Individuals Round 1 States 2010 (No Calculators)

3 pts 1. The State Math Meet is sponsored by MAML, the Maine Association of Math Leagues. How many of the possible arrangements of the four letters in MAML do not have the two M's next to each other?

Ans. _____

4 pts 2. Find the value of constant k so that the solution to $|k - 2x| < 7$ is $3 < x < 10$.

Ans. _____

5 pts 3. Maine is divided into 16 counties. The towns of Southwest Harbor, Solon, Upton, Baileyville, and Millinocket are in all different counties. Find the probability that 5 different counties selected at random will contain at least 2 of the 5 named towns. Express answer as a fraction reduced to lowest form.

Ans. _____

Individuals Round 2 States 2010 (No Calculators)

3 pts 1. How many whole numbers less than or equal to 100 satisfy:

$$(x - 1)^2 = (1 - x)^2$$

Ans. _____

4 pts 2. In right triangle ABC the tangent of angle A is $\frac{24}{7}$. What is the cosine of the complement of angle A?

Ans. _____

5 pts 3. A circle with center on the y-axis passes through the points (-7, -6) and (20, 3). The circle intersects the positive x-axis at $(a, 0)$. Find a.

Ans. _____

Individuals Round 3 States 2010 (No Calculators)

3 pts 1. If P = iV and $V = i$R, find P, if $i = 5$ and R = 22.

Ans. _____

4 pts 2. If $\cos^2 \theta = \frac{3}{4}$, find the maximum possible value of $1 - \sin \theta$.

Ans. _____

5 pts 3. The sides of a triangle are 17, 25, and 28. Find the length of the altitude to the longest side.

Ans. _____

Individuals Round 4 States 2010 (No Calculators)

3 pts 1. Find the probability a person turning 17 years old in 2010 will have a birthday in April. Assume the same number of people are born each day, and express your answer as a common fraction in simplest form.

Ans. _____

4 pts 2. What integer is not in the domain of the function

$$f(x) = \frac{\sqrt{x^2 - 2x + 1}}{x^4 + 4x^3 + 6x^2 + 4x + 1}$$

Ans. _____

5 pts 3. Find a in terms of b and c in simplest form, if

$$5a^2 - 20b^2 = 12abc.$$

Ans. _____

Individuals Round 5 States 2010 (No Calculators)

3 pts 1. If an inch worm crawls an inch 8 times a minute, how many hours will it take the inch worm to travel half a mile?

Ans. _____

4 pts 2. An infinite geometric series has a first term of 8 and a sum of 6. Find the 4^{th} term of the series.

Ans. _____

5 pts 3. In conducting a recent study, a survey firm divided Maine into two parts. The study found that 11% of the teenagers in Southern Maine owned a pair of mittens and 23% of the teenagers in Northern Maine owned a pair of mittens. Statewide, 17.36% owned mittens. What percent of Maine's teenagers live in what the survey firm called Northern Maine?

Ans. _____

Individuals Round 6 States 2010 (No Calculators)

3 pts 1. Sara's favorite place to sit and rest while climbing the Beachcroft Trail to Heugenot Head in Acadia National Park is by a pine tree on a mountainside right at the 14^{th} switchback. A "switchback" is a place marked on the trail where a climber can switch and go back down the mountain. When coming back down the mountain from the top her favorite spot is the 7^{th} switchback from the top. How many switchbacks are on the trail?

Ans. _____

4 pts 2. How many distinct equilateral triangles can be formed by drawing three segments between vertices of a cube?

Ans. _____

5 pts 3. Suppose $\frac{1}{x}$ is added to $\frac{1}{100}$ and the sum is expressed as a fully reduced common fraction. For how many values of x selected from the set {1, 2, 3, 4, … , 99} will the denominator of the sum equal 100x.

Ans. _____

Team Round 1 States 2010 (You may use calculators)

4 pts 1. Trig McMaml had a life span of 2.7 billion seconds.
What was Trig's age on his last birthday? **Ans.** _____

4 pts 2. Find the greatest integer x, such that $x^7 < 100{,}000{,}000$. **Ans.** _____

6 pts 3. How many natural numbers when written as numerals in base 8 and as numerals in base 12 have exactly three digits in both of them? **Ans.** _____

6 pts 4. The equation $x^4 - 6x^3 + tx^2 + wx - 40 = 0$ has two roots $4 + \sqrt{11}$ and $4 - \sqrt{11}$. Find the integer value of $t + w$.

Ans. _____

6 pts 5. If $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = \frac{1}{a}$, find a in simplest form. **Ans.** _____

8 pts 6. Find the sum of the positive factors of 720. **Ans.** _____

8 pts 7. A right triangle has a hypotenuse of 8 and altitude to the hypotenuse of 2. Find the perimeter of the triangle.

Ans. _____

8 pts 8. Determine which of the statements below are always true. Add the points given after each of the statements. Write the sum as your answer.

- $\sec\theta = \frac{1}{\sin\theta}$ (1 pt)
- $\sqrt{360}$ is a rational number (2 pts)
- $\log\left(\frac{1}{x}\right) = -\log x$ (4 pts)
- The number of primes is infinite (8 pts)
- $i^{-1} = -i$ (16 pts
- $\pi > e$ (32 pts)
- $\log x < x$ for all $x > 0$ (64 pts)
- 2010 has 32 integer factors (128 pts)
- $\sqrt{A} < A$ for all $A > 0$ (256 pts)
- $f(x) = \frac{4}{x}$ is its own inverse (512 pts)

Ans. _____

Team Round 2 States 2010 (You may use calculators)

4 pts 1. Both diagonals are drawn in a 10 by 16 rectangle, forming 4 non-overlapping triangles. Find the largest area of any of these triangles.

Ans. _____

4 pts 2. An 850 foot guy wire is attached to the top of an 840 foot radio tower. How many feet at the most away from the base of the tower will the wire be able to touch the ground? Assume the ground is level.

Ans. _____

6 pts 3. The first two entries in a particular row of Pascal's Triangle are 1 and 2010. Find the next entry in this row.

Ans. _____

6 pts 4. In the mini-Sudoku puzzle shown, each row of 4, each column of 4, and each of the 2 by 2 boxes must contain all the numbers 1, 2, 3, 4. The puzzle doesn't have enough information for a unique solution. Find the sum of all possible entries into the box labeled "X" that are part of a proper solution.

1			
			2
		3	
	X		

Ans. _____

6 pts 5. Find the probability the 8^{th} power of a randomly selected whole number will have a unit's digit of 1.

Ans. _____

8 pts 6. Consider the set of all 4^{th} degree polynomials with real number coefficients. To each polynomial, assign an ordered pair of whole numbers (r, c), where r = the number of distinct real roots of the polynomial and c = the number of distinct complex roots. How many unique (r, c) ordered pairs will be assigned?

Ans. _____

8 pts 7. A right triangle has hypotenuse 85 and integer leg lengths. Find the sum of all possible lengths for the shorter leg of this triangle.

Ans. _____

8 pts 8. Mr. Math runs a whole number Shop in which he sells primes for \$3 apiece, composites for \$2 apiece, and the numbers 0 and 1 for \$1 apiece. Elmer buys a set of distinct whole numbers for \$25. What is the least possible sum of the numbers in Elmer's set?

Ans. _____

Seat A Blue Relay States 2010

Solve for x: $\left\|4-x\right|-\left|x+2\right\| = 0$

Pass back: $A^2 + 2A + 1$ A = Your answer.

Seat B Blue Relay States 2010

Paint goes on sale next week at 40% off the regular price. How many gallons can be purchased next week for what 6 gallons would cost this week?

Pass back: $\frac{2B}{X}$ B = Your answer. X = The number you will receive.

Seat C Blue Relay States 2010

The area of rectangle R is 35 and the area of rectangle Q is 42. If the width of Q is 4/3 times the width of R, by what per cent is the length of Q less than the length of R? (Your answer C is amount of the per cent; that is C%.

Pass back: 2CX C = Your answer. X = The number you will receive.

Seat D Blue Relay States 2010

Solve for x: $\frac{25}{2}x^2 - 10x + 2 = 0$

Pass back: DX D = Your answer. X = The number you will receive.

Seat E Blue Relay States 2010

The first three terms of a sequence are -211, -202, and -193. Which term is 356?

Pass in: E – X E = Your answer. X = The number you will receive.

Seat A Green Relay States 2010

Solve for x: $\|x+7|-|1-x\| = 0$

Pass back: $A^2 + 4A + 4$ A = Your answer.

Seat B Green Relay States 2010

Paint goes on sale next week at 25% off the regular price. How many gallons can be purchased next week for what 6 gallons cost this week?

Pass back: $\frac{2B}{X+1}$ B = Your answer. X = The number you will receive.

Seat C Green Relay States 2010

The area of a rectangle R is 45 and the area of rectangle Q is 54. If the width of Q is 3/2 times the width of R, by what per cent is the length of Q less than the length of R? (Your answer C is the amount of the per cent; that is C%).

Pass back: CX C = Your answer. X = The number you will receive.

Seat D Green Relay States 2010

Solve for x: $\frac{9}{2}x^2 - 6x + 2 = 0$

Pass back: $\frac{X}{D}$ D = Your answer. X = The number you will receive.

Seat E Green Relay States 2010

The first three terms of a sequence are -307, -296, and -285. Which term in the sequence is 210?

Pass in: $\frac{X}{E}$ E = Your answer. X = The number you will receive.

Seat A Pink Relay States 2010

Solve for q: $\frac{2-7q}{4} = \frac{7-2q}{4}$

Pass back: A^4 A = your answer.

Seat B Pink Relay States 2010

There are 3 teaspoons in a tablespoon, 4 tablespoons in a quarter cup, 2 cups in a pint, 2 pints in a quart, 4 quarts in a gallon, and 5280 feet in a mile. Elmer's street rod gets 8 miles per gallon of gas. How many feet can Elmer drive on a teaspoon of gas?

Pass back: B – X B = Your answer. X = The number you will receive.

Seat C Pink Relay States 2010

The surface area of a cube is 36. Find the length of the longest interior diagonal.

Pass back: $\frac{X}{C^2}$ C = Your answer. X = The number you will receive.

Seat D Pink Relay States 2010

There are two values of x satisfying the following proportion: x is to 40 as 45 is to x. Find the larger of these two values.

Pass back: $\frac{D^2}{X^2}$ D = your answer. X = the number you will receive.

Seat E Pink Relay States 2010

How many odd 7-digit numbers, each less than 3,000,000, can be made from the set consisting of {1, 2, 3, 4, 5, 6, 7}, if the digits are distinct?

Pass in: $\frac{X}{E}$ E = Your answer. X = The number you will receive.

Seat A Yellow Relay States 2010

Solve for q: $\frac{5-3q}{53}=\frac{15-13q}{53}$

Pass back: A^4 - 1 A = Your answer.

Seat B Yellow Relay States 2010

There are 3 teaspoons in a tablespoon, 4 tablespoons in a quarter cup, 2 cups in a pint, 2 pints in a quart, 4 quarts in a gallon, and 5280 feet in a mile. Alva's street rod gets 16 miles per gallon of gas. How many feet can Alva drive on a teaspoon of gas?

Pass back: B – X + 1 B = Your answer. X = The number you will receive.

Seat C Yellow Relay States 2010

The surface area of a cube is 72. Find the length of the longest interior diagonal.

Pass back: $\frac{2X}{C}$ C = Your answer. X = The number you will receive.

Seat D Yellow Relay States 2010

There are two values of x satisfying the following proportion: x is to 36 as 49 is to x. Find the lesser of these two values.

Pass back: X + D D = Your answer. X = The number you will receive.

Seat E Yellow Relay States 2010

How many even 7-digit numbers, each less than 3,000,000, can be formed from the set consisting of {1, 2, 3, 4, 5, 6, 7}, if the digits are distinct?

Pass in: $\frac{E}{|X|}$ E = Your answer. X = The number you will receive.

Solutions – Individuals Round 1

1. The M's can be separated if they are in positions 1 and 3, 1 and 4, or 2 and 4. The A and L can be in 2 orders for each. **Ans. 6**

2. If $|k-2x| < 7$, then -7 < k – 2x , 7 ➔ -7 – k < -2x < 7 – k ➔ $\frac{k+7}{2} > x > \frac{k-7}{2}$.

So $\frac{k+7}{2} = 10$ or $\frac{k-7}{2} = 3$. Both imply that k = 13. **Ans. 13**

3. Probability 0 towns: $\frac{{}_{11}C_5}{{}_{16}C_5} = \frac{462}{4368}$. Probability 1 town: $\frac{{}_{11}C_4 \cdot {}_5C_1}{{}_{16}C_5} = \frac{330 \cdot 5}{4368} = \frac{1650}{4368}$.

Probability 2 or more towns: 1 - $\frac{462+1650}{4368} = \frac{2256}{4368} = \frac{141}{273} = \frac{47}{91}$. **Ans.** $\frac{47}{91}$

Individuals Round 2

1. Since all numbers work, there are 101 whole numbers less or equal to 100. **Ans. 101**

2. By the Pythagorean Thm., the hypotenuse is 25. The cosine of the complement of A would be he same as the sine of A, which is 24/25.

Ans. 24/25

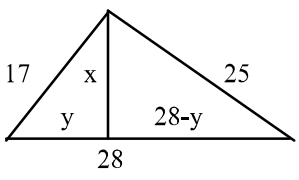
3. The perpendicular bisector of any chord passes through the center. The midpoint of the chord is $(\frac{13}{2}, -\frac{3}{2})$ and the slope is 1/3, so the center lies on $y + \frac{3}{2} = -3\left(x - \frac{13}{2}\right)$. If x = 0, then y = 18, so the center is at (0, 18). The distance to either point is 25, so the equation of the circle is $x^2 + (y-18)^2 = 25^2$. For y = 0, $x^2 = 625 - 324 = 301$. **Ans.** $\sqrt{301}$

Individuals Round 3

1. $V = (5)(22) = 110$. P = (5)(110) = 550. **Ans. 550**

2. $\cos^2\theta = \frac{3}{4}$ ➔ $\sin^2\theta = \frac{1}{4}$ ➔ $\sin\theta = \pm\frac{1}{2}$. 1 – (- ½) = 1½ . **Ans. 1½**

3. Let the altitude to the longest side be x as in the figure.
Then (1) $x^2 + y^2 = 289$ and (2) $x^2 + (28-y)^2 = 625$.
In (2): $x^2 + 784 - 56y + y^2 = 625$. Subtracting (1) from (2):
784 – 56y = 336. 448 = 56y. ➔ y = 8. Thus x = 15.



Ans. 15

Individuals Round 4

1. There are 30 days in April and 2010 is not a leap year, thus $\frac{30}{365} = \frac{6}{73}$. **Ans.** $\frac{6}{73}$

2. Since the numerator is $(x-1)^2$ it is never negative. The denominator factors into $(x+1)^4$. When x = -1, the denominator is zero. **Ans. -1**

3. $5a^2 - 20b^2 = 12abc$ ➔ $5a^2 - 12abc - 20b^2 = 0$. By the quadratic formula:

$$a = \frac{12bc \pm \sqrt{144b^2c^2 - 4(5)(-20b^2)}}{10} = \frac{12bc \pm 4b\sqrt{9c^2+25}}{10} =$$ **Ans.** $\frac{6bc \pm 2b\sqrt{9c^2+25}}{5}$

Individuals Round 5

1. 8 in/min = 480 in/hr = 40 ft/hr. 2640 ft ÷ 40 ft/hr = 66 hours. **Ans. 66 hr**

2. $\frac{8}{1-r} = 6$ ➔ 8 = 6 – 6r. r = -1/3. 4th term: $8(-1/3)^3 = -8/27$. **Ans. -8/27**

3. Let N = the fraction of the teenagers living in Northern Maine. Then using the weight averaging principle, which is the formula for doing mixture/solution word problems:
(.23)N + (.11)(1 – N) = .1736(1). ➔ 2300N + 1100 – 1100N = 1736 ➔ 1200N = 636.
$N = \frac{636}{1200} = \frac{106}{200} = \frac{53}{100}$. **Ans. 53%**

Individuals Round 6

1. There are 13 switchbacks below Sara's favorite spot and 6 above: 13 + 1 + 6. **Ans. 20**

2. The plane of each triangle makes a tetrahedron with one of the vertices of the cube. There will be 8 such tetrahedrons that can be made, from the 8 vertices. **Ans. 8**

3. $\frac{1}{x} + \frac{1}{100} = \frac{100+x}{100x}$. If n divides 100x, it must divide 100 and x for the fraction to be reducible. This is true if and only if 100 and x are relatively prime. So x cannot be a multiple of 2, 4, or 5. There are 49 even numbers and 10 multiples of 5, which are not also multiples of 2. 99 – 59 = 40. **Ans. 40**

Team Round 1

1. 2,700,000,000 ÷ 3600 sec/hr ÷ 24 hr/day ÷ 365.25 days/year = 85.558. **Ans. 85**

2. $x^7 < 100{,}000{,}000 \rightarrow 7 \log x = 8 \rightarrow \log x = 8/7 \rightarrow x = 13.895$ **Ans. 13**

3. Minimum in base 12 is $100_{12} = 144$. Maximum in base 8 is 777_8 or $1000_8 - 1 = 511$. $511 - 144 = 367$. $367 + 1 = 368$. **Ans. 368**

4. The two roots $4 + \sqrt{11}$ and $4 - \sqrt{11}$ came from the equation $(x-4)^2 = 11$ or $x^2 - 8x + 5 = 0$. Dividing this into $x^4 - 6x^3 + tx^2 + wx - 40 = 0$:

$$
\begin{array}{r}
x^2 + 2x - 8 \\
x^2 - 8x + 5 \;\overline{)\; x^4 - 6x^3 + tx^2 + wx - 40} \\
\underline{x^4 - 8x^3 + 5x^2} \\
2x^3 - (t-5)x^2 + wx \\
\underline{2x^3 - 16x^2 + 10x} \\
(t+11)x^2 + (w-10)x - 40 \\
\underline{-8x^2 + 64x - 40} \\
(t+19)x^2 + (w-74)x + 0
\end{array}
$$

In order for these to be roots, then $t + 19$ must $= 0$. So $t = -19$.

And $w - 74$ must $= 0$. So $w = 74$.

$t + w = 55$.

In the last term of quotient, -8 has to multiply 5 to get -40.

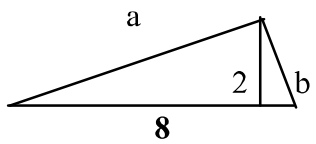
Ans. 55

5. $\dfrac{1}{\log_3 x} + \dfrac{1}{\log_4 x} + \dfrac{1}{\log_5 x} = \dfrac{1}{\frac{\log x}{\log 3}} + \dfrac{1}{\frac{\log x}{\log 4}} + \dfrac{1}{\frac{\log x}{\log 5}} = \dfrac{\log 3}{\log x} + \dfrac{\log 4}{\log x} + \dfrac{\log 5}{\log x} = \log_x 3 + \log_x 4 + \log_x 5 =$

$\log_x 60 = \dfrac{\log 60}{\log x} = \dfrac{1}{\frac{\log x}{\log 60}} = \dfrac{1}{\log_{60} x} = \dfrac{1}{a}$. Thus $a = \log_{60} x$. **Ans. $a = \log_{60} x$**

6. There are two ways to find the sum of the factors of 720. One is to write all the ordered pairs of two numbers that multiply to get 720. As the first number in the ordered pairs increases, the second number in the ordered pairs decreases. Thus (1, 720), (2, 360), (3, 240), (4, 180), (5, 144), (6, 120), (8, 90), (9, 80), (10, 72), (12, 60), (15, 48), (16, 45), (18, 40), (20, 36), (24, 30). Adding these yields 2418. The second way to find the sum is through the prime factors, which is also involved in finding the number of factors. Prime factorizing 720: $3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 3^2 \cdot 2^4 \cdot 5^1$. The sum of the factors of 720 through its factors is: $\dfrac{3^3-1}{3-1} \cdot \dfrac{2^5-1}{2-1} \cdot \dfrac{5^2-1}{5-1} = 13 \cdot 31 \cdot 6 = 2418$. **Ans. 2418**

7. In the figure, the 3 triangles are similar. Thus $\dfrac{a}{8} = \dfrac{2}{b}$ or $ab = 16$. $a^2 + b^2 = 64$. $(a+b)^2 = a^2 + 2ab + b^2 = 64 + 32 = 96$. Thus $a + b = \sqrt{96} = 4\sqrt{6}$. **Ans. $8 + 4\sqrt{6}$**



8. The following points are true: $(4 + 16 + 64 + 8 + 32 + 128 + 512) = 764$. **Ans. 764**

Team Round 2

1. Each triangle has ¼ bh for area = 40. **Ans. 40**

2. Using the Pythagorean Theorem the distance is 130 ft. **Ans. 130 ft**

3. The first two entries are $\binom{2010}{0}$ and $\binom{2010}{1}$. The 3rd is $\binom{2010}{2}$. **Ans. 2, 019, 045**

4. The box beside the 1 has to be a 2 and the box below the 3 has to be a 2. So x cannot be 2. It can be all others 1, 3, 4. The sum is 8. **Ans. 8**

5. The number has to have an odd unit's digit, but cannot be 5. Of 1, 3, 7, 9 all will produce a 1 at the 8th power. 4 out of 10. **Ans. 2/5** or **.4**

6. There can be up to 4 distinct roots according to the Fundamental Theorem of Algebra. Since the coefficients are real, any complex roots must come in pairs. The possibilities are: (4, 0), (3, 0), (2, 0), (1, 0), (2, 2), (1, 2), (0, 2) and (0, 4). The following are possible (r, c) ordered pairs and examples of p(x): (0, 4): x^4+1; (2, 2): $x^4 - 1$; (1, 2): $(x-1)^2(x^2+1)$; (0, 2): $(x^2+1)^2$; (4, 0): (x-1)(x-2)(x-3)(x-4); (3,0): $(x-1)^2(x-2)(x-3)$; (2,0): $(x^2-1)^2$; (1, 0): x^4. **Ans. 8**

7. $85 = 5 \cdot 17$, so multiples of 3-4-5 and 8-15-17 work. Now find all *a,b* with gcf (a, b) = 1 and one of a, b even such that $a^2 + b^2 = 85$. Then $|a^2 - b^2|$, 2ab, $a^2 + b^2$ will be the Fundamental Pythagorean Triples. There are two: (9, 2) and (7, 6). The four triangles are: 51 – 68 – 85, 40 – 75 – 85, 36 – 77 – 85, 13 – 84 – 85. Add shorter legs: 51 + 40 + 36 + 13 = 140. **Ans. 140**

8. Students will find that adding the numbers 0 through 9 and 11 will make $25. So the least possible sum is 56. **Ans. 56**

Blue Relay Seat A

x = 1 makes the absolute value = 0. A = 1. Pass: $1^2 + 2(1) + 1 = 4$. **Ans. A = 1, Pass: 4**

Blue Relay Seat B

40% off is equivalent of a factor of 0.6. 6/0.6 = 10. B = 10 , X = 4. Pass: $\frac{2B}{X} = \frac{2(10)}{4} = 5$.

Ans. B = 10 Pass: 5

Blue Relay Seat C

For rectangle R, width times length equals WL = 35. For rectangle Q, (4/3 W)(kL) = 42. $\frac{4}{3}k = \frac{6}{5}$ ➔ k = 0.9. Thus, the length is 10% less. C = 10, X = 5. Pass: 2CX = 2(10)(5) = 100. **Ans. C = 10 Pass 100**

Blue Relay Seat D

$\frac{25}{2}x^2 - 10x + 2 = 0$ ➔ $25x^2 - 20x + 4 = 0$ ➔ $(5x - 2)^2 = 0$. D = 2/5, X = 100. Pass DX: (2/5)(100) = 40. **Ans. D = 2/5 Pass: 40**

Blue Relay Seat E

356 = -211 + (n – 1)9 ➔ 567 = (n – 1)9 ➔ 63 = n – 1. E = 64, X = 40. Pass: E – X = 24. **Ans. E = 64 Pass: 24**

Green Relay Seat A

x = -3 makes the absolute value = 0. Pass back: $(-3)^2 + 4(-3) + 4 = 1$. **Ans. A = -3, Pass: 1**

Green Relay Seat B

25% off is equivalent of a factor of .75. 6/0.75 = 8. B = 8, X = 1. Pass: $\frac{2B}{X+1} = \frac{2(8)}{(1)+1} = 8$ **Ans. B = 8 Pass: 8**

Green Relay Seat C

For rectangle R, width times Length equals WL = 45. For rectangle Q, (3/2 W)(kL) = 54. $\frac{3}{2}k = \frac{6}{5}$ ➔ k = 0.8. Thus the length is 20% less. C = 20, X = 8. Pass: CX = (20)(8) = 160 **Ans. C = 20 Pass: 160**

Green Relay Seat D

$\frac{9}{2}x^2 - 6x + 2 = 0$ ➔ $9x^2 - 12x + 4 = 0$ ➔ $(3x - 2)^2 = 0$. C = 2/3, X = 160.

Pass: $\frac{X}{D} = \frac{160}{\frac{2}{3}} = 240$. **Ans. D = 2/3 Pass: 240**

Green Relay Seat E

210 = -307 + (n – 1) 11 ➔ 517 = (n – 1)11 ➔ 47 = n – 1. E = 48, X = 240.

Pass: $\frac{X}{E} = \frac{240}{48} = 5$. **Ans. E = 48 Pass: 5**

Pink Relay Seat A

$2 - 7q = 7 - 2q$ ➔ $5q = -5$ ➔ $q = -1$. A = 1. Pass: $(1-)^4 = 1$. **Ans. A = -1 Pass: 1**

Pink Relay Seat B

8(5280)ft/gal, 3(4)(4)(2)(2)(4) tsp/ gal ➔ $\frac{8 \cdot 5280 ft}{3 \cdot 4 \cdot 4 \cdot 2 \cdot 2 \cdot 4 tsp} = \frac{8 \cdot 660}{3 \cdot 4 \cdot 4 \cdot 2} = \frac{220}{5} = 55 ft / tsp$.

B = 55, X =1. Pass: B – X = 54. **Ans. B = 55 Pass 54**

Pink Relay Seat C

Let s = side length. Then $6s^2 = 36$ and $s = \sqrt{6}$. The long interior diagonal = $\sqrt{6} \cdot \sqrt{3} = 3\sqrt{2}$.

C = $3\sqrt{2}$, X = 54. Pass: $\frac{X}{C^2} = \frac{54}{18} = 3$. **Ans. C = $3\sqrt{2}$ Pass: 3**

Pink Relay Seat D

$\frac{x}{40} = \frac{45}{x}$ ➔ $x^2 = 40(45)$ ➔ $x = \pm 30\sqrt{2}$. D = $30\sqrt{2}$, X = 3. Pass: $\frac{D^2}{X^2} = \frac{1800}{9} = 200$.

Ans. $30\sqrt{2}$ Pass: 200

Pink Relay Seat E

First digit is 1 or 2. If 1, the possibilities for the last digit to be odd is 3, 5, 7 (3 possibilities) the rest of the digits is 5! If 2, there are 4 possibilities for the last digit and 5! for the rest of the digits. That makes (3 + 4)5! = 840. E = 840, X = 200. Pass: $\frac{X}{E} = \frac{200}{840} = \frac{5}{21}$

Ans. E = 840 Pass 5/21

Yellow Relay Seat A

$5 - 3q = 15 - 13q$ ➔ $10q = 10$. A = 1, Pass: $A^4 - 1 = 0$. **Ans. 1 Pass: 0**

Yellow Relay Seat B

110 ft/tsp (Look at Yellow Seat A). A = 110, X = 0. Pass: B – X + 1 = 111.

Ans. B = 110 Pass: 111

Yellow Relay Seat C

$6s^2 = 72$. $s = 2\sqrt{3}$. The long interior diagonal = $2\sqrt{3}\ \sqrt{3} = 6$. C = 6, X = 111.

Pass: $\frac{2X}{C} = \frac{2(111)}{6} = 37$. **Ans. C = 6 Pass: 37**

Yellow Relay Seat D

$\frac{x}{36} = \frac{49}{x}$ ➔ $x^2 = 36(49)$ ➔ $x = \pm 42$. D = -42, X = 37. Pass: X + D = -5.

Ans. D = -42 Pass: -5

Yellow Relay Seat E

The first digit is 1 or 2. If 1, the last digit has to be 2, 4, or 6 (3 possibilities) and 5! for the rest of the digits. If 2, the last digit can be 4 or 6 (2 possibilities) and 5! for the rest.

Thus there are (3 + 2)5! = 600 numbers. E = 600, X = -5. Pass: $\frac{E}{|X|} = \frac{600}{5} = 120$

Ans. E = 600 Pass: 120

Answer Sheet – States 2010

Individuals Round 1

1. **6**
2. **13** or **k = 13**
3. **47/91**

Individuals Round 2

1. **101**
2. **24/25**
3. $\sqrt{301}$

Individuals Round 3

1. **550 or P = 550**
2. **1.5 or 3/2 or** $1\frac{1}{2}$
3. **15**

Individuals Round 4

1. **6/73**
2. **-1 or x ≠ -1 or x = -1**
3. $\frac{6bc \pm 2b\sqrt{9c^2+25}}{5}$

Individuals Round 5

1. **66** or **66 hrs**
2. **– 8/27**
3. **53** or **53%**

Individuals Round 6

1. **20**
2. **8**
3. **40**

Team Round 1

1. **85**
2. **13**
3. **368**
4. **55**
5. $\log_{60}x$
6. **2418**
7. $8 + 4\sqrt{6}$ or **17.7980**
8. **764**

Team Round 2

1. **40**
2. **130** or **130 ft**
3. **2,019,045**
4. **8**
5. **2/5** or **0.4**
6. **8**
7. **140**
8. **56**

Blue Relay		Green Relay		Pink Relay		Yellow Relay	
A = 1	Pass: 4	A = -3	Pass: 1	A = -1	Pass: 1	A = 1	Pass: 0
B = 10	Pass: 5	B = 8	Pass: 8	B = 55	Pass: 54	B = 110	Pass: 100
C = 10	Pass: 100	C = 20	Pass: 160	C = $3\sqrt{2}$	Pass: 3	C = 6	Pass: 37
D = 2/5	Pass: 40	D = 2/3	Pass: 240	D = $30\sqrt{2}$	Pass: 200	D = -42	Pass: -5
E = 64	Pass: 24	E = 48	Pass: 5	E = 840	Pass: 5/21	E = 600	Pass: 120