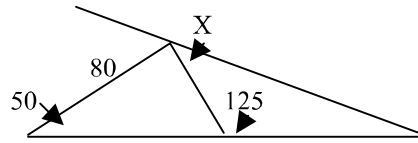


Individuals Round 1 States 2012

3 pts 1. Find the value of x in the figure:
All measurements are in degrees.



Ans. _____

4 pts 2. When the expression $(15^6)(20^5)(55^7)$ is evaluated, it ends with a string of consecutive zero's. How many zero's are in the string?

Ans. _____

5 pts 3. If $t_1 = 1$, $t_2 = -1$ and $t_n = \left(\frac{n-3}{n-1}\right)t_{n-2}$, for $n \geq 3$, find the value of t_{2012} .

Ans. _____

Individuals Round 2 States 2012

3 pts 1. A vender sells large sodas for \$0.70 each and small sodas for \$0.50 each. One afternoon he sold 1000 sodas for a total of \$580. How many large sodas did he sell?

Ans. _____

4 pts 2. On a circle ten points $A_1, A_2, A_3, \dots, A_{10}$ are equally spaced. If C is the center of the circle, what is the measure in degrees of angle A_1A_5C ? The points are ordered numerically clockwise around the circle, according to their subscripts, back to A_1 .

Ans. _____

5 pts 3. If $P = 3^{2000} + 3^{-2000}$ and $Q = 3^{2000} - 3^{-2000}$, what is the value of $P^2 - Q^2$.

Ans. _____

Individuals Round 3 States 2012

3 pts 1. What is the number of solutions (x, y) of the equation $3x + y = 100$, where x and y are positive integers?

Ans. _____

4 pts 2. If $f(x) = x^2 - 3x - 5$, what are the values of k such that $f(k) = k$?

Ans. _____

5 pts 3. The equation $y = x^2 + 2ax + a$ represents a parabola for all real values of a . Each of these parabolas passes through a common point. Determine the coordinates of the point.

Ans. _____

Individuals Round 4 States 2012

3 pts 1. 9 cubes are placed on a table snugly to form a square prism, one cube high. A 10th cube is placed on top of the center cube so that the bottom face of the 10th cube coincides with the top face of the middle cube. What is the total number of faces of the cubes that are exposed?

Ans. _____

4 pts 2. Let a and b be the distinct real numbers for which $\frac{a}{b} + \frac{a+10b}{b+10a} = 2$. Find $\frac{a}{b}$.

Ans. _____

5 pts 3. A die that has the numbers 1, 2, 3, 4, 6, 8 on its faces is rolled. If an odd number appears on the top face, all the odd numbers on the die are replaced with their doubles. If an even number appears on the top face, all even numbers on the die are replaced with their halves. What is the probability that a “2” will appear on the top face of the second roll of the die?

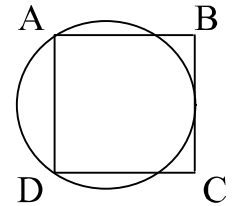
Ans. _____

Individuals Round 5 States 2012

3 pts 1. The triangle with vertices A(6, 1), B(4, 1) and C(4, 4) is rotated 90 degrees clockwise around B. What are the coordinates of the image of C (the point where C is located after the rotation is complete)? Express your answer in ordered pair form.

Ans. _____

4 pts 2. Square ABCD has sides of length 14. A circle is drawn through A and D so that it is tangent to side BC as shown. What is the radius of the circle?



Ans. _____

5 pts 3. Let $f(x) = \sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$. Express $f(x)$ as a single term of the cosine function.

Ans. _____

Individuals Round 6 States 2012

3 pts 1. How many factors of 192 are larger than 10?

Ans. _____

4 pts 2. How many 4-digit numbers of distinct digits can be made from the digits 1, 2, 3, 4, 5, 0?

Ans. _____

5 pts 3. Given that $a + \frac{1}{a} = 3$, what is the value of $\left| a - \frac{1}{a} \right|$?

Ans. _____

Team Round 1 States 2012

4 pts 1. The population of two cities were equal at the end of 2005. The population of city A decreased by 2.9% in 2006, increased by 8.9% in 2007, and increased by 6.9% in 2008. The population in city B increased by r percent in each of the three years. If the populations of the towns were equal at the end of 2008, find the value of r correct to the nearest tenth of a percent.

(1) **Ans.** _____ 4 pts

4 pts 2. Pete and a group of his friends took a bus trip. Each person paid the driver with the same combination of 9 coins and some bills. If the bus driver received \$8.41 in coins from the group, how many nickels did he receive?

(2) **Ans.** _____ 4 pts

6 pts 3. If $(a + b + c + d + e + f + g + h + j)^2$ is expanded and simplified, how many different terms are in the final answer?

(3) **Ans.** _____ 6 pts

6 pts 4. A large square has a smaller square cut from one of its corners in such a way that the area of the remaining section equals the area of the square removed. If x represents the side of the removed square and y represents the length of the side of the large square, find the ratio of x to y .

(4) **Ans.** _____ 6 pts

6 pts 5. Find $i + 2i^2 + 3i^3 + \dots + 2012i^{2012}$ in simplest form, where $i = \sqrt{-1}$.

(5) **Ans.** _____ 6 pts

8 pts 6. If $2x^4 - 3x^3 + ax^2 + 7x + b$ is divisible by $x^2 + x - 2$, find $\frac{a}{b}$.

(6) **Ans.** _____ 8 pts

8 pts 7. The altitudes of a triangle are 12, 15, and 20. What is the measure of the largest angle of this triangle?

(7) **Ans.** _____ 8 pts

8 pts 8. A communications satellite is in an elliptical orbit around Earth. The Earth's center is one focus of the orbit. The eccentricity of the orbit is 0.394, and the satellite is 14,300 kilometers from Earth's center at its closest point in the orbit. What is the satellite's distance from Earth's center at the farthest point in its orbit? Find your answer to the nearest hundred km.

(8) **Ans.** _____ 8 pts

Team Round 2 States 2012

4 pts 1. In a three-digit number, the hundred's digit is greater than 5, the ten's digit is greater than 4, but less than 8, and the unit's digit is prime. How many three-digit numbers satisfy all these conditions?

(1) Ans. _____ 4 pts

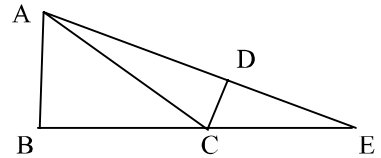
4 pts 2. Nine children, Andy, Betty, Cathy, Deb, Ed, Fred, Guy, Helen, and Igor sit clockwise in a circle as listed. They want to pick a leader by counting clockwise to 5, have that person leave, then have the next person start the count to 5 and the that fifth person leave the circle, and so on until only one is left, the leader. If Andy wants to be the leader, then who should be the first one to start the count?

(2) Ans. _____ 4 pts

6 pts 3. Find the coordinates, both positive, of the vertex of the isosceles triangle having as its base the line segment determined by the points (8, 3) and (-3, 3), if its area is 66.

(3) Ans. _____ 6 pts

6 pts 4. In the diagram $\angle ABC$ and $\angle CDE$ are right angles. $AB = 4$, $BC = 5$ and $CE = 3$. Find the length of \overline{CD} . Give exact answer in simplest form.



(4) Ans. _____ 6 pts

6 pts 5. Simplify: $\frac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} - \frac{1}{2-x}}$.

(5) Ans. _____ 6 pts

8 pts 6. Find the three prime factors of $2^{22} + 1$. (6) Ans. _____ 8 pts

8 pts 7. During a party everyone shook hands with everyone else. A straggler arrives and shakes hands with only those people he knows. Altogether 68 handshakes occurred. How many people at the party did the straggler know?

(7) Ans. _____ 8 pts

8 pts 8. Three rugs have a combined area of 200 square meters. By overlapping the rugs to cover a floor area 140 square meters, the area that is covered by exactly two layers of rug is 24 square meters. What is the area covered by three layers of rug?

(8) Ans. _____ 8 pts

Seat A Blue Relay States 2012

If $x + y = 5$ and $xy = 3$, what is the value of $x^2 + y^2$?

Pass back: $\frac{A-3}{4}$ A = Your answer

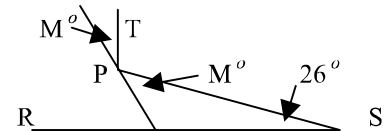
Seat B Blue Relay States 2012

Dan drove from his home to a friend's house at 50 mph. If he had driven 10 mph faster, he would have gotten there 2 hours sooner. How long did it take at the faster speed?

Pass back: $B + \frac{1}{2} X$ B = Your answer. X = The number that you will receive.

Seat C Blue Relay States 2012

A beam of light shines from point S, reflects off a surface at point P, and reaches point T so that \overline{PT} is perpendicular to \overline{RS} . Find the value of M.



Pass back: $\frac{C-X}{2}$ C = Your answer. X = The number you will receive.

Seat D Blue Relay States 2012

The line $y = 3 - x$ intersects the parabola $y = 3x - x^2$ in two points (x_1, y_1) and (x_2, y_2) . Find $y_1 + y_2$.

Pass back: $\frac{X}{2D}$ D = your answer. X = The number you will receive.

Seat E Blue Relay States 2012

Two positive numbers may be inserted between 3 and 9 such that the first three numbers in the sequence form a geometric progression and the last three numbers form an arithmetic progression. Find the sum of these two positive numbers.

Pass in: $2E + X$ E = Your answer. X = The number you will receive.

Seat A Green Relay States 2012

If $x + y = 6$ and $xy = 2$, what is the value of $x^2 + y^2$?

Pass back: $\frac{A}{4}$ A = your answer.

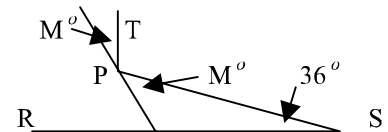
Seat B Green Relay States 2012

Dan drove from his home to a friend's house at 50 mph. If he had driven 10 mph faster, he would have gotten there 2 hours sooner. How long did it take at the slower speed?

Pass back: $B + X + 1$ B = Your answer. X = The number you will receive.

Seat C Green Relay States 2012

A beam of light shines from point S, reflects off a surface at point P, and reaches point T so that \overline{PT} is perpendicular to \overline{RS} . Find the value of M.



Pass back: $C - X$ C = Your answer. X = The number you will receive.

Seat D Green Relay States 2012

The line $y = x - 8$ intersects the parabola $y = 3x - x^2$ in two points (x_1, y_1) and (x_2, y_2) . Find $y_1 + y_2$.

Pass back: $\frac{D}{2} + X$ D = Your answer. X = The number you will receive.

Seat E Green Relay States 2012

Two numbers may be inserted between 2 and 20 such that the first three numbers in the sequence form a geometric progression and the last three terms form an arithmetic progression. Find the sum of these two positive numbers.

Pass back: $E - 2.5X$ E = Your answer. X = The number you will receive.

Seat A Pink Relay States 2012

If $A * B = A^B - B$, find the value of $(2 * 3) * 4$.

Pass back: $\frac{A}{3^3}$ $A =$ Your answer.

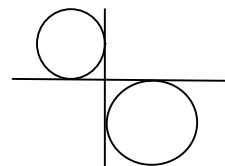
Seat B Pink Relay States 2012

A man invest some money at 6 percent interest and three times as much at 3 percent. If the total income from the two investments is \$315, how much is invested at the higher rate?

Pass back: $X - \frac{B}{100}$ $B =$ Your answer. $X =$ The number you will receive.

Seat C Pink Relay States 2012

In the diagram, the tangents to the two circles intersect at 90° angles. If the radius of the smaller circle is 2 and the radius of the larger is 6, what is the exact distance between the centers of the two circles?



Pass back: $C^2 - X^2$ $C =$ Your answer. $X =$ The number you will receive.

Seat D Pink Relay States 2012

Solve for X: $4^x - 4^{x-1} = 12$.

Pass back: $\frac{X}{4} + D$ $D =$ Your answer. $X =$ The number you will receive.

Seat E Pink Relay States

The first two terms of an arithmetic progression are $\log_2 3$ and $\log_2 9$. If the sixth term is X, what is the numerical value of 2^X ?

Pass back: $30X - E$ $E =$ Your answer. $X =$ The number you will receive.

Seat A Yellow Relay States 2012

If $A * B = A^B - B$, find the value of $(4 * 2) * 2$.

Pass back: $\sqrt{A+2}$ A = Your answer.

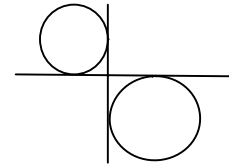
Seat B Yellow Relay States 2012

A man invests some money at 8 percent interest and twice as much at 6 percent. If the total income from the two investments is \$250, how much is invested at the lower rate?

Pass back: $\frac{B}{100} - X$ B = Your answer. X = The number you will receive.

Seat C Yellow Relay States 2012

In the diagram, the tangents to the two circles intersect at 90° angles. If the radius of the smaller circle is 3 and the radius of the larger is 4, what is the exact distance between the centers of the two circles?



Pass back: $X^2 - C^2$ C = Your answer. X = The number you will receive.

Seat D Yellow Relay States 2012

Solve for x, if $3^x - 3^{x-1} = 18$.

Pass back: $\frac{D+X}{2}$ D = Your answer. X = The number you will receive.

Seat E Yellow Relay States 2012

The first two terms of an arithmetic progression are $\log_2 3$ and $\log_2 9$. If the fifth term is X, what is the numerical value of 2^X ?

Pass in: $E - 5X$ E = Your answer. X = The number you will receive.

Solutions – Individuals Round I

1. The angle adjacent to X is 75° , since the exterior angle is 125° . So $x = 25^\circ$ **Ans. 25°**
2. $(15^6) = 3^6(5^6)$ $(20^5) = 2^{10}(5^5)$ $(55^7) = 11^7(5^7)$. Out of these factors $2^{10}(5^{18})$ **Ans.10**
3. Since $t_1 = 1$, $t_2 = -1$, the pattern follows: 1, -1, 0, -1/3, 0, -1/5, ... thus every even term is $-1/(n-1)$. So $t_{2011} = -1/2011$. **Ans. -1/2011**

Individuals – Round 2

1. $.7x + .5(1000-x) = 580 \rightarrow .2x = 80 \rightarrow x = 400$. **Ans. 400**
2. $\angle A_1A_5C$ is an inscribed angle intercepting one of the 36° arcs, **Ans. 18°**
3. $P = \frac{3^{4000} + 1}{3^{2000}}$, $Q = \frac{3^{4000} - 1}{3^{2000}}$. $P^2 = \frac{3^{8000} + 2(3^{4000}) + 1}{3^{4000}}$, $Q^2 = \frac{3^{8000} - 2(3^{4000}) + 1}{3^{4000}}$.
Thus $P^2 - Q^2 = \frac{4(3^{4000})}{3^{4000}} = 4$. **Ans. 4**

Individuals – Round 3

1. The solutions are: (1,97), (2, 94), (3, 91), ... (33, 1). **Ans. 33**
2. $k^2 - 3k - 5 = k \rightarrow k^2 - 4k - 5 = 0 \rightarrow (k-5)(k+1) = 0$. **Ans. 5 or -1**
3. Since a is real then you can plug in almost any value for a and use the two parabolic equations to get result. Using $a = 0$, $y = x^2$, Using $a = 1$, $y = x^2 + 2x + 1$. Thus:
 $x^2 = x^2 + 2x + 1 \rightarrow 0 = 2x + 1$, so $x = -1/2$. Plugging back in: $y = 1/4$. **Ans. (-1/2, 1/4)**

Individuals – Round 4

1. Each corner has 3 open faces (12). Each side center has 2 (8). Top has 5. **Ans. 25**
2. $\frac{a}{b} + \frac{a+10b}{b+10a} = 2 \rightarrow ab + 10a^2 + ab + 10b^2 = 2b^2 + 20ab \rightarrow 10a^2 - 18ab + 8b^2 = 0$ or
 $5a^2 - 9ab + 4b^2 = 0$. Factoring: $(5a - 4b)(a - b) = 0 \rightarrow 5a = 4b \rightarrow a/b = 4/5$. **Ans. 4/5**
3. Favorable outcomes result from (1) odd on first die and 2 on the second or (2) even on the first die and 2 on the second. (1): odd on the first die: (1/3). Result of odd, the second die will be (2,2,6,4,6,8). Prob of 2 on 2nd die = 1/3. Thus 1/3 times 1/3 = 1/9.
(2): even on first die: 2/3. Result of even, the second die will be (1,1,3,2,3,4). Prob. of 2 = 1/6. Thus 2/3 times 1/6 = 1/9. Prob. of (1) or (2) together: $1/9 + 1/9 = 2/9$. **Ans. 2/9**

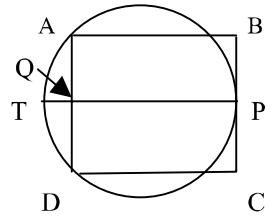
Individuals – Round 5

1. B(4, 1) is the vertex of the right Δ . C is 3 units up. Rotating C 90° clockwise puts it 3 units to the right of B, which would be at (7, 1). **Ans. (7, 1)**

2. Draw a segment from P the point of tangency, to the midpoint Q of side AD and continuing to meet the circle at T. Using $AQ \cdot QD = TQ \cdot PQ$: $7(7) = TQ(14)$ or $TQ = 3.5$.

Thus the diameter of the circle is 17.5 and the radius is 8.75.

Ans. 8.75



$$\begin{aligned}
 3. \quad & \sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x} \rightarrow \sqrt{(1 - \cos^2 x)^2 + 4\cos^2 x} - \sqrt{(1 - \sin^2 x)^2 + 4\sin^2 x} \rightarrow \\
 & \sqrt{1 - 2\cos^2 x + \cos^4 x + 4\cos^2 x} - \sqrt{1 - 2\sin^2 x + \sin^4 x + 4\sin^2 x} \rightarrow \\
 & \sqrt{1 + 2\cos^2 x + \cos^4 x} - \sqrt{1 + 2\sin^2 x + \sin^4 x} \rightarrow \sqrt{(1 + \cos^2 x)^2} - \sqrt{(1 + \sin^2 x)^2} = \\
 & 1 + \cos^2 x - (1 + \sin^2 x) = \cos^2 x - \sin^2 x = \cos 2x. \quad \text{Ans. } \cos 2x
 \end{aligned}$$

Individuals - Round 6

1. $192 = 1 \cdot 192 = 2 \cdot 96 = 3 \cdot 64 = 4 \cdot 48 = 6 \cdot 32 = 8 \cdot 24 = 12 \cdot 16$. 8 are larger than 10. **Ans. 8**

2. If the last digit is 0, then there are $5 \cdot 4 \cdot 3 \cdot 1 = 60$ 4-digit numbers. If the last digit is not 0, and the first digit is not 0, then there are $4 \cdot 4 \cdot 3 \cdot 5 = 240$ digit numbers. **Ans. 300**

3. Squaring $(a + 1/a) = 3$, we get $a^2 + 2 + 1/a^2 = 9$, so $a^2 + 1/a^2 = 7$.

Since $(a - 1/a)^2 = a^2 - 2 + 1/a^2 = [a^2 + 1/a^2] - 2$. Then $(a - 1/a)^2 = 7 - 2 = 5$.

Square rooting both sides and only taking the positive root yields $|a - 1/a| = \sqrt{5}$. **Ans. $\sqrt{5}$**

Team – Round 1

1. $(0.971)(1.089)(1.069) = r^3 \rightarrow r = 1.041697 \rightarrow r = 1.042$ **Ans. 4.2%**

2. $841 = 29^2$, there must have been 29 people paying 29 cents. 29cents made up of 9 coins can only be 5 nickels and 4 pennies. $5(29) = 145$. **Ans. 145**

3. Using patterns: $(a)^2 = 1$ term, $(a + b)^2 = 3$ terms, $(a + b + c)^2 = 6$ terms when multiplied out. Pattern: 1, 3, 6, (2 between 1st 2 terms, 3 between the next two terms) continuing the sequence: 10, 15, 21, 28, 36, 45, for 9 letters of the alphabet. **Ans. 45**

4. The large square has area of y^2 . So the rectangle on the left and the rectangle underneath the square have the same area as the removed square or

$$y(y - x) + (y - x)x = x^2 \text{ or } y^2 - x^2 = x^2 \rightarrow y^2 = 2x^2 \rightarrow \frac{x^2}{y^2} = \frac{1}{2}. \text{ So } \frac{x}{y} = \frac{1}{\sqrt{2}}. \text{ Ans. } 1:\sqrt{2}$$

5. Real parts: $-2 + 4 - 6 + 8 - \dots - 2010 + 2012 = -2 + (4 - 6) + (8 - 10) + \dots + (2008 - 2010) + 2012 = -2(503) + 2012 = 1006$.

Imaginary parts: $(i - 3i) + (5i - 7i) + \dots + (2009i - 2011i) = -2i(503) = -1006i$.

Ans. 1006 - 1006i

$$\begin{array}{r}
 2x^2 - 5x + (a+9) \\
 \hline
 2x^4 - 3x^3 + ax^2 + 7x + b \\
 2x^4 + 2x^3 - 4x^2 \\
 \hline
 -5x^3 + (a+4)x^2 + 7x \\
 -5x^3 - 5x^2 + 10x \\
 \hline
 (a+9)x^2 - 3x + b \\
 (a+9)x^2 + (a+9)x - 2(a+9) \\
 \hline
 (-a-12)x + 2a + 18 + b
 \end{array}$$

So $-a - 12 = 0$ or $a = -12$

and $2a + 18 + b = 0$ or
 $b = -2(-12) - 18 = 6$

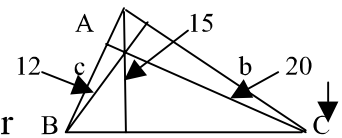
$a/b = -12/6 = -2$

Ans. -2

7. Using the figure, the area of the triangle from the corresponding bases and altitudes if $\frac{1}{2}(12a) = \frac{1}{2}(15b) = \frac{1}{2}(20c)$ and thusly

$12a = 15b = 20c$. Multiplying by $\frac{1}{60}$: $\frac{a}{5} = \frac{b}{4} = \frac{c}{3}$. $a = 5r, b = 4r, c = 3r$

for some constant. Thus $a:b:c = 5:4:3$ making $\triangle ABC$ a right \triangle . Thus its largest angle measures 90° .

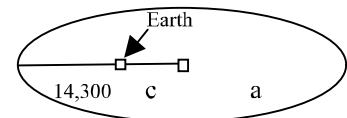


Ans. 90°

8. At right, $\text{ecc.} = \frac{c}{a} = 0.394, a = c + 14,300. \frac{c}{c+14,300} = 0.394$.

Thus $c = .394c + 5634.2$ or $0.606C = 5634.2 \rightarrow c = 9297$.

Distance needed: $c + c + 14,300 = 32894.7$. Rounded: 32,900.



Ans. 32,900

Team - Round 2

1. There are 4 choices for the hundred's digit (6, 7, 8, 9). There are 3 choices for the ten's digit (5, 6, 7). There are 4 choices for the unit's digit (2, 3, 5, 7). $4 \cdot 3 \cdot 4 = 48$. **Ans. 48**

2. Put the Letters on a circle and start anywhere to see who is left. If you start with A, E goes first, A second, G third, D fourth, C fifth, I seventh, B eighth. H is standing. If you shift the start so A is left, Cathy should start the counting. **Ans. Cathy**

3. The midpoint of the base which is horizontal and 11 units long is $(2\frac{1}{2}, 3)$. Since the area is 66, the height has to be 12. Therefore the vertex is $(2\frac{1}{2}, 15)$. **Ans. $(2\frac{1}{2}, 15)$**

4. $\triangle ABC$ is a right \triangle with sides 4, 8 and $4\sqrt{5}$. $\triangle CDE$ is a similar right triangle such that

$$\frac{4}{4\sqrt{5}} = \frac{CD}{3} \rightarrow 4\sqrt{5} CD = 12 \rightarrow CD = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

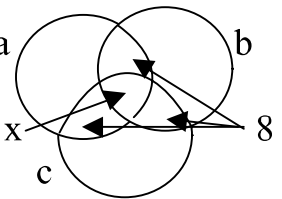
Ans. $\frac{3\sqrt{5}}{5}$

$$5. \frac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} - \frac{1}{2-x}} \rightarrow \frac{\frac{3(x+2)}{(x-2)(x+2)} - \frac{6}{(x-2)(x+2)}}{\frac{3(x-2)}{(x+2)(x-2)} + \frac{1}{(x-2)(x+2)}} \rightarrow \frac{3x+6-6}{3x-6+x+2} = \frac{3x}{4x-4} \quad \text{Ans. } \frac{3x}{4x-4}$$

6. $2^{22} + 1$ is in the form $a^2 + b^2$. $a^2 + b^2$ doesn't factor, but if we insert $2ab$, it does. So $2^{22} + 2(2^{11}) + 1 - 2(2^{11}) = (2^{11} + 1)^2 - 2^{12}$. This now factors into $[(2^{11} + 1) - 2^6][(2^{11} + 1) + 2^6] = (1985)(2113) = (5)(397)(2113)$. **Ans. (5)(397)(2113)**

7. Making 2 handshakes for n people is ${}_n C_2 = \frac{N!}{(N-R)!R!} = \frac{N(N-1)}{2}$. $\frac{n(n-1)}{2} + s = 68$ is result after straggler arrives. If $\frac{11(10)}{2} = 55$ then $s = 13$; $n \neq 13$. If $\frac{12(11)}{2} = 66$, then $s = 2$; That's ok. $\frac{13(12)}{2} = 78$ too big. **Ans. 2**

8. Using a Venn diagram and splitting the 24 sq. m. into 3 equal parts a , b , c yields the picture shown. So (1) $a + b + c + 24 = 140$. (2) $a + 3(16) + 3x + b + c = 200$. Subtracting (1) and (2): $24 + 2x = 60$. $2x = 36$, so $x = 18$. **Ans. 18**

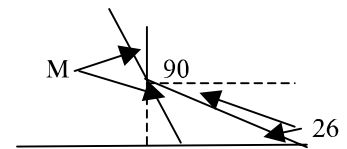


Blue Relay

Seat A: $(x + y)^2 = 25 \rightarrow x^2 + 2xy + y^2 = 25$. Since $xy = 3$, $2xy = 6$ and thus $x^2 + 6 + y^2 = 25 \rightarrow x^2 + y^2 = 19$. Pass: $\frac{A-3}{4} = \frac{19-3}{4} = 4$. **Ans. A = 19, Pass 4**

Seat B: (1) $50T = D$, (2) $60(T - 2) = D$. $50T = 60T - 120 \rightarrow T = 12$ and $T - 2 = 10$. Pass: $B + \frac{1}{2}X = 10 + \frac{1}{2}(4) = 12$. **Ans. B = 10, Pass 12**

Seat C: In the figure: $M + 90 + 26 + M = 180 \rightarrow 116 + 2M = 180$
 $2M = 64$, so $M = 32$. Pass: $\frac{C-X}{2} = \frac{32-12}{2} = 10$



Ans. C = 32, Pass 10

Seat D: $3 - x = 3x - x^2 \rightarrow x^2 - 4x + 3 = 0 \rightarrow (x-3)(x-1) = 0$. $(3, 0)$ and $(1, 2)$ are the two points. Sum of y -coordinates = 2. Pass: $\frac{X}{2D} = \frac{10}{2(2)} = 2.5$. **Ans. D = 2, Pass 2.5**

Seat E: In the geometric sequence $3, 3r, 3r^2, 9$. Arithmetic sequence: $3r^2 - 3r = 9 - 3r^2$, $6r^2 - 3r - 9 = 0 \rightarrow 2r^2 - r - 3 = 0 \rightarrow (2r-3)(r+1) = 0$, so $r = 3/2$. Thus $3r = 4.5$ and $3r^2 = 6.75$. Sum = 11.25. Pass: $2E + X = 2(11.25) + 2.5 = 25$. **Ans. 11.25, Pass 25**

Green Relay

Seat A: Refer to Blue Seat A: $36 - 4 = 32$. Pass: $A/4 = 32/4 = 8$. **Ans. A = 32, Pass 8**

Seat B: Refer to Blue Seat B: $T = 12 = B$. Pass: $B + X + 1 = 12 + 8 + 1 = 21$.
Ans. B = 12, Pass 21

Seat C: Refer to Blue Seat C: $126 + 2M = 180$, $M = 27$. Pass: $C - M = 27 - 21 = 6$
Ans. C = 27, Pass 6

Seat D: $x - 8 = 3x - x^2 \rightarrow x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0$, so $x = 4$ or -2 . Plugging back in yields $(4, -4)$ and $(-2, -10)$. Sum of y-coordinates: -14 . Pass: $\frac{D}{2} + X = \frac{-14}{2} + 6 = -1$
Ans. D = -14, Pass -1

Seat E: Geometric sequence $2, 2r, 2r^2, 20$. Arithmetic sequence: $2r^2 - 2r = 20 - 2r^2 \rightarrow 4r^2 - 2r - 20 = 0$ or $2r^2 - r - 10 = 0 \rightarrow (2r - 5)(r + 2) = 0$. Thus $r = 2.5$. $2(2.5) = 5$, and $2(2.5)^2 = 12.5$. Sum: 17.5 . Pass: $E - 2.5X = 17.5 - 2.5(-1) = 20$. **Ans. E = 17.5, Pass 20**

Pink Relay

Seat A: $2 * 3 = 2^3 - 3 = 5$. $5 * 4 = 5^4 - 4 = 621$. Pass: $\frac{A}{27} = \frac{621}{27} = 23$.
Ans. A = 621, Pass 23

Seat B: $.06a + .03(3a) = 315 \rightarrow 15a = 31500 \rightarrow a = 2100$. Pass: $X - \frac{B}{100} = 23 - \frac{2100}{100} = 2$
Ans. B = 2100, Pass 2

Seat C: You are looking for the diagonal of an 8 by 8 square $= 8\sqrt{2}$. Pass: $C^2 - X = (8\sqrt{2})^2 - 2^2 = 128 - 4 = 124$.
Ans. C = $8\sqrt{2}$, Pass 124

Seat D: $4^x - 4^{x-1} = 12 \rightarrow 4^x - \frac{1}{4}(4^x) = 12 \rightarrow \frac{3}{4}(4^x) = 12 \rightarrow 4^x = 16$, so $X = 2$.
Pass: $\frac{X}{4} + D = \frac{124}{4} + 2 = 33$. **Ans. D = 2, Pass 33**

Seat E: Sequence start: $\log_2 3, \log_2 9, \dots$ Sequence transformed $\log_2 3, 2 \log_2 3, 3 \log_2 3, 4 \log_2 3, 5 \log_2 3, 6 \log_2 3$ to 6 terms. The sixth term is X. Thus $2^X = 2^{6 \log_2 3}$. This is hard to see, but it becomes $3^6 = 729$. Pass: $30X - E = 30(33) - 729 = 990 - 729 = 261$.
Ans. E = 729, Pass 261

Yellow Relay

Seat A: $4 * 2 = 4^2 - 2 = 14$. $14 * 2 = 14^2 - 2 = 196 - 2 = 194$. Pass: $\sqrt{A+2} = \sqrt{194+2} = \sqrt{196} = 14$.
Ans. A = 194, Pass 14

Seat B: $.06a + .08(1/2 a) = 250 \rightarrow 10a = 25000$ so $a = 2500$.

Pass: $\frac{B}{100} - X = \frac{2500}{100} - 14 = 11$

Ans. B = 2500. Pass 11

Seat C: Refer to Pink C. $C = 7\sqrt{2}$. Pass: $X^2 - C^2 = 11^2 - (7\sqrt{2})^2 = 121 - 98 = 23$

Ans. C = $7\sqrt{2}$, Pass 23

Seat D: $3^x - 3^{x-1} = 18 \rightarrow 3^x - \frac{1}{3}(3^x) = 18 \rightarrow \frac{2}{3}(3^x) = 18 \rightarrow 3^x = 27$, so $X = 3$.

Pass: $\frac{D+X}{2} = \frac{3+23}{2} = 13$.

Ans. D = 3, Pass 13

Seat E: Refer to Pink E. $E = 243$. Pass: $E - 5X = 243 - 5(13) = 243 - 65 = 178$.

Ans. E = 243, Pass 178

Answer Sheet – States 2012

Individuals Round 1

1. 25 or 25°
2. 10
3. $-1/2011$

Individuals Round 2

1. 400
2. 18 or 18°
3. 4

Individuals Round 3

1. 33
2. 5 or -1
3. $(-1/2, 1/4)$

Individuals Round 4

1. 25
2. $4/5$
3. $2/9$

Individuals Round 5

1. (7, 1)
2. 8.75 or $8\frac{3}{4}$
3. $\cos 2x$

Individuals Round 6

1. 8
2. 300
3. $\sqrt{5}$

Relays:

Blue	Ans	Pass	Green	Ans	Pass	Pink	Ans	Pass	Yellow	Ans	Pass
A	19	4	A	32	8	A	621	23	A	194	14
B	10	12	B	12	21	B	2100	2	B	2500	11
C	32	10	C	27	6	C	$8\sqrt{2}$	124	C	$7\sqrt{2}$	23
D	2	2.5	D	-14	-1	D	2	33	D	3	13
E	11.25	25	E	17.5	20	E	729	261	E	243	178

Team Round 1

1. 4.2 or 4.2%
2. 145
3. 45 or 45 terms
4. $1:\sqrt{2}$ or $\sqrt{2}:2$
5. $1006 - 1006i$
6. -2
7. 90°
8. 32,900 or 32,900 km

Team Round 2

1. 48
2. Cathy
3. (2.5, 15)
4. $\frac{3\sqrt{5}}{5}$
5. $\frac{3x}{4x-4}$
6. (5)(397)(2113)
7. 2
8. 18 or 18 sq m

