Circles and Circumference:

Key Concept — **Special Segments in a Circle** — For Your FOLDABLE

A **radius** (plural radii) is a segment with endpoints at the center and on the circle.

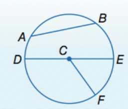
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of ⊙C.

A **chord** is a segment with endpoints on the circle.

Examples $\overline{AB}$ and $\overline{DE}$ are chords of ⊙C.

A **diameter** of a circle is a chord that passes through the center and is made up of collinear radii.

Example $\overline{DE}$ is a diameter of ⊙C. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.



Key Concept — **Radius and Diameter Relationships** — For Your FOLDABLE

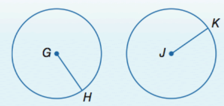
If a circle has radius r and diameter d, the following relationships are true.

Radius Formula $r = \frac{d}{2}$ or $r = \frac{1}{2}d$

Diameter Formula $d = 2r$

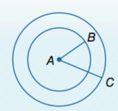
Key Concept — **Circle Pairs** — For Your FOLDABLE

Two circles are **congruent circles** if and only if they have congruent radii.



Example $\overline{GH} \cong \overline{JK}$, so ⊙$G$ ≅ ⊙J.

Concentric circles are coplanar circles that have the same center.



Example ⊙A with radius $\overline{AB}$ and ⊙A with radius $\overline{AC}$ are concentric.

Key Concept — For Your FOLDABLE

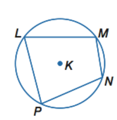
Circumference

Words If a circle has diameter d or radius r, the circumference C equals the diameter times pi or twice the radius times pi.

Symbols $C = \pi d$ or $C = 2\pi r$

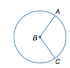
A polygon is **inscribed** in a circle if all of its vertices lie on the circle. A circle is **circumscribed** about a polygon if it contains all the vertices of the polygon.

- Quadrilateral *LMNP* is *inscribed in* ⊙K.
- Circle *K* is *circumscribed about* quadrilateral *LMNP*.



Measuring Angles and Arcs:

Angles and Arcs A **central angle** of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle. $\angle ABC$ is a central angle of $\odot B$.

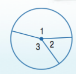


Key Concept — Sum of Central Angles

For Your FOLDABLE

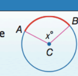
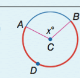
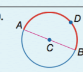
Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$



Key Concept — Arcs and Arc Measure

For Your FOLDABLE

Arc	Measure
A **minor arc** is the shortest arc connecting two endpoints on a circle.	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\widehat{AB} = m\angle ACB = x$ 
A **major arc** is the longest arc connecting two endpoints on a circle.	The measure of a major arc is greater than 180 and equal to 360 minus the measure of the minor arc with the same endpoints. $m\widehat{ADB} = 360 - m\widehat{AB} = 360 - x$ 
A **semicircle** is an arc with endpoints that lie on a diameter.	The measure of a semicircle is 180. $m\widehat{ADB} = 180$ 

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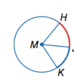
Adjacent arcs are arcs in a circle that have exactly one point in common. In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.

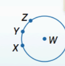


Postulate 10.1 — Arc Addition Postulate

For Your FOLDABLE

Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$



Arc Length **Arc length** is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

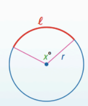
Key Concept — Arc Length

For Your FOLDABLE

Words The ratio of the **length of an arc ℓ** to the **circumference** of the circle is equal to the ratio of the **degree measure of the arc** to 360.

Proportion $\frac{\ell}{2\pi r} = \frac{x}{360}$ or

Equation $\ell = \frac{x}{360} \cdot 2\pi r$



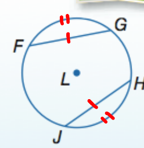
Arcs and Chords:

Theorem 10.2

For Your FOLDABLE

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\overline{FG} \cong \overline{HJ}$ if and only if $\widehat{FG} \cong \widehat{HJ}$.

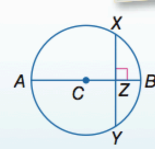


Theorems

For Your FOLDABLE

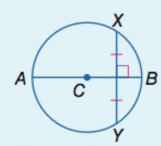
10.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.

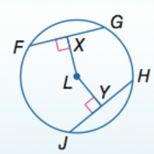


Theorem 10.5

For Your FOLDABLE

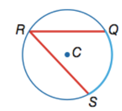
Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example $LX = LY$ if and only if $\overline{FG} \cong \overline{JH}$.



Inscribed Angles:

Inscribed Angles Notice that the angle formed by each streamer appears to be a right angle, no matter where point P is placed along the arch. An **inscribed angle** has a vertex on a circle and sides that contain chords of the circle. In $\odot C$, $\angle QRS$ is an inscribed angle.



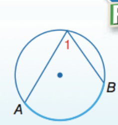
An **intercepted arc** has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In $\odot C$, minor arc $\widehat{QS}$ is intercepted by $\angle QRS$.

Theorem 10.6 Inscribed Angle Theorem

For Your FOLDABLE

Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Example $m\angle 1 = \frac{1}{2}m\widehat{AB}$ and $m\widehat{AB} = 2m\angle 1$

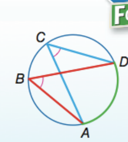


Theorem 10.7

For Your FOLDABLE

Words If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.

Example $\angle B$ and $\angle C$ both intercept $\widehat{AD}$. So, $\angle B \cong \angle C$.



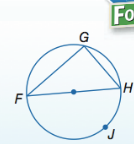
Theorem 10.8

For Your FOLDABLE

Words An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.

Example If $\widehat{FJH}$ is a semicircle, then $m\angle G = 90$.

If $m\angle G = 90$, then $\widehat{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.

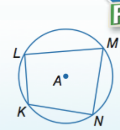


Theorem 10.9

For Your FOLDABLE

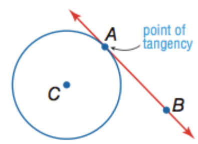
Words If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Example If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.

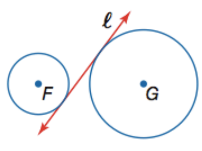


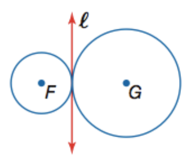
Tangents:

Tangents A **tangent** is a line in the same plane as a circle that intersects the circle in exactly one point, called the **point of tangency**. $\overleftrightarrow{AB}$ is tangent to $\odot C$ at point A. $\overline{AB}$ and $\overrightarrow{AB}$ are also called tangents.



A **common tangent** is a line, ray, or segment that is tangent to two circles in the same plane. In each figure below, line ℓ is a common tangent of circles F and G.



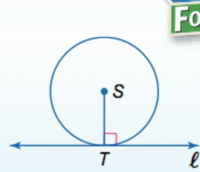


Theorem 10.10

For Your FOLDABLE

Words In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

Example Line ℓ is tangent to $\odot S$ if and only if $\ell \perp \overline{ST}$.

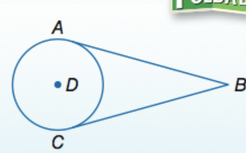


Theorem 10.11

For Your FOLDABLE

Words If two segments from the same exterior point are tangent to a circle, then they are congruent.

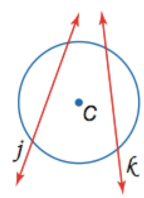
Example If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.



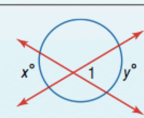
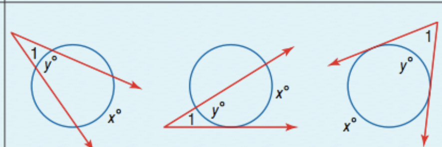
Secants, Tangents, and Angle Measure:

Intersections On or Inside a Circle A **secant** is a line that intersects a circle in exactly two points. Lines *j* and *k* are secants of ⊙*C*.

When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.



Concept Summary — **Circle and Angle Relationships** — For Your FOLDABLE

Vertex of Angle	Model(s)	Angle Measure
on the circle		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

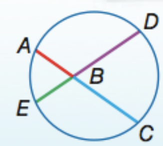
Special Segments in a Circle:

Theorem 10.15 Segments of Chords Theorem

For Your FOLDABLE

Words If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

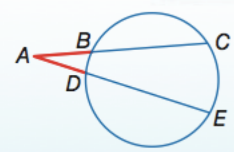
Example $AB \cdot BC = DB \cdot BE$



Theorem 10.16 Secant Segments Theorem

For Your FOLDABLE

Words If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.

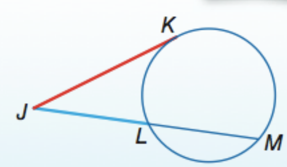


Example $AC \cdot AB = AE \cdot AD$

Theorem 10.17

For Your FOLDABLE

Words If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.



Example $JK^2 = JL \cdot JM$