le x < 2\pi$.

Ans. ______________________

8 pts 6. For how many n in {1, 2, 3, … , 100} is the ten's digit of n^2 odd?

Ans. _________________

8 pts 7. In a geometric series of positive terms, the difference between the fifth and the fourth term is 576, and the difference between the second and the first term is 9. What is the sum of the first 5 terms of the series?

Ans. _________________

8 pts 8. In $\triangle ABC$, $m\angle A = 35$, AB = 18, and BC = 14. Find the sum of all possible measures of $\overline{AC}$. Round your answer to nearest thousandth.

Ans. _________________

Team Round 2 (You may use calculators) States 2009

4 pts 1. If A = $\begin{bmatrix} 5 & -1 & 2 \\ -3 & 5 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 5 & 1 \end{bmatrix}$, find AB. **Ans.** ______________

4 pts 2. The sum of all but one of the interior angles of a convex polygon is 2570^o. Find the measure of that angle.

Ans. ____________________

6 pts 3. Find the area of the polygon formed by $2|x| + |y| = 4$.

Ans. ______________________

6 pts 4. If f(x) = $\frac{1}{1-x^2}$ and g(x) = $\sqrt{3x-1}$, find the domain of g(f(x)).

Ans. ____________________________________

6 pts 5. Find all values of x that would make the following true: $||x-4|-2x| < 7$.

Ans. ____

8 pts 6. If $\log_8 3 = p$ and $\log_3 5 = q$, then in terms of p and q find $\log_{10} 5$ with no logs.

Ans. __________________________________

8 pts 7. 8 one inch diameter right cylindrical holes are drilled down through a cube's top face. If the cube is 8 inches high, and the cube is then dipped in paint, what is the area of the surface covered with paint?

Ans. ___________________________

8 pts 8. (a, 2a), (2a, a), (3, 3) form the vertices of a triangle which has a perimeter of $4\sqrt{2}$.
Find all possible values of a.

Ans. ______________________

Seat A Blue Relay States 2009

Find the smallest integer that satisfies $|3x-5|<x$.
Pass back: $A^2 - 2A + 3$ A = Your answer.

Seat B Blue Relay States 2009

The sum of three consecutive even integers is 66. Find the largest of the three.

Pass back: $B - X - 2$ B = Your answer. X = The number you will receive.

Seat C Blue Relay States 2009

The radius of a circle is 4. Find the ratio of the area to the circumference.

Pass back: CX C = Your answer. X = The number you will receive.

Seat D Blue Relay States 2009

Find the largest value of x such that $\frac{1}{x-1} - \frac{x-2}{5} = 1$.

Pass back: $\frac{X}{D}$ D = Your answer. X = The number you will receive.

Seat E Blue Relay States 2009

Let $\log b = 24$, $\log w = .24$ and $\log_b w = x$. Find x.

Pass back: $\frac{X}{E}$ E = Your answer. X = The number you will receive.

Seat A Green Relay States 2009

Find the largest integer that satisfies $|3x-5|<x$.

Pass back: A^2 - 2A +2 A = Your answer.

Seat B Green Relay States 2009

The sum of three consecutive even integers is 66. Find the smallest of these integers.

Pass back: B – X B = Your answer. X = The number you will receive.

Seat C Green Relay States 2009

The radius of a circle is 6. Find the ratio of the area of the circle to its circumference.

Pass back: CX C = Your answer. X = The number you will receive.

Seat D Green Relay States 2009

Find the largest value of x such that $\frac{2}{x-2}-\frac{x-3}{2}=2$.

Pass back: $\frac{X}{D}$ D = Your answer. X = The number you will receive.

Seat E Green Relay States 2009

Log B = 42, log W = .42, $\log_W$ B = x. Find x.

Pass back: E – 4X E = Your answer. X = The number you will receive.

Seat A Pink Relay States 2009

The sum of two numbers is 35. Their difference is 5. What is their product?

Pass back: $\frac{A}{6}$ A = Your answer.

Seat B Pink Relay States 2009

Peter is 13 years older than Mary. In 19 years Mary will be 3/4 of Peter's age. How old will Peter be in 2 years?

Pass back: $\frac{BX}{5}$ B = Your answer. X = The number you will receive.

Seat C Pink Relay States 2009

From point P the secant segment to a circle is 7 units long. The chord made by the secant segment is 21 units long. How long is the tangent segment from P to the circle?

Pass back: $\frac{X}{C}$ C = Your answer. X = The number you will receive.

Seat D Pink Relay States 2009

Find the smallest value of m for which $m^3 + 3m^2 - m - 3 = 0$.

Pass back: 2X – 3D D = Your answer. X = The number you will receive.

Seat E Pink Relay States 2009

Find the largest value for θ, where $90^o \le \theta \le 180^o$ and $2\cos^2\theta - \cos\theta - 1 = 0$.

Pass back: E – 2X E = Your answer. X = The number you will receive.

Seat A Yellow Relay States 2009

The sum of two numbers is 55. Their difference is 5. What is their product?

Pass back: $\frac{A}{10}$ A = Your answer.

Seat B Yellow Relay States 2009

Peter is 13 years older than Mary. In 19 years Mary will be 3/4 as old as Peter. How old will Mary be in 5 years?

Pass back: X + B B = Your answer. X = The number you will receive.

Seat C Yellow Relay States 2009

A tangent segment from point P to a circle is 14 units long. A secant segment to the circle from P is 7 units long. How long is the chord made by the secant segment?

Pass back: X – 4C C = Your answer. X = The number you will receive.

Seat D Yellow Relay States 2009

Find the largest value for *m* such that $m^3 + 3m^2 - m - 3 = 0$.

Pass back: $\frac{X}{D+1}$ D = Your answer. X = The number you will receive.

Seat E Yellow Relay States 2009

Find the largest value for θ, where $180^o \leq \theta < 360^o$ and $2\cos^2\theta - \cos\theta - 1 = 0$.

Pass back: $\frac{E}{X}$ E = Your answer. X = The number you will receive.

Solutions – Individuals Round 1 States 2009

1. mean = 5, mode = 3, median = 3. **Ans. 11**

2. (1) $x + y - z = 8$ Adding (1) and (2): $2x - 3z = 8$. (3) – (1): $x + 2z = 4$.
(2) $x - y - 2z = 0$ Mult. The 2nd of these by -2: $-2x - 4z = -8$. Adding this one to the
(3) $2x + y + z = 12$ other: $-7z = 0$. Thus $z = 0$. $x = 4$ and $y = 4$. **Ans. 8**

3. $\frac{-1+3i}{-2+i} \div \frac{-1+i}{1+i} = \frac{-1+3i}{-2+i} \cdot \frac{1+i}{-1+i} = \frac{-4+2i}{1-3i} \cdot \frac{1+3i}{1+3i} = \frac{-10-10i}{10} = -1-i = a+bi$. **Ans. -2**

Individuals – Round 2

1. (1) $A + B + C = 180$, (2) $C = 90 - \frac{1}{2}B$, (3) $A + C + 20 = B$. Subbing (2) into (3): $A + (90 - \frac{1}{2}B) + 20 = B$ thus (4) $A = 1\frac{1}{2}B - 110$. Subbing (4) and (2) into (1): $(1\frac{1}{2}B - 110) + B + (90 - \frac{1}{2}B) = 180$. Thus $2B = 200$, $B = 100$. C and A = 40. **Ans. 40**

2. Plugging (3, -1) into $3x - 4y = c$, makes $3x - 4y = 13$. Distance = $\frac{|13-12|}{\sqrt{9+16}}$ **Ans. 1/5**

3. $4x^2 + y^2 - 8x - 4y = 28 \Rightarrow 4(x^2 - 2x + 1) + (y^2 - 4y + 4) = 28 + 4 + 4 = 36$
$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{36} = 1$. Area is $ab\pi = 3(6)\pi = 18\pi$. **Ans. 18π**

Individuals – Round 3

1. New volume is $(\frac{6}{5}L)(\frac{6}{5}W)(\frac{6}{5}H) = \frac{216}{125}(20) = \frac{216 \cdot 4}{25} = \frac{216 \cdot 4 \cdot 4}{25 \cdot 4} = \frac{3456}{100}$ **Ans. 34.56**

2. $8^{1/2} \cdot 4^{1/3} = 2^{3/2} \cdot 2^{2/3} = 2^{13/6} = \sqrt[6]{2^{13}} = \sqrt[a]{b^c}$. Then a + b + c = 21. **Ans. 21**

3. Since the three triangles are all similar, and the corresponding hypotenuses of $\triangle ABE$ is twice as long as that of $\triangle BEC$, then the area of $\triangle BEC$ is 4 times the area of $\triangle BEC$. The area of $\triangle ADC$ is the sum of their areas. **Ans. 1/10 or 1:10**

Individuals – Round 4

1. Each of the 6 triangles that make up the hexagon is equilateral. Each side is 6 and each altitude of the triangle or apothem of the hexagon is $3\sqrt{3}$. Area = $\frac{1}{2}(3\sqrt{3})36$. **Ans. $54\sqrt{3}$**

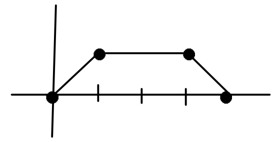
2. $\frac{7!}{2!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{2!3!} = 7 \cdot 6 \cdot 5 \cdot 2 = 420$ **Ans. 420**

3. $\frac{{}_3C_2 \cdot {}_7C_2}{{}_{10}C_4} = \frac{3 \cdot 7 \cdot 3}{(10 \cdot 9 \cdot 8 \cdot 7)/(4 \cdot 3 \cdot 2)} = \frac{3 \cdot 7 \cdot 3}{5 \cdot 3 \cdot 2 \cdot 7} = \frac{3}{10}$ **Ans. $\frac{3}{10}$**

Individuals – Round 5

1. $\frac{3}{2}x - \frac{1}{2}x^2 < 1 \Rightarrow x^2 - 3x + 2 > 0 \Rightarrow (x-2)(x-1) > 0.$ **Ans. x < 1 or x > 2**

2. The line y = x + k is parallel to side AB thus will form a parallelogram. The area of the trapezoid is ½ (1)(4 + 2) = 3. The area of the parallelogram is 1½. Since the height is 1, the base has to be 1½. So the y-intercept has to be the same as the x-intercept. **Ans. 1½ or k = 1½**



3. $f(x) = (2x-1)(x-2)(x-3) = 2x^2 - \ldots - 6$. Since the y-intercept is -6, one would have to multiply each term of the polynomial by $-\frac{10}{6}$ to get a constant of 10. Multiplying the lead coefficient 2 by $-\frac{10}{6}$ yields $-\frac{10}{3}$. **Ans.** $-\frac{10}{3}$

Individuals – Round 6

1. $\frac{3}{5}+\frac{4}{6}+\frac{5}{7} = \frac{208}{105}$. **Ans.** $\frac{208}{105}$

2. $242 = 2 + (n-1)3 \Rightarrow 240 = (n-1)3 \Rightarrow 80 = n - 1$. **Ans. 81**

3. If x is the largest of the three numbers and y is one of the other numbers, then y = ½x would be the smallest that y could be. The other number would also have to be ½x. Then x + ½x + ½x = 1 ➔ 2x = 1, thus x = ½.
The product: $\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{32}$. **Ans.** $\frac{1}{32}$

Team – Round 1

1. $\frac{2}{2-\frac{1}{1-x}} = 6 \Rightarrow \frac{2}{\frac{1-2x}{1-x}} = 6 \Rightarrow \frac{2-2x}{1-2x} = 6 \Rightarrow 2 - 2x = 6 - 12x.\ 10x = 4.$ **Ans. 2/5**

2. The smallest value for f(x) is when x = 2, f(x) will be 4. **Ans. All Reals ≥ 4**

3. The x-coordinate of the vertex is at $-\frac{b}{2a} = -\frac{\sqrt{3}}{4}$. The y-coordinate is:

$$2\left(-\frac{\sqrt{3}}{4}\right)^2 + \sqrt{3}\cdot\left(\frac{-\sqrt{3}}{4}\right) + 5 = \frac{3}{8} - \frac{3}{4} + 5 = 4\frac{5}{8}$$

Ans. $\left(-\frac{\sqrt{3}}{4}, 4\frac{5}{8}\right)$

4. $2x^2 + 2x\sqrt{3} - 3 = 2x$ ➔ $2x^2 + (2\sqrt{3} - 2)x - 3 = 0$.

The sum of the solutions is $\frac{2-2\sqrt{3}}{2} = 1 - \sqrt{3}$. **Ans. 1 - $\sqrt{3}$**

5. $2\sin^2 x + \cos x - 1 = 0$ ➔ $2(1-\cos^2 x) + \cos x - 1 = 0$ ➔ $2\cos^2 x - \cos x - 1 = 0$ ➔ $(2\cos x + 1)(\cos x - 1) = 0$. $\cos x = -\frac{1}{2}$ at 120^o and 240^o. $\cos x = 1$ at 0^o. In radian measure from $0 \le x < 2\pi$, these are 0, $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. **Ans. 0, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$**

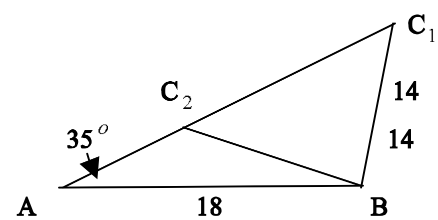
6. $4^2 = 16$, $6^2 = 36$, $14^2 = 196$, $16^2 = 256$. Each number which ends in 4 or 6 will have an odd ten digit. Students using calculators should see the pattern. **Ans. 20**

7. The first 5 terms of the geometric series are *a, ar,* ar^2, ar^3, and ar^4. $Ar^4 - ar^3 = 576$ Thus (1) $a(r^4 - r^3) = 576$. Likewise $ar - a = 9$. Thus $a(r-1) = 9$ or (2) $a = \frac{9}{r-1}$.

Plugging (2) into (1): $\frac{9}{r-1}\cdot(r^4 - r^3) = 576$ ➔ $9r^3 = 576$ ➔ $r^3 = 64$ and $r = 4$. In (2), a = 3.

The first 5 terms are 3, 12, 48, 192, and 768. Their sum is 1023. **Ans. 1023**

8. In the figure, let acute $\angle C_2C_1B = x$, then $\frac{\sin x}{18} = \frac{\sin 35}{14}$ and x = 47.51510. Thus m $\angle AC_2B = 132.484895$, m$\angle ABC_2 = 12.51510$ and m$\angle ABC_1 = 97.484895$.



In $\triangle ABC_2$: $\frac{14}{\sin 35} = \frac{AC_2}{\sin ABC_1}$, thus $AC_2 = 5.289193$.

In $\triangle ABC_1$: $\frac{14}{\sin 35} = \frac{AC_1}{\sin ABC_1}$, thus $AC_1 = 24.2002782$. The sum: **Ans. 29.489**

Team – Round 2

1. $\begin{bmatrix} 5 & -1 & 2 \\ -3 & 5 & 4 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 15-1+10 & 20-2+2 \\ -9+5+20 & -12+10+4 \end{bmatrix} = \begin{bmatrix} 24 & 20 \\ 16 & 2 \end{bmatrix}$. **Ans.** $\begin{bmatrix} 24 & 20 \\ 16 & 2 \end{bmatrix}$

2. 2570/180 = 14 remainder 50. 180 – 50 = 130. **Ans. 180 or 180^o**

3. If x = 0, then y = 4 or -4. If y = 0, then x = 2 or -2. Area is ½ the product of the diagonals since they are perpendicular. Area = ½ (8)(4) = 16. **Ans. 16**

4. $g(f(x)) = \sqrt{3\left(\frac{1}{1-x^2}\right)-1} = \sqrt{\frac{3-(1-x^2)}{1-x^2}} = \sqrt{\frac{2+x^2}{1-x^2}}$. The numerator is always (+). The denominator is (+) when $-1 < x < 1$. **Ans. -1 < all reals < 1**

5. $||x-4|-2x| < 7$: Either $|x-4-2x|<7$ in which case (1) $|-x-4|<7$ or $|4-x-2x|<7$ in which case (2) $|4-3x|<7$. Finding critical points: in (1): –x – 4 = 7 so x = -11, or x + 4 = 7, so x = 3. In (2): 4 – 3x = 7 so x = -1, or 3x – 4 = 7 so x = $3\frac{2}{3}$. –o------o▬▬●▬▬o—
Plugging in interval points and checking the original   -11 -1 3 $3\frac{2}{3}$
Inequality: - 12 ➔ $|16+24|<7$, no -2 ➔ $|6+4|<7$, no
0 ➔ $|4|<7$, yes $3\frac{1}{3}$ ➔ $|\frac{2}{3}-6\frac{2}{3}|<7$, yes 4 ➔ $|0-8|<7$, no 3 ➔ $|1-6|<7$, yes

Ans. 1 < x < $3\frac{2}{3}$

6. $\log_8 3 = p$ ➔ $\frac{\log 3}{\log 8} = p$ ➔ (1) $\log_{10} 3 = p \log_{10} 8$. $\log_3 5 = q$ ➔ (2) $\frac{\log 5}{\log 3} = q$ ➔ $\log_{10} 5 = q \log_{10} 3$ ➔ $\log_{10} 5 = qp \log_{10} 8 = 3pq \log_{10} 2 = 3pq \log_{10} 10/5 = 3pq(\log_{10} - \log_{10} 5)$.
$\text{Log}_{10} 5 = 3pq - 3pq \log_{10} 5$ ➔ $\log_{10} 5(1 + pq) = 3pq$. So $\log_{10} 5 = \frac{3pq}{1+3pq}$. **Ans.** $\frac{3pq}{1+3pq}$

7. Each circle has an area of $\frac{1}{4}\pi$. 8 circles taken away from the upper and lower base areas: Area = $8 \cdot 8 \cdot 6$ - 16 circles = 384 – 4π. Each cylinder's lateral area is $\pi(8) = 8\pi$. All 8 will make 64π. Total area: 384 – 4π + 64π = 384 + 60π. **Ans. 384 + 60π**

8. The distance fro (a, 2a) to (2a, a) is $\sqrt{(a-2a)^2+(2a-2)^2} = a\sqrt{2}$. The distance from (3, 3) to the other two points is the same, thus: $2\sqrt{(a-3)^2+(2a-3)^2} = 2\sqrt{5a^2-18a+18}$. Thus:
$a\sqrt{2} + 2\sqrt{5a^2-18a+18} = 4\sqrt{2}$ ➔ $2\sqrt{5a^2-18a+18} = (4-a)\sqrt{2}$.
$4(5a^2 - 18a + 18 = 2(a^2 - 8a + 16)$ ➔ $20a^2 - 72a + 72 = 2a^2 - 16a + 32$ ➔
$18a^2 - 56a + 40 = 0$ ➔ $9a^2 - 28a + 20 = 0$ ➔ $(9a – 10)(a – 2) = 0$. **Ans. 2 or 10/9**

Blue Relay

Seat A Since $|3x-5|<x$, then critical points are at (1) 3x – 5 = x or (2) 5 – 3x = x.
In (1): x = $2\frac{1}{2}$. In (2): x = $1\frac{1}{4}$. Only values that satisfy are $1\frac{1}{4}$ < x < $2\frac{1}{2}$. A = 2.
Pass back: $A^2 - 2A + 3 = 4 – 4 + 3 = 3$. **Ans. A = 2 Pass: 3**

Seat B x + x + 2 + x + 4 = 66. ➔ 3x + 6 = 66. x = 20. x = 4 = 24.
Pass back: B – X – 2 ➔ 24 – 3 – 2 = 19. **Ans: B = 24 Pass: 19**

Seat C $\frac{\pi r^2}{2\pi r} = \frac{16\pi}{8\pi} = 2$. Pass: CX = (2)(19) = 38. **Ans: C = 19 Pass: 38**

Seat D $\frac{1}{x-1} - \frac{x-2}{5} = 1$ ➔ 5 – (x – 1)(x – 1) = 5(x – 1) ➔ $5 - x^2 + 3x - 2 = 5x - 5$.
$0 = x^2 + 2x - 8 = (x + 4)(x - 2)$. x = -4 or 2. Pass: $\frac{X}{D} = \frac{38}{2} = 19$. **Ans: D = 2 Pass: 19**

Seat E log w = .24, log b = 24 ➔ $\frac{\log w}{\log b} = \frac{.24}{24}$ ➔ $\log_b w = .01$. **Ans: E = .01 Pass: 1900**

Green Relay

Seat A From Blue A: A = 2. Pass back: $A^2 - 2A + 2 = 4 - 4 + 2 = 2$. **Ans. A = 2 Pass: 2**

Seat B From Blue B: B = 20. Pass back: B – X = 20 – 2 = 18. **Ans: B = 20 Pass: 18**

Seat C $\frac{\pi r^2}{2\pi r} = \frac{36\pi}{12\pi} = 3$. Pass back: CX = (3)(18) = 54. **Ans: 3 Pass: 54**

Seat D $\frac{2}{x-2} - \frac{x-3}{2} = 2$ ➔ 4 – (x – 2)(x – 3) = 4(x – 2) ➔ 4 – (x^2 - 5x – 6) = 4x – 8 ➔

0 = x^2 - x – 6 = (x – 3)(x + 2), x = 3 or -2. D = 3. Pass back: $\frac{X}{D} = \frac{54}{3} = 18$.

Ans: D = 3 Pass: 18

Seat E Reciprocal of Blue E: E = 100. Pass back: E – 4X = 100 – 4(18) = 28.

Ans: E = 100 Pass: 28

Pink Relay

Seat A (1) x + y = 35 (2) x – y = 5. (1) + (2): 2x = 40, x = 20. y = 15. xy = 300 = A

Pass back: $\frac{A}{6} = \frac{300}{6} = 50$. **Ans: A = 300 Pass: 50**

Seat B ¾ (x + 13 + 19) = x + 19 ➔ 3x + 39 + 57 = 4x + 76 ➔ 20 = x. Peter is 33. In 2 years 35. B = 35. Pass back: $\frac{BX}{5} = \frac{35 \cdot 50}{5} = 350$ **Ans: B = 35 Pass: 350**

Seat C 7(28) = $\tan^2$ ➔ tan = 14. C = 14. Pass back: $\frac{X}{C} = \frac{350}{14} = 25$. C = 25.

Ans: C = 14 Pass: 25

Seat D $m^3 + 3m^2 - m - 3 = 0$ ➔ $m^2(m + 3) - (m + 3) = 0$ ➔ $(m^2 - 1)(m + 3) = 0$ ➔ (m – 1)(m + 1)(m + 3) = 0. D = -3. Pass back: 2X – 3D = 2(25) – 3(-3) = 59.

Ans: D = -3 Pass: 59

Seat E $2\cos^2\theta - \cos\theta - 1 = 0$ ➔ $(2\cos\theta + 1)(\cos\theta - 1) = 0$. $\cos\theta = -1/2$ or 1.
$\theta = 120^\circ$. E = 120. Pass back: E – 2X = 120 – 118 = 2 **Ans: E = 120 Pass: 2**

Yellow Relay

Seat A (1) x + y = 55 (2) x – y = 5. (1) + (2): 2x = 60, x = 30 and y = 25. xy = 750.
A = 750. Pass back: $\frac{A}{10} = \frac{750}{10} = 75$. **Ans: A = 750 Pass: 75**

Seat B ¾ (x + 13 + 19) = x + 19 ➔ 3x + 39 + 57 = 4x + 76 ➔ 20 = x, Mary's age. In 5 years she will be 25. B = 25. Pass back: X + B = 75 + 25 = 100. **Ans: B = 25 Pass: 100**

Seat C From pink C: C = 21. Pass back: X – 4C = 100 – 84 = 16. **Ans: C =21 Pass: 16**

Seat D From Pink D: D = 1. Pass back: $\frac{X}{D+1}=\frac{16}{1+1}$ = 8. **Ans: D = 1 Pass: 8**

Seat E From Pink E: E = 240. Pass back: $\frac{E}{X}=\frac{240}{8}$ = 30. **Ans: E = 240 Pass: 30**

Answer Sheet – States 2009

Individuals Round 1
1. 11
2. 8
3. -2

Individuals Round 2
1. 40 or 40^o
2. 1/5 or .2
3. 18π

Individuals Round 3
1. 34.56 or 34.560
2. 21
3. 1:10 or 1/10 or 1 to 10

Individuals Round 4
1. $54\sqrt{3}$ or $54\sqrt{3}$ units^2
   or $54\sqrt{3}\ u^2$
2. 420
3. 3/10 or 3 out of 10
   or .3 or 30%

Individuals Round 5
1. x < 1 or x > 2
   or $(-\infty,1) \cup (2, \infty)$
2. – 3/2 or -1½ or -1.5
3. – 10/3 or – 3⅓

Individuals Round 6
1. 208/105
2. 81 or 81st or term 81
3. 1/32 or 0.03125

Team Round 1
1. 2/5
2. All Reals $\geq$ 4 $[4, \infty)$
3. $\left(-\frac{\sqrt{3}}{4}, 4\,5/8\right)$
4. $1 - \sqrt{3}$ -.7321
5. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ 0, 2.0944, 4.1888
6. 20
7. 1023
8. 29.489

Team Round 2
1. $\begin{bmatrix} 24 & 20 \\ 16 & 2 \end{bmatrix}$
2. 130 or 130^o
3. 16
4. -1 < all reals <1
5. $-1 < x < 3\,2/3$
6. $\frac{3pq}{1+3pq}$
7. $384 + 60\pi$ 572.4956
8. 10/9 1.1111 1 1/9

Blue Relay	Green Relay	Pink Relay	Yellow Relay